

**Student-Generated Representations of Special and General Relativity
in an Interdisciplinary College Course**

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Abstract:

The goal of the ongoing study *Symmetry and Aesthetics in Contemporary Physics* is to increase access to physics for all learners by using arts-based teaching strategies, along with analyses of readings and mathematical descriptions, in an interdisciplinary college physics course. In this paper examples of student-generated representations (SGRs) of topics in Special Relativity (SR) and General Relativity (GR) are presented, both in response to specific reading assignments and as final projects in which the students chose to further explore the concepts on their own. It is proposed that: SGRs, along with students' written commentaries, provide feedback to an instructor as to whether students understood a concept as well as the effectiveness of the instruction; the use of SGRs help students develop their meta-representational competence (MRC) throughout a course; and that SGRs and arts-based teaching strategies provide pathways by which non-traditional learners can access abstract concepts in physics, as well as means by which physics students can explore concepts in non-traditional ways.

1. Rationale and theoretical framework

Results of a number of studies (Hazari, et al., 2010; Blickenstaff, 2005, e.g.) suggest that the standard introductory physics curriculum may be a deterrent to studying physics for many students, particularly females. In spite of small gains by women and American minorities in physics-related jobs in the U.S., the covert discrimination in the physics and engineering workplace remains. Thus, two initial questions motivated this study:

1. How can we improve access to physics for all learners by re-inventing the introductory curriculum so as to be interesting, relevant and meaningful in young people's lives?
2. What teaching strategies can we develop that allow students to utilize their full capacities of creativity, imagination, and inquiry so as to make physics appealing and accessible to a more diverse population of learners?

Levrini (1999) suggests that introducing contemporary ideas about space and time into physics education early on can provide a powerful means of drawing diverse learners into the study of physics. Results of the ROSE study in Europe suggest that contemporary questions dealing with spacetime and cosmology are equally attractive for female and male adolescents, and thus should be part of introductory physics courses, even in high school (Sjøberg & Schreiner, 2007; 2010). Osborne (1990) suggests that the lack of appeal that physics has for many students is because traditional physics education fails to address the interesting questions that motivate the practice of physics, namely ontology (the nature of reality) and

epistemology (how knowledge is produced). Thus, if the physics community truly wants to improve diversity among its members, a high priority should be to explore new curricular pathways by which non-traditional, currently under-represented learners can access contemporary physics in meaningful ways.

Symmetry in physics refers to the concept of ‘sameness within change,’ and is the basis of all the laws of physics. Symmetry is the set of rules that allow us to define the invariance of a system under rotations, reflections, and translations. Historically, when physicists have confronted an apparent paradox it has been resolved by finding the symmetry which explains away the paradox by a change of perspective. Thus, the search for deeper symmetries in Nature propels the advancements in contemporary physics. Symmetry is also an important concept in human perception, biology, evolution, neuroscience, and chemistry. Symmetry and asymmetry are central to our aesthetic experiences in the arts, and thus provide a natural foundation for an interdisciplinary physics course that incorporates arts-based teaching strategies. Teaching beginning students about Relativity and curved spacetime brings them face to face with some of the ontological questions that motivate contemporary physics at the largest scales. Moreover, teaching about Relativity in its historic context provides the iconic example of how symmetry has come to play a fundamental role in the development of contemporary physics. It also brings to the fore Professor Emmy Noether, whose theorems have played a seminal, yet little known, role in the development of new physical theories, and thus foregrounds the discrimination that women in physics have faced historically, and continue to face.

Maxine Greene’s model of Aesthetic Education provides the philosophical framework for using arts in the teaching of physics. Greene describes education as a “process of enabling persons to become different” by which “the learner must break with the taken-for-granted, . . . and look through the lenses of various ways of knowing, seeing, and feeling in a conscious endeavor to impose different orders upon experience” (Greene, 2001, p. 5). Although Greene’s original model of Aesthetic Education focused on using arts to teach humanities, the enthusiastic reactions of this author’s students, both via anonymous evaluations and direct communication, suggest that the arts provide a powerful means of bringing to life the objective, abstract world of physics and mathematics.

The practices of Aesthetic Education and arts-based teaching strategies naturally encourage the development of *meta-representational competence* (MRC), which includes having students invent or design creative representations of concepts in science, explain and critique representations, understand how representations function in conveying ideas, and participate in the work of scientists (diSessa, 2004).

The present study investigates two applications of SGR and MRC in teaching physics:

1. How students use creative representations to make sense of and communicate their understanding of concepts which are counter-intuitive to our everyday experiences; and
2. How students’ drawings serve as both assessments of their understanding as well as feedback to the instructor when a reading assignment or other teaching strategy is not bringing students to the desired learning.

The subjects of the students’ representations are the symmetries of Lorentz Invariance (Special Relativity) and General Covariance (General Relativity). These symmetries explain the apparent contradictions to our every-day experience that would be noticed by observers in relative motion at high speeds in the absence of gravity (Special Relativity) and observers in differently-curved regions of spacetime, such as close to a black hole (General Relativity).

2. Methods

Overall, the course emphasizes both math and the arts as interdependent semiotic systems for making sense out of and interacting with the universe. While math provides the language with which to probe the logic of the phenomenological universe, the arts provide languages with which to express the aesthetic side of math and science, and explore the individual's relationship with the cosmos. The progression of arts-based assignments begins with students drawing their visualization of an article about the nature of science by Einstein in order to understand the way in which *they* process information (discussed in van der Veen, 2012), and ends with students creating a final 'physics work of art' that is intended to teach a concept of their choice to the public, and is displayed in one of the campus art galleries. Throughout the course students are exposed to practicing physicists and artists by visiting the art gallery in the Institute for Theoretical Physics on campus and a lab in which 3-D immersive simulations of concepts in science are developed, and having guest lectures by theoretical physicists. In my lecture-presentations I utilize examples of artists who have attempted to represent concepts in Relativity (most notably Picasso and Escher), as well as computer-generated visualizations and simulations.

Each art assignment is motivated by a drawing prompt, and is based on concepts that have been explored in the readings and class discussions. Students share their work in class first in small groups, then with the whole class, and are required to turn in a written explanation of their artistic representations. Member checking is done via dialog between the artist and the audience of other students plus the instructor, and the instructor's feedback on the students' written explanations.

For the final project the students choose a topic that they wish to explore in greater depth, that will be displayed in one of the campus art galleries, and that they will explain to visitors at a public reception. By the time they get to the final project, students have had experience with using art to explore physics, observed physics-based art produced by professionals, and have developed sufficient meta-representational competence to create their physics work of art.

4. Data

Symmetry in physics refers to the concept of 'sameness within change' and is the basis of all the laws of physics. Historically, when physicists have confronted an apparent paradox, it has been resolved by finding the underlying symmetry – the invariant quantity – that remains constant under a change of coordinates. The apparent contradiction to our everyday experience that emerged from Special Relativity is that measurements of lengths and time intervals are not absolute, but depend on the velocity of the observer. Only the speed of light is constant for all observers. Lorentz Invariance explains that any two observers moving relative to each other at a constant velocity will always measure the same physical reality when they observe each other. The symmetry rule that governs these measurements is called the Lorentz Transformation. The radical break with classical (Newtonian) physics is that space and time are not independent, but are really a four-dimensional continuum which Einstein called spacetime. General Covariance is the symmetry of General Relativity that allows observers in accelerating reference frames to interpret the different physical realities they measure as being due to the presence of a gravitational field.

The data presented here include responses to specific assignments dealing with Lorentz Invariance and General Covariance, as well as examples of students' final projects.

4.1. Students' representations of concepts from Special Relativity

The first two examples were drawn in response to reading a short section about the Lorentz Transformation from Einstein's book entitled *Relativity – The Special and the General Theory – A Clear Explanation that Anyone Can Understand*, which he intended for a general audience. Prior to assigning this reading, students had a lecture-presentation on the development of Special Relativity in the historic context of Maxwell's electromagnetic theory.

Example 1: *Will the snake get cut?*

Figure 1 was drawn by a fourth-year physics major (male). His correct summary of the text indicated that he understood the concepts from previous courses. For his visualization, he chose to represent his version of the familiar paradox posed to physics students of whether a 10-meter pole moving close to the speed of light would fit inside a 10-meter-long barn. The caption says:

Snake moving at $.99c$ – In its reference frame, the cutters appear to be closer together, and will easily cut the snake in 3. But, in the cutters' reference frame, the snake seems shorter, short enough not to be cut. Which happens?

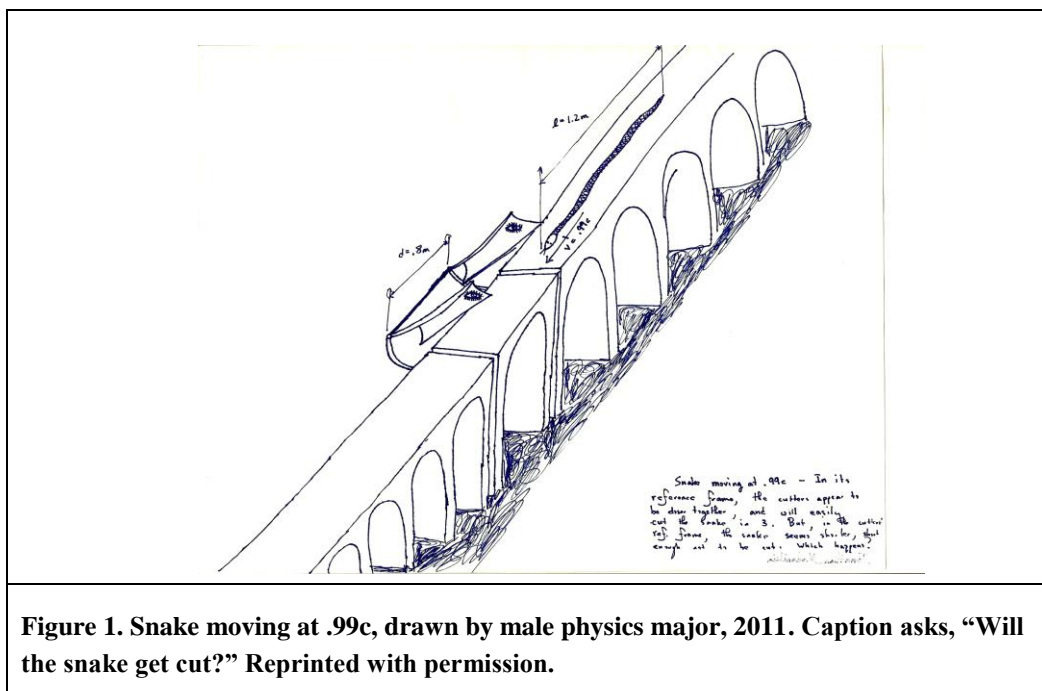


Figure 1. Snake moving at $.99c$, drawn by male physics major, 2011. Caption asks, “Will the snake get cut?” Reprinted with permission.

Drawing scientific ideas: Student-generated representations (SGR) as means of sense-making, communicating ideas, and meta-representational competence. AERA Symposium, 2013.

Example 2: *K and K'*

Figure 2 was drawn by a second year political science major (female), who had not studied any prior physics. Her written explanation indicates that she had some difficulty understanding the reading, while her drawing suggests that she understood the basic ideas but did not understand the . She wrote:

Disclaimer: I had a lot of trouble understanding any of this, but this is what I do understand. K is a reference frame...for example if I was on a train and I threw a ball, to me the ball goes up and down, but from the earth perspective the ball moves horizontally. Einstein looked at a light pulse in both reference frames <...>. X is the distance traveled, c is the speed of light and t is the time. The assumption, that if you look at ct light in one reference frame, time and distance change. The only thing that stays the same is c...I think that's the idea.

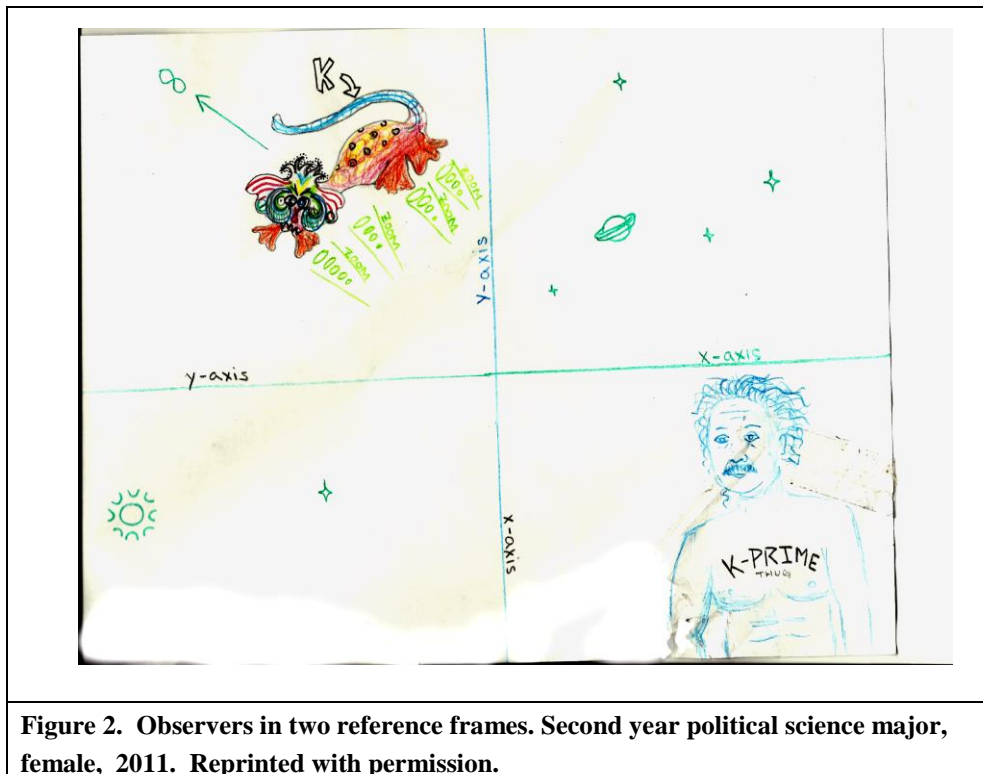


Figure 2. Observers in two reference frames. Second year political science major, female, 2011. Reprinted with permission.

Her statement “if you look at ct light in one reference frame” is her attempt to re-state in her own words Einstein’s mathematical description of the distance traveled by a light pulse as “ $x = ct$ ”, as she correctly interpreted the symbols x , c , and t in her previous sentence.

The students explained their drawings in class, and answered questions posed by other students and the instructor. Einstein’s derivation was presented in detail in class, along with a more intuitive derivation of the Lorentz Transformation using the thought experiment of a light clock on a train moving at a constant velocity close to the speed of light, as observed by a person on the train and one on the platform. The feedback to the instructor was that it would have been more efficient to present the derivation in class

first, rather than have the students ‘discover’ the conclusions on their own through the reading. This feedback was expressed in some of their anonymous exit card comments for that day:

Thank you for going through the derivation with us. It was so much easier with your visuals!

The Einstein reading was too difficult. It was much easier to grasp when you did the equations and I could follow the math in real time. I did understand the concept in class.

I like the light clock explanation of SR a lot better..

Explaining the equations... kinda went over my head. But in the end kinda clicked together. I really thought the star exploding → we see it 2 billion years later thing is awesome.

I am glad we went over his equations in class, because now at least I understand the concept! But I do not think I could figure it out on my own. <...> It reminds me of painting because when things are far away from you they appear close together on the picture plane. Excited for more art!

The reason for presenting these mixed results is to illustrate how SGRs and students’ written explanations and commentaries not only help *them* understand concepts, but also provide important feedback to the *instructor* that can be used to refine instruction – a crucial part of developing any new curriculum. This reading has since been dropped from the syllabus.

The next two examples are final projects, in which students chose to explore the idea of the light cone – one of the central concepts to emerge from Einstein’s Special Relativity. First proposed by Hermann Minkowski, the light cone is a visualization of a cross section in time and one spatial dimension of the propagation of light from a single ‘event.’

Example 3: *What’s so special about Special Relativity?*

In the student’s description, he poses the question: If a person in the dark emits a flash of light, he perceives himself to be at the center of an expanding sphere of light, but another person running away from the first will also perceive himself to be at the center of an expanding sphere of light. How can this be? The explanation is that motion (assumed to be close to the speed of light) can rotate your perception of space and time.



Figure 3. Wire sculpture depicting the intersection of the light cones of two observers in relative motion. 3rd year physics major, male, 2013. Reprinted with permission.

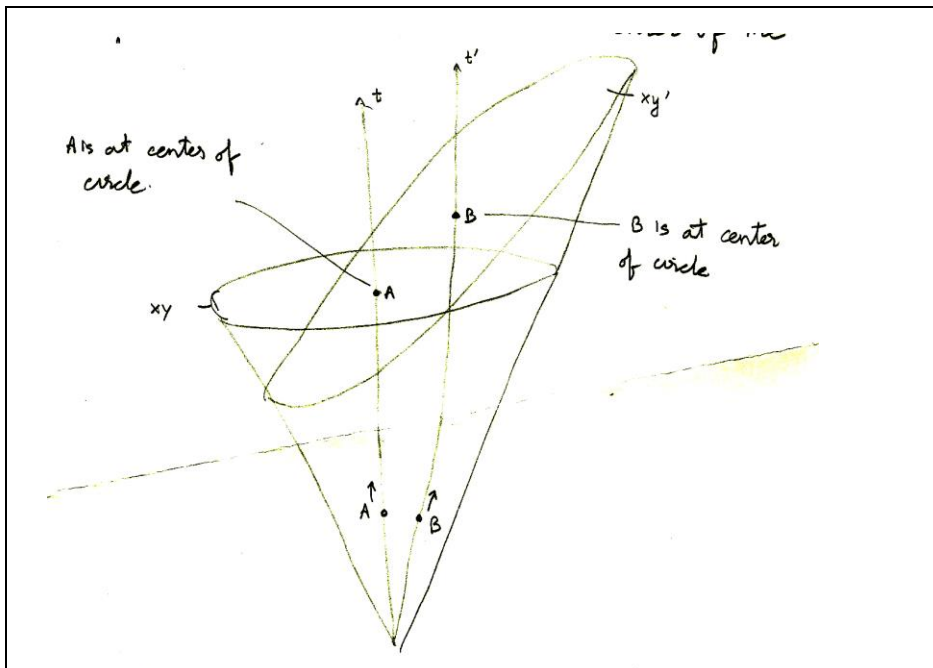


Figure 4. Diagram explaining the sculpture shown in Figure 3. Reprinted with permission.

Example 4: *Light cone in two dimensions*

The second exploration of the light cone was done by a second year computer science major. He writes,

The Principle of Relativity has done away with the notions of Absolute time or Absolute space, replacing them with the light cone, whose one Absolute defining feature is the speed of light. <...> For me, the light cone is a symbol that represents not only how modern physics has changed our perception of space and time, but also causality, existence and the physical limitations of what is possible for us to detect and know.



Figure 5. Light cone. Ink on paper; textures and colors generated by a Python script. Computer science major, second year, male, 2013. Reprinted with permission.

4.2. Students' representations of concepts from General Relativity

The following examples are of students' final projects, in which they chose to explore and represent curved spacetime.

Example 5: *Gravity installation*



Figure 6. Hanging sculpture depicting the curvature of spacetime by mass, by female art major, 3rd year, 2007. Reprinted with permission.

This student, an art major, created an installation depicting the way mass bends spacetime. She wrote a long explanation of her creative process, which she described as her way of resolving her inner struggle to accept the physics point of view, as well as her triumph of finally understanding the notion of curved space. She writes,

My own feelings about the subjects became tested and I started to mistrust the findings of Science based on mathematical equations after reading Feynman's explanation of gravities effect on space-time. I started to think that math was a convenient language used to invent descriptions of our universe and reality. I still have not made any conclusion of the subject. <...> The piece I created for class was as much influenced by ideas and concepts that I had been struggling with in class, as it was an expression of something existing inside myself needing expression.

After describing the symbolism of the materials (knitted squares to represent the fabric of spacetime, black beans to create the warping of spacetime by mass), she writes,

The creation of the final project was important for me to be able to digest many of the concepts that we discussed in class. It gave me the ability develop an understanding and a personal reference to concepts that I found challenging. This was an invaluable part of my study of physics and I greatly appreciated the opportunity to learn in an integrated manner.

Example 6: *Illustration of the principle of General Covariance*



Figure 7. Booklet of transparencies illustrating the principle of General Covariance by showing the way an outside observer might interpret the relative distortion of an image in regions of spacetime with different metrics by understanding the rule that maps one image into the next. Female art major, first year, 2013. Reprinted with permission.

This student, also an art major, chose to explore the concept of general covariance. She writes,

This piece was inspired by the concept of general covariance. Nine drawings were drawn on nine unique grids. The image of the man and woman is distorted and layered one on top of the other. <...> Each drawing is warped, as spacetime will do, yet still preserves a system and basic foundation.

Example 7: *Curved spacetime from different points of view*

This piece, created by a computer science major, is also an exploration of curved spacetime and the different physical realities seen by different observers. He writes,

This piece...from afar looks like an ordinary picture, but as one zooms in on it many details and textures not previously seen come into focus. The portrait itself is a depiction of curved spacetime in the universe and examines some of the clustering effects of space that gravity causes

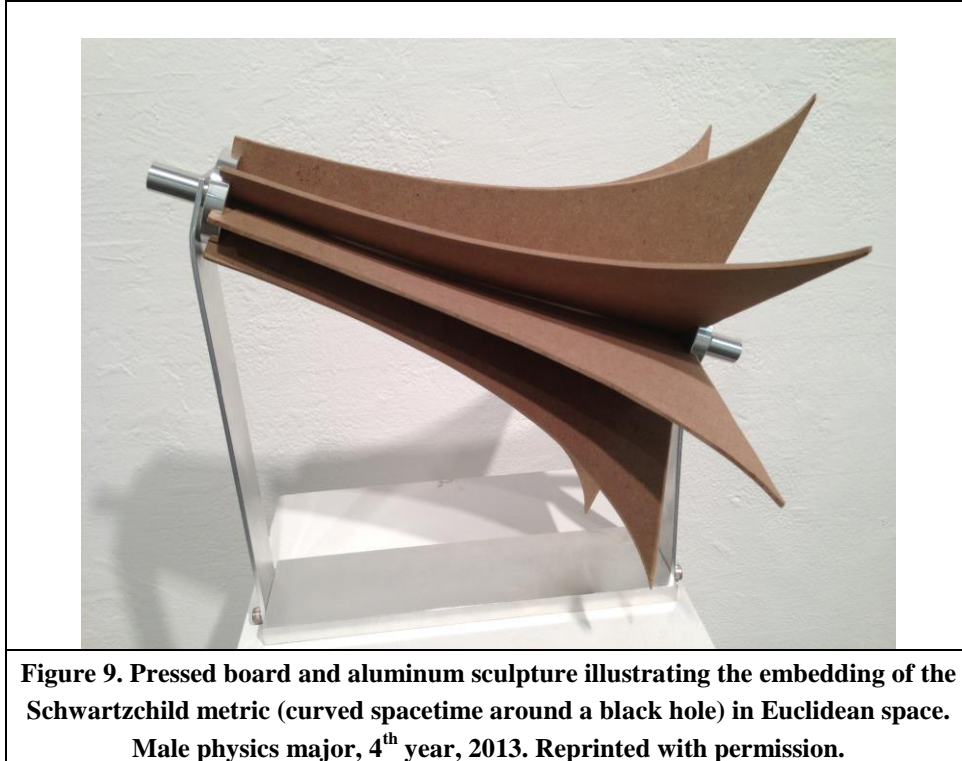


Figure 8. Image representing spacetime distorted by the presence of mass printed on glossy paper, cut into squares and attached to styrofoam blocks of different heights, so that when viewed from different angles and distances, a different perspective is attained. Male computer science major, 2nd year, 2013. Reprinted with permission.

Example 8: *Minkowski metric embedded in Euclidean space*

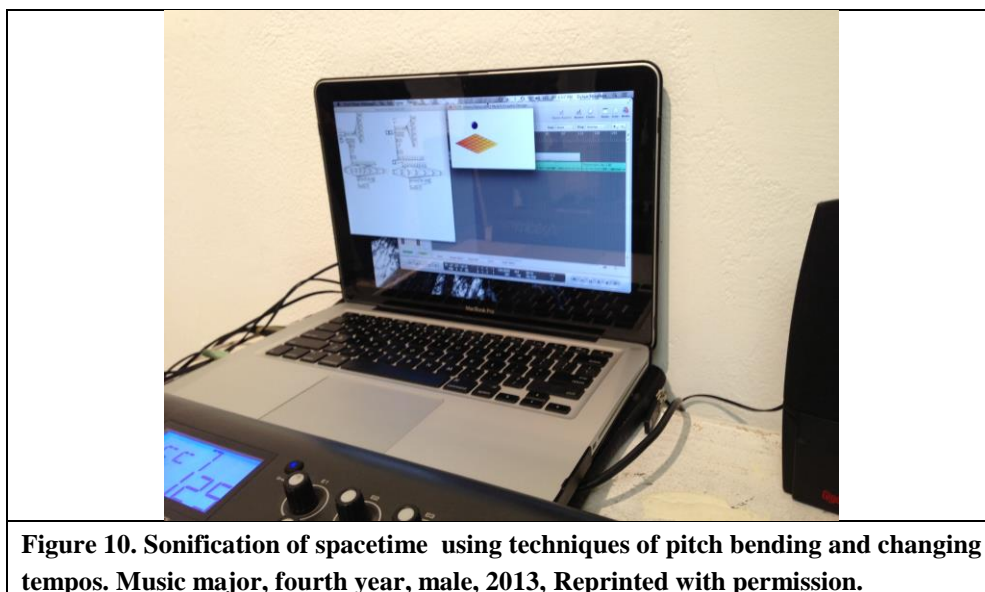
This sculpture was created by a fourth year physics major in the machine shop, using particle board and scrap aluminum that he found. He writes,

For my art piece I wanted to explore a way to represent curved spacetime. I was looking for inspiration and found a really cool paper about how to embed curved spacetime metrics into Euclidean space. The strategy that was used was to equate the geodesics of the metric to the geodesics of Euclidean space, which are just straight lines. After some mathematical manipulation equations can be solved to produce a curve that is rotated 360 degrees to create a 2 dimensional surface. <...> I chose to use the equations to embed the Schwarzschild metric into Euclidean space because it is a very important metric that describes spacetime around a spherical mass, such as stars, planets, and black holes.



Example 9: *Musical representation of spacetime continuum*

A number of students over the six years that the course has been offered have chosen to create musical renditions, with synthesizers or acoustic instruments, for their physics works of art. For completeness, I have included one example (by way of a photo of the equipment used), along with the student's explanation.



He writes,

I wish to explore how to represent curved space-time in sonic form. The slowing of time and space is a concept still new to me, and I hope by exploring through my favored medium, sound, I can better understand it and help others understand it. The idea of sinking into a black hole and having time and space distort itself into an unfamiliar reality is intriguing. I want to illustrate how a single sequence can be modified in terms of tempo, velocity, pitch, and symmetry to represent curved spacetime. My goal is to have people recognize the connection between my music and the concept of spacetime. This may provide a different method of understanding.

5. Discussion

This course began as an experiment to develop an alternative to the standard introductory curriculum that would make physics more appealing and accessible to a broad spectrum of learners, including students who might be curious about physics, but avoid the traditional large introductory classes. Over the six years I have taught the class, I have continued to refine the curriculum and improve my own understanding of how to use what I have called *arts-based teaching strategies* to help students visualize abstract concepts, as well as use students' creative representations to assess their understanding of the concepts. The course is an elective, offered through the Interdisciplinary Studies program of the College of Creative Studies (CCS) at the University of California, Santa Barbara. CCS is the smallest of the colleges in the university, in which class sizes are capped at twelve to fifteen students, and interdisciplinary, experimental approaches are encouraged. The course is open to all students, but most who elect to take it are enrolled in CCS. My enrollment has ranged from a minimum of eight to a maximum of twelve, so that after six years I have accumulated a sample size of $N = 65$.

Overall, students' reactions to the curriculum and methodology have been quite positive in all years, as indicated by their final evaluations. In 2007, I administered the Maryland Physics Expectation Survey (MPEX) (Redish, Saul, & Steinberg, 1998). The students in my class demonstrated significant gains in attitudes toward physics as compared with students in the original survey of 1500 undergraduates, whose attitudes towards physics declined after a one-year introductory course (van der Veen, 2007).

The term *meta-representational competence* coined by diSessa (2004) is described succinctly by Heggarty (2011) as “the ability to choose the optimal external representation for a task, use novel external representations productively, and invent new representations as necessary” (p. 1240). The data presented in this study are all examples of novel representations developed by students. The progression of the assignments, from those that were done in response to specific drawing prompts, to the final projects in which students chose the topic as well as the medium in which to work, suggests that through the course students gained in their meta-representational competence as well as their understanding of the concepts. Heggarty recommends that “more attention should be paid to teaching people to use, design, and critique external spatial representations, in addition to training their internal visualization abilities” (p. 1241), a recommendation which I support. DiSessa (2004) suggests that “learning may implicate developing one's own personally effective representations for dealing with a conceptual domain” (p. 299) – a statement which is definitely supported by the outcomes of the course in the form of the students' final ‘physics works of art’ as well as their comments and continued high evaluations of the course.

6. Results and Significance

The results of the ongoing study of this course suggest:

- 1) Using SGRs in undergraduate physics education promotes students' conceptual understanding of abstract concepts and serve as feedback to the instructor as to the effectiveness of the teaching; and
- 2) An introductory curriculum which incorporates arts-based teaching strategies and reflects the motivating principles of physics today has the potential to make physics accessible to a diverse population of learners. This outcome was predicted by diSessa (2004):

The fact that MRC differs in “feel” compared to much of science as conventionally taught may be particularly important in engaging students from segments of the population who have been systematically underrepresented in scientific careers (diSessa, 2004, p. 300).

While I cannot claim to have ‘converted’ college students to become physics majors, students’ comments strongly suggest that the use of arts-based teaching strategies in an interdisciplinary setting, in the context of a curriculum that begins with contemporary ideas in physics (instead of Newtonian mechanics) have the potential to promote access to physics for students who may never have taken a more traditional physics course in their entire education.

Overall the course initiated a strong interest in physics – before I had always thought it was beyond me, but now I can’t get enough of it for art inspiration, etc. (male art major, 2007).

I learned so much from this course, and I have never taken physics before. .. Jatila definitely encouraged thoughtful original work, urging us to think about our own experiences when we tried to interpret physics. (anonymous evaluation, 2007).

I found this course to be wonderfully exciting. I had to overcome and grapple with a lot of struggles with math and understanding the language of science. I wish that this class would continue and I could continue to study the maths and science in such an integrated way (female art major, 2007).

I don’t think I’ve ever been quite so stimulated by a class here as much as your class has done for me. I thought the material was awesome and the readings were great although difficult at times. Thank you for opening up my mind more! (anonymous student comment, 2011).

Thank you for one of the craziest (cool) classes I’ve taken my whole time being here. Not many classes care about what you think but in your class I felt like my understanding was the whole point (female political science major, 2011; artist who drew Figure 2).

It is really refreshing to see an unconventional approach to physics and view the world around us from both points of view. My mind was opened up to so much this quarter. <...> These ideas really apply to art but no where does art teach it or explore the questions that one might have. I was really glad to make my final project, it helped me develop my view of the interconnectedness of things (female art major, 2011).

I’m really glad that I got to take this class & that classes like this exist. I feel like I learned a lot & that I will retain it because I enjoyed learning it, and I think it is useful & really interesting information. It was awesome to learn about something so unlike what I normally study, and from so many different perspectives (anonymous student, 2012).

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As a physics major in my senior year, I've felt that our curriculum, though very demanding and informative, lacks the history of how the laws came to be, as well as the thought process that was taken to get to that point. Your class seems to be one that will fill those holes, and I am truly excited to learn more about it (anonymous 'exit card comment,' 2013).

In conclusion, I suggest that

- Having students design their own representations of abstract concepts in physics, and explain their representations to non-experts, helps them develop meta-representational competence;
- Teaching physics in an interdisciplinary setting in which high importance is placed on students designing their own representations improves self-confidence regarding the study of physics for arts and humanities students who might otherwise avoid a traditional introductory physics course;
- SGRs, along with students' written and verbal explanations, can serve as an alternative form of assessment to traditional tests and problem sets, that give deeper insight into students' understanding of concepts as well as the way students process information;
- SGRs, along with students' written explanations provide valuable feedback to the instructor as to the effectiveness of his/her instruction, which the instructor can use in refining the course and assignments;
- The use of arts-based teaching strategies and open-ended assignments that encourage students' creativity has the potential to increase access to physics, and thus attract a broader population of learners to study physics.

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