Symmetry and Aesthetics in Contemporary Physics

CS-10, Winter, 2015

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CLASS 2: EINSTEIN & REALITY
INTRO TO SYMMETRY
AS A DYNAMIC CONCEPT

Agenda:

1. **Discuss Einstein's article *Physics & Reality***
2. **Share and discuss your drawings of Einstein's article**
3. **Presentation: Drawing as a window into how we think**
4. ***~* Short Break *~***
5. **Introduction to Symmetry, Group Theory, and examples of symmetries in music, dance, and the universe**

re-visit Max Tegmark:

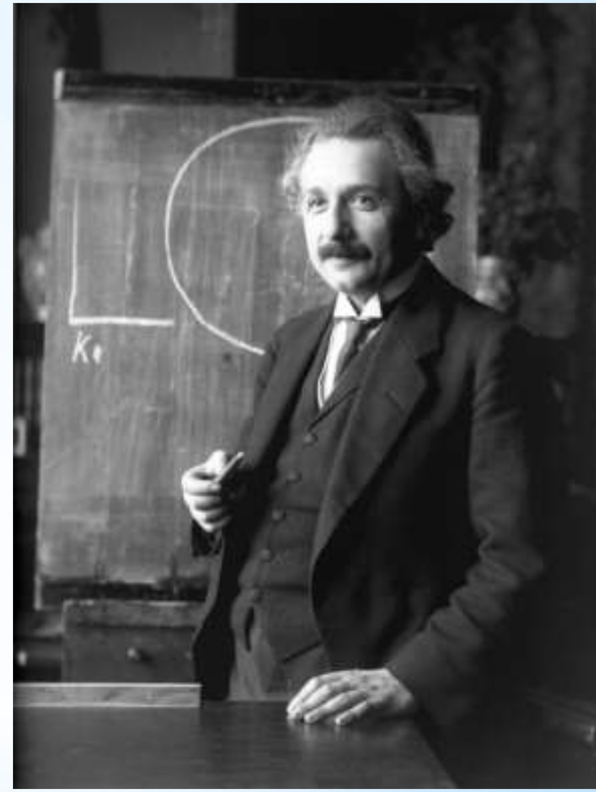
Discussion of *Physics & Reality* by Albert Einstein

1. **How does Einstein define *reality*?**
2. **What is his definition (description) of the scientific system?**
3. **How does this relate to what you have learned about the scientific method, attributed to Newton?**
4. **Give some examples which illustrate his description of the scientific system.**

Do you agree with Einstein? Do you feel that his assessment of the Nature of Science is relevant today, a century later?

Drawing as a window into how we think

The whole of science is nothing more than a refinement of everyday thinking. It is for this reason that the critical thinking of the physicist cannot possibly be restricted to the examination of the concepts of his own specific field. He cannot proceed without considering critically a much more difficult problem, the problem of analyzing the nature of everyday thinking.

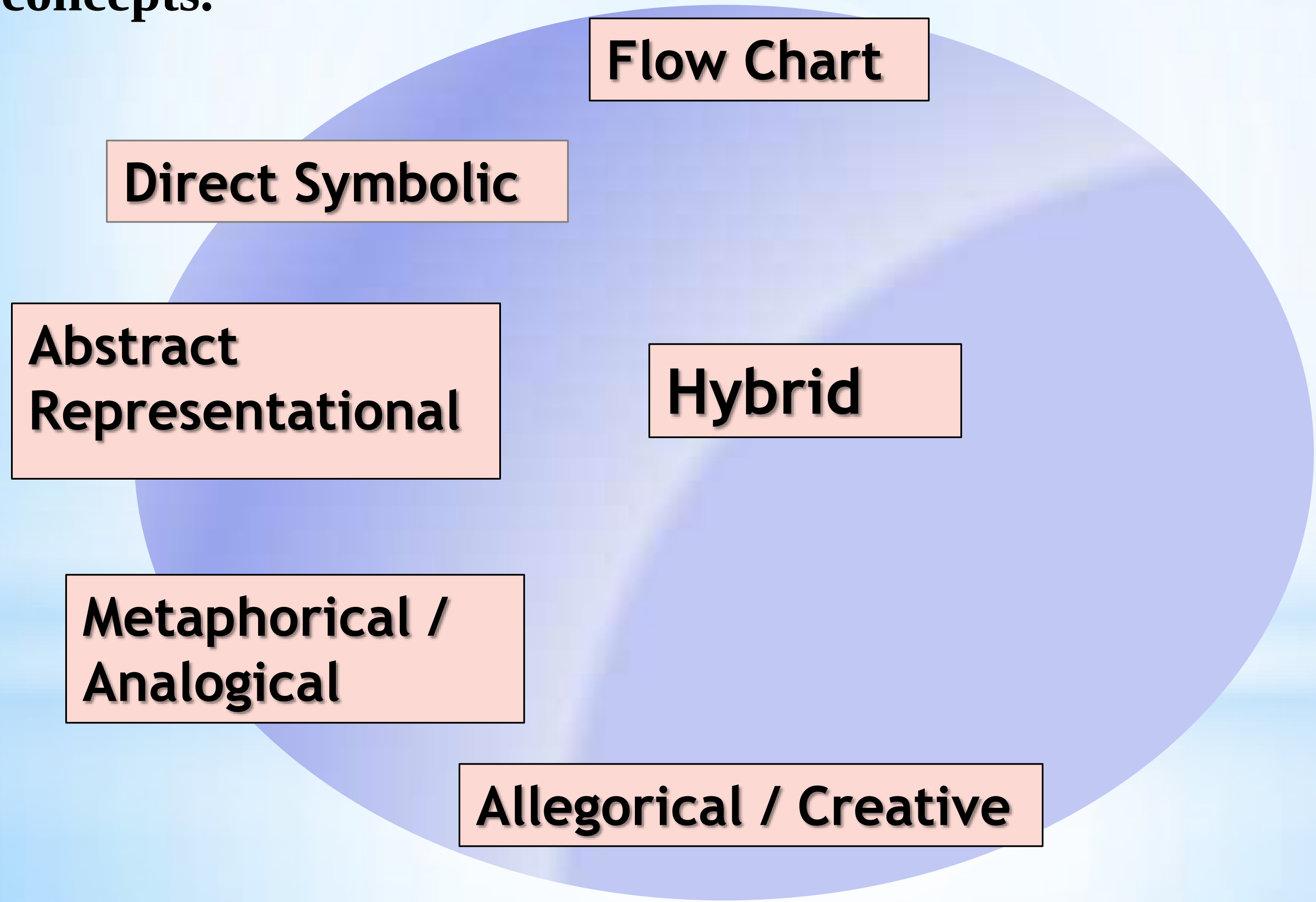


How did you visualize the concepts Einstein describes?

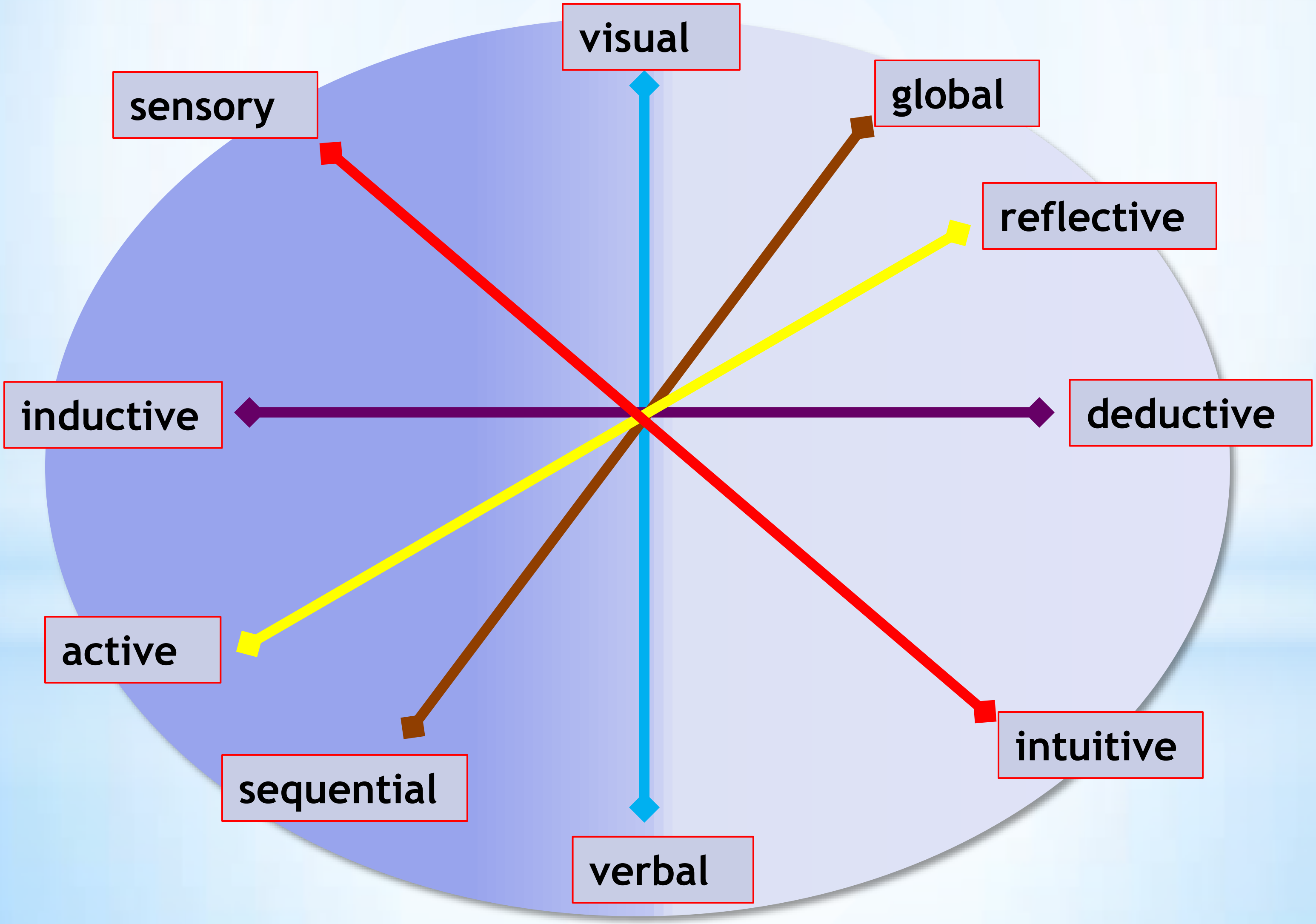
Share as a class:

Explain the symbols in your drawing, and what aspect of the article you are depicting.

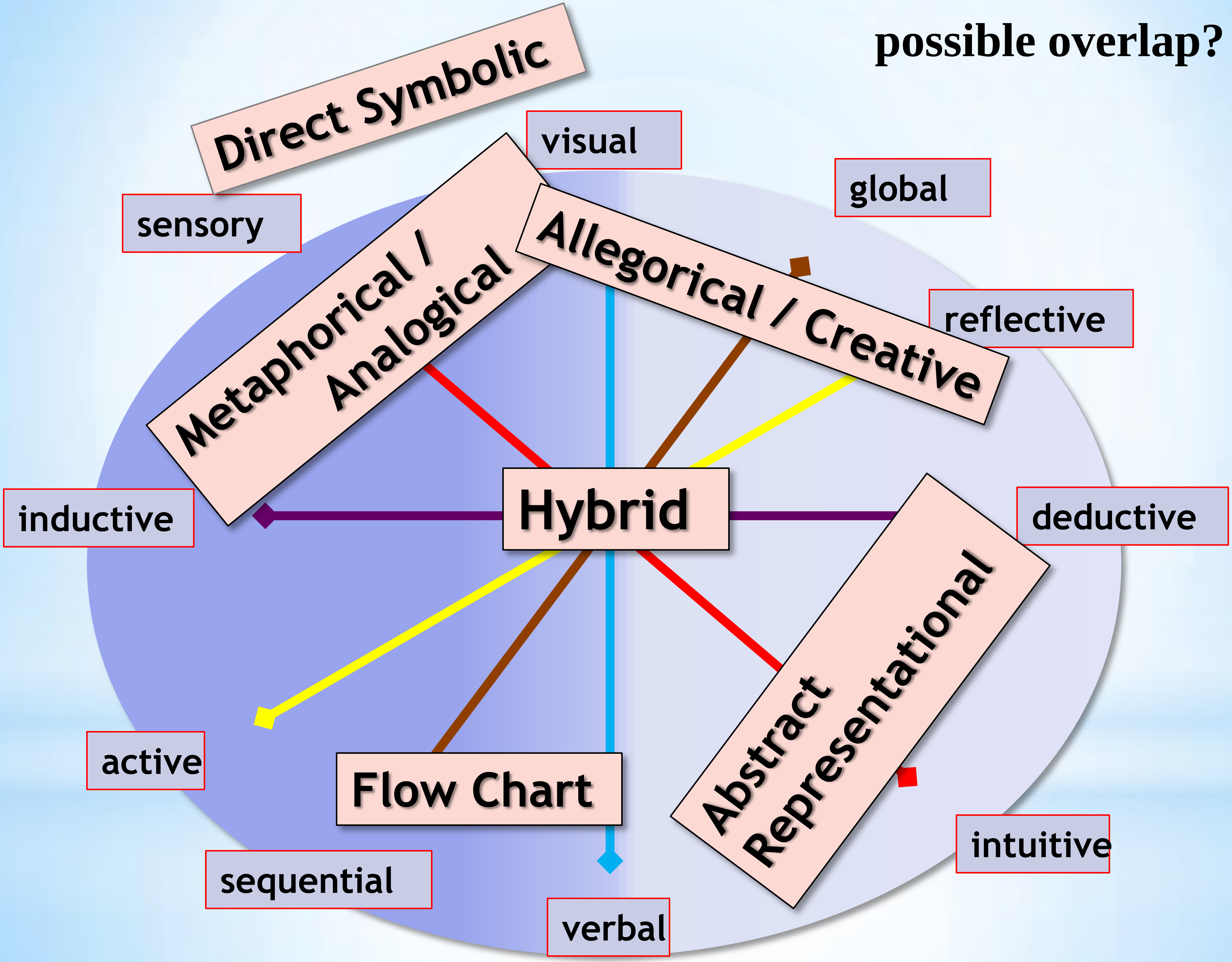
Sharing my research: I find that people’s drawings tend to fall into general categories which indicate how they visualize concepts.



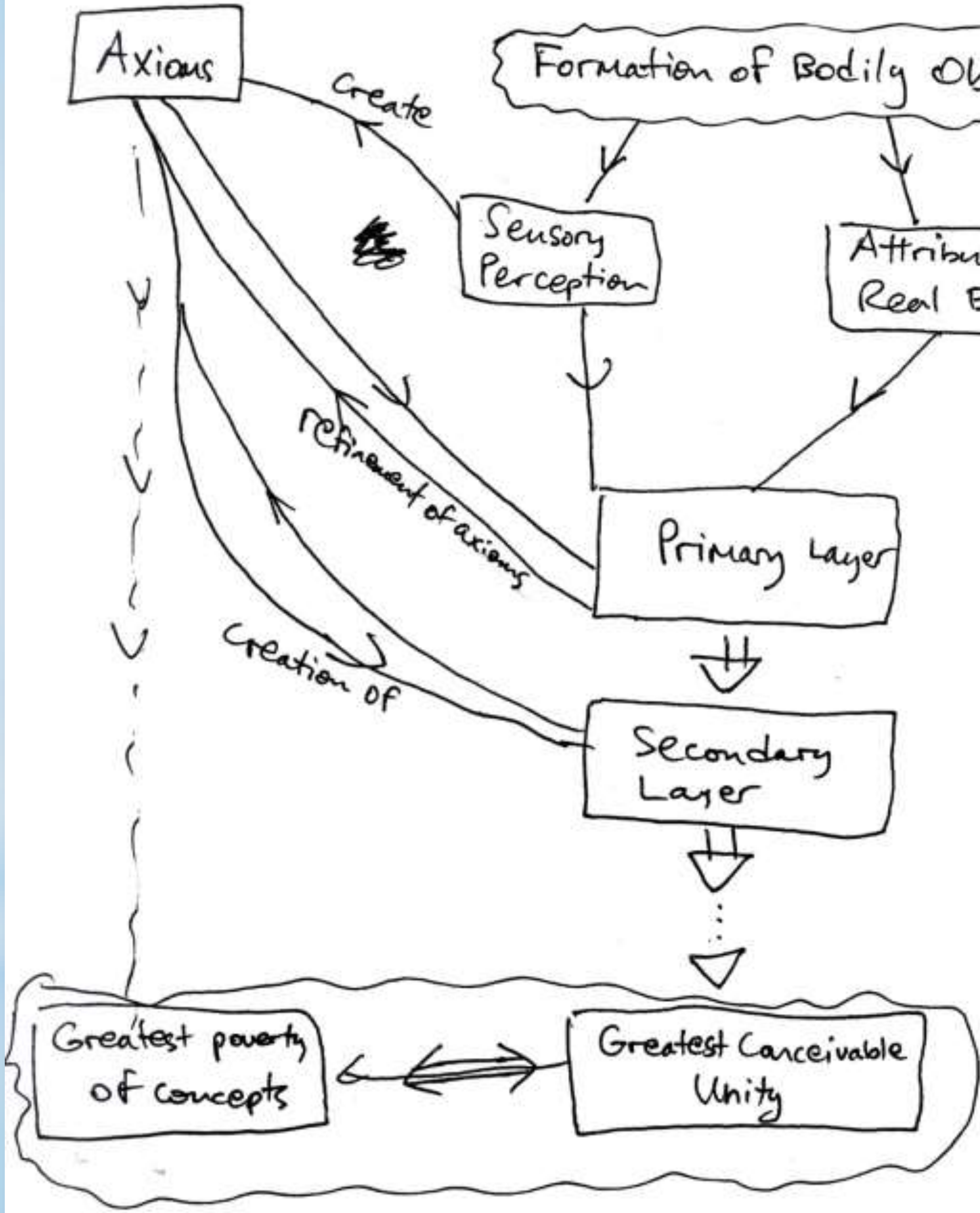
Types of learners from a study of engineering students



possible overlap?



Examples of students' drawings from previous years



It is fairly easy to draw this as there is something fundamentally simple going on: Using logic to refine logic

Science according to Einstein

Layer 1

 +  = everyday science

data — laws

Layer 2

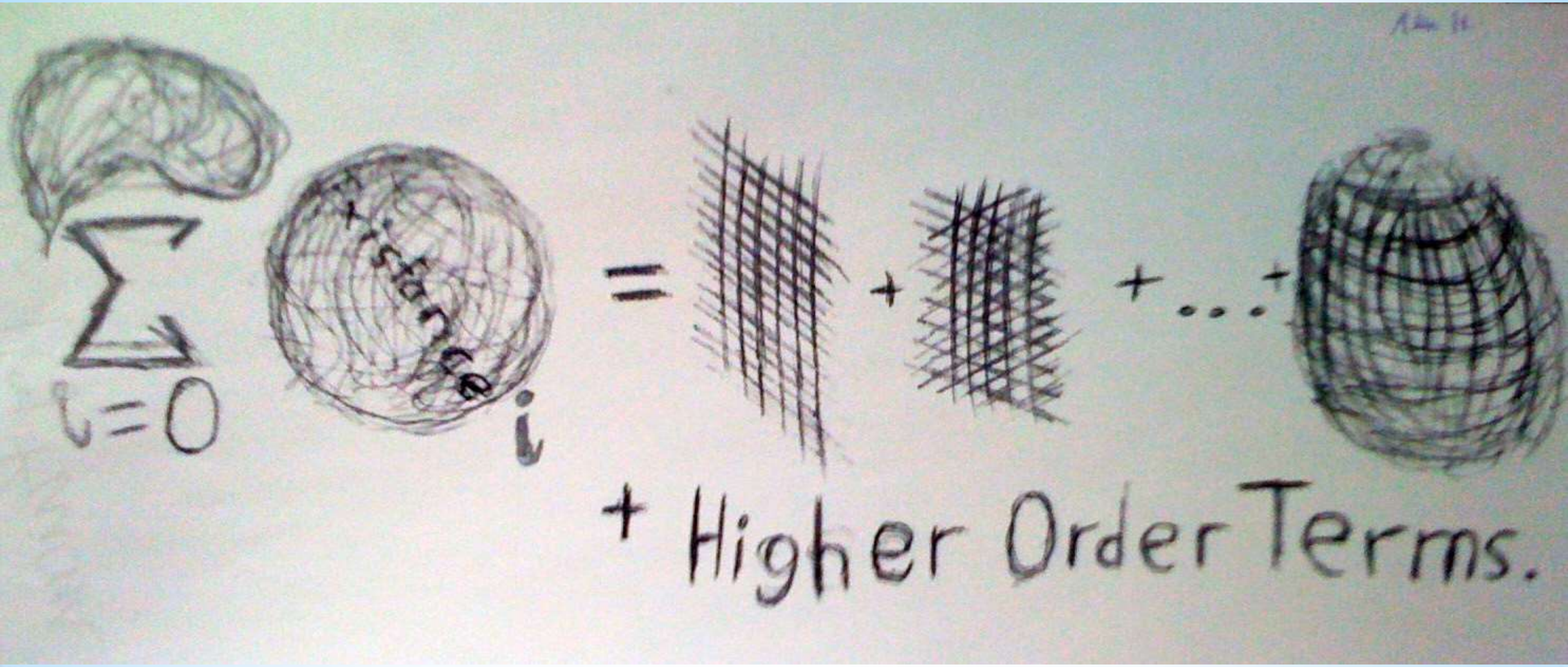
 +  +  = theory

data — laws — thinking

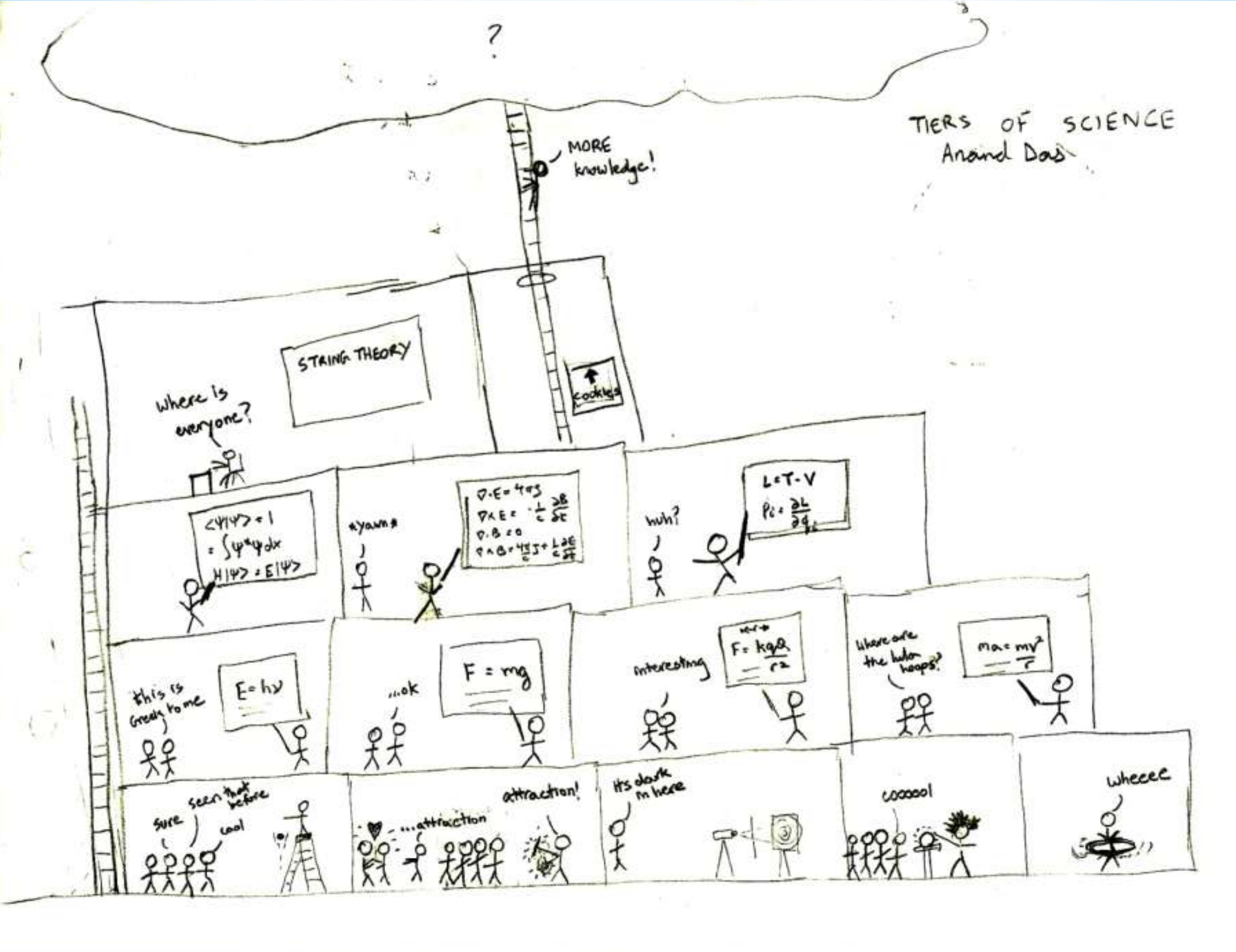
Layer 3

theory +  = projection

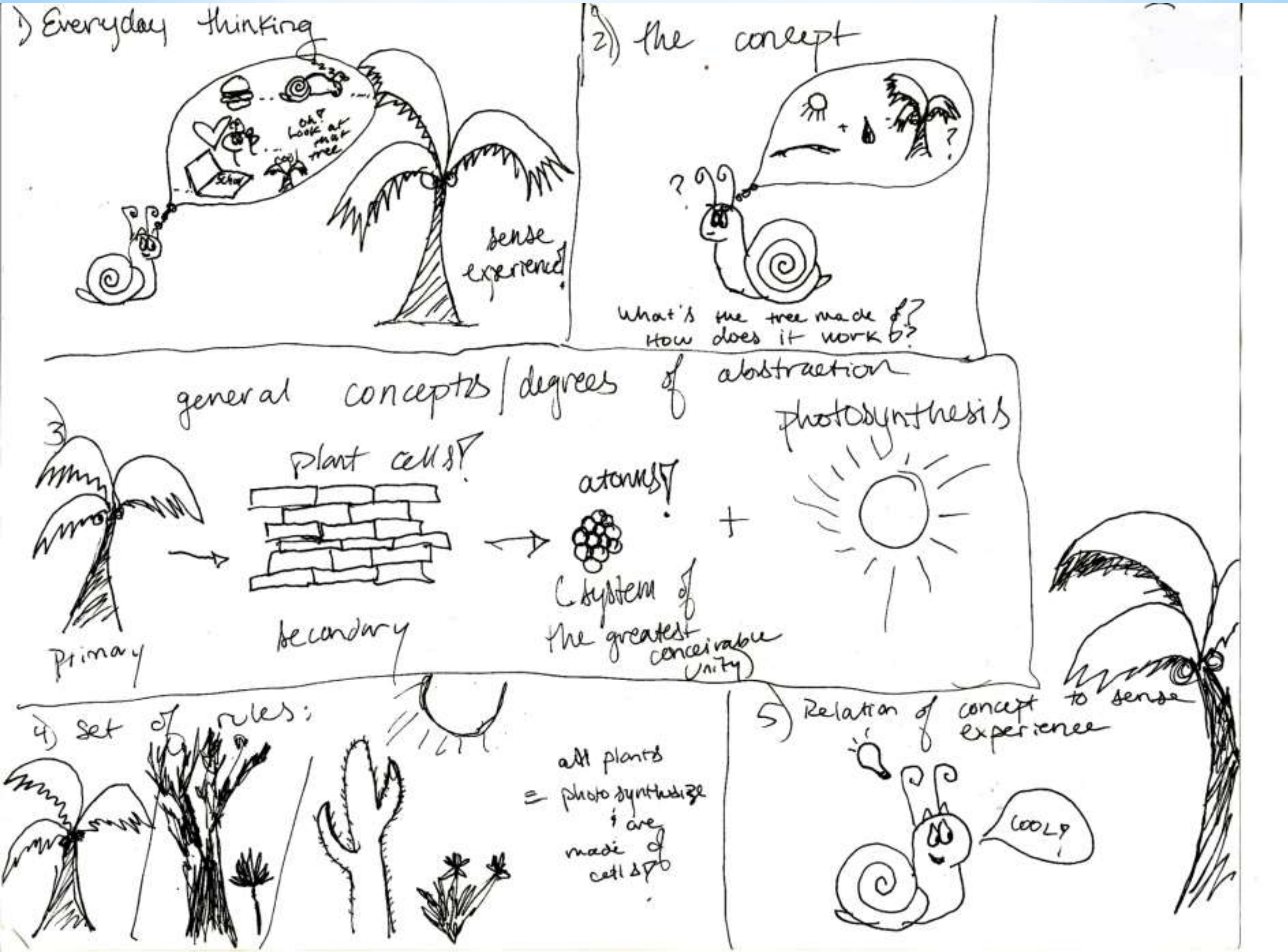
data that is imaginary

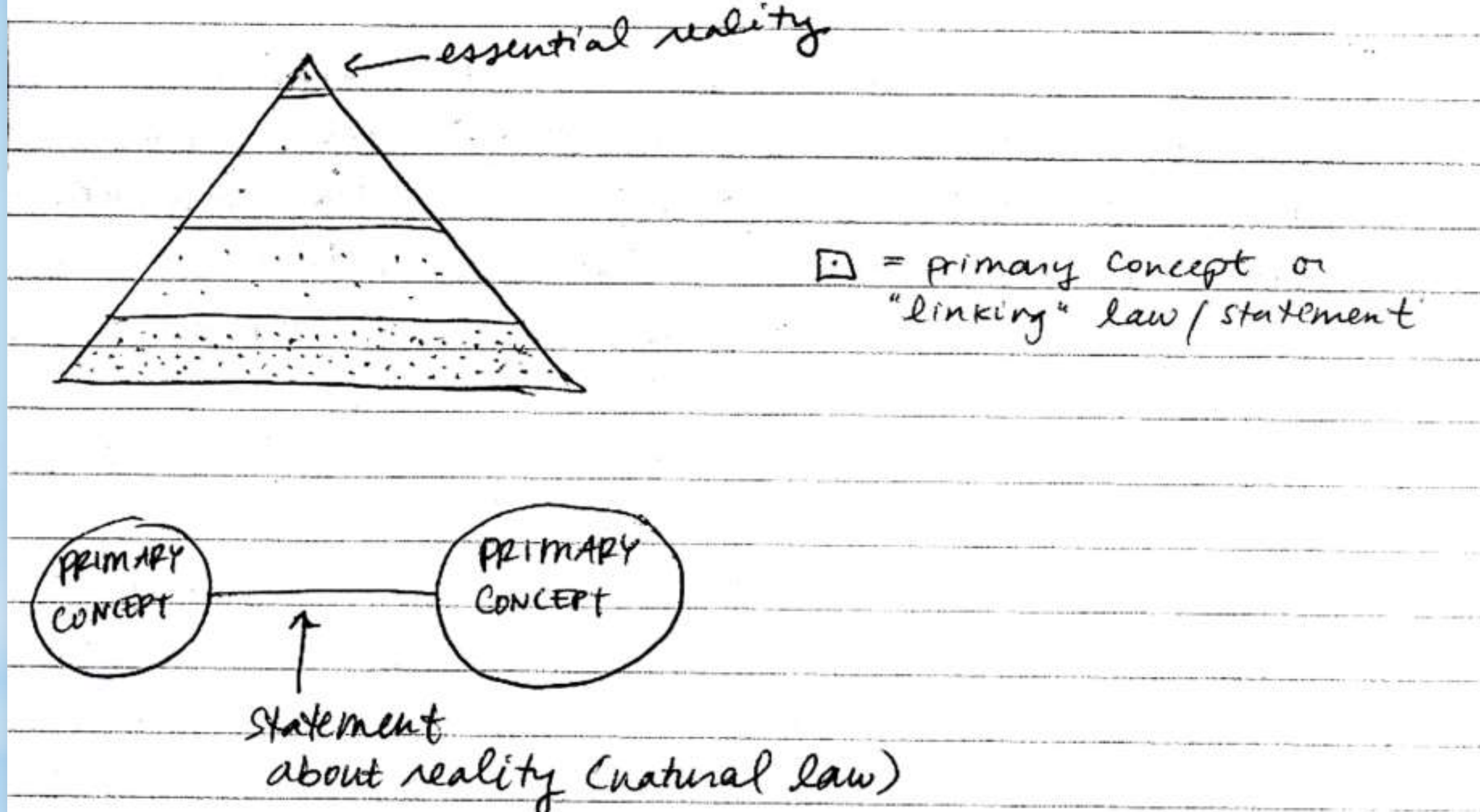


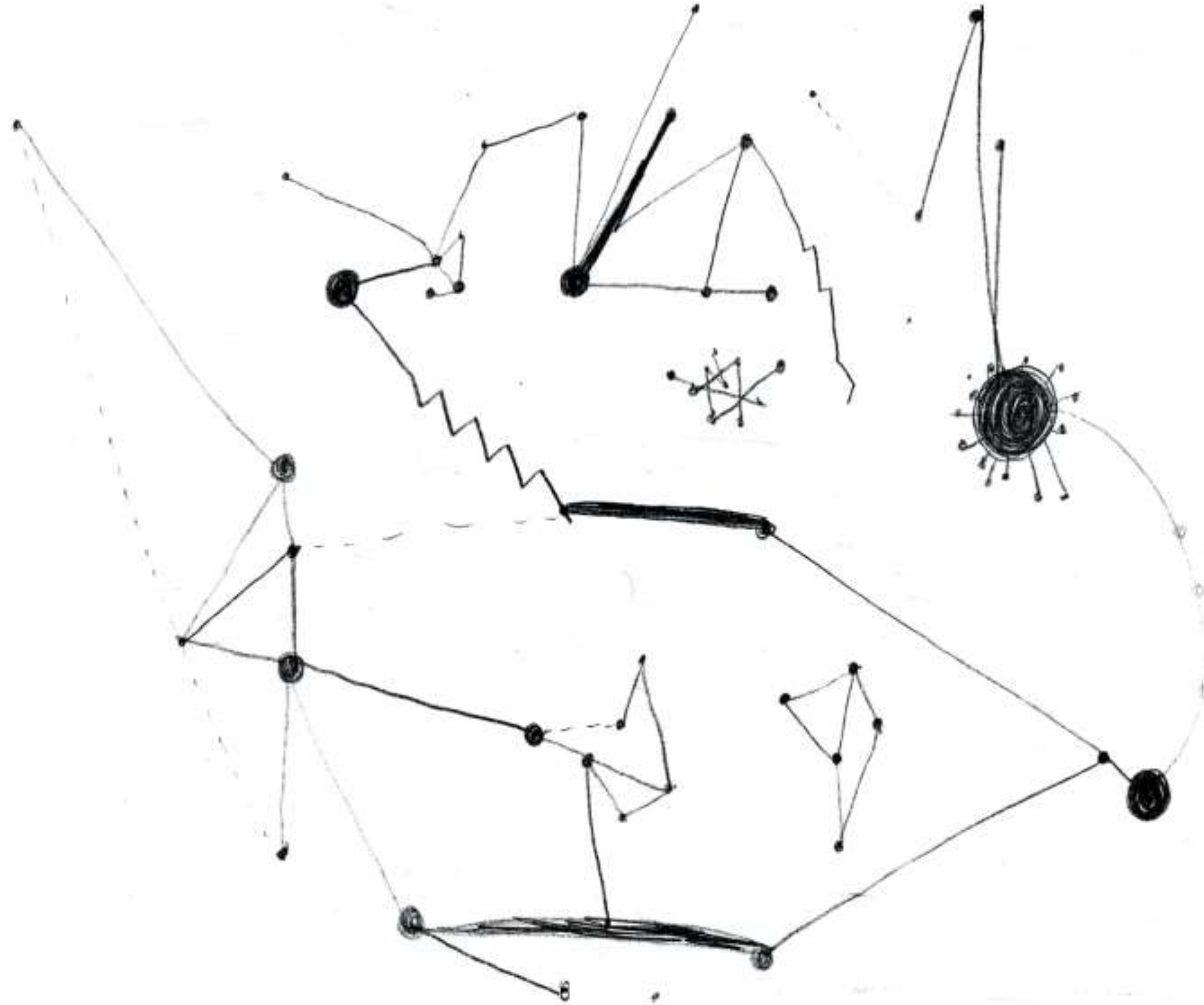
i=0
i
+ Higher Order Terms.

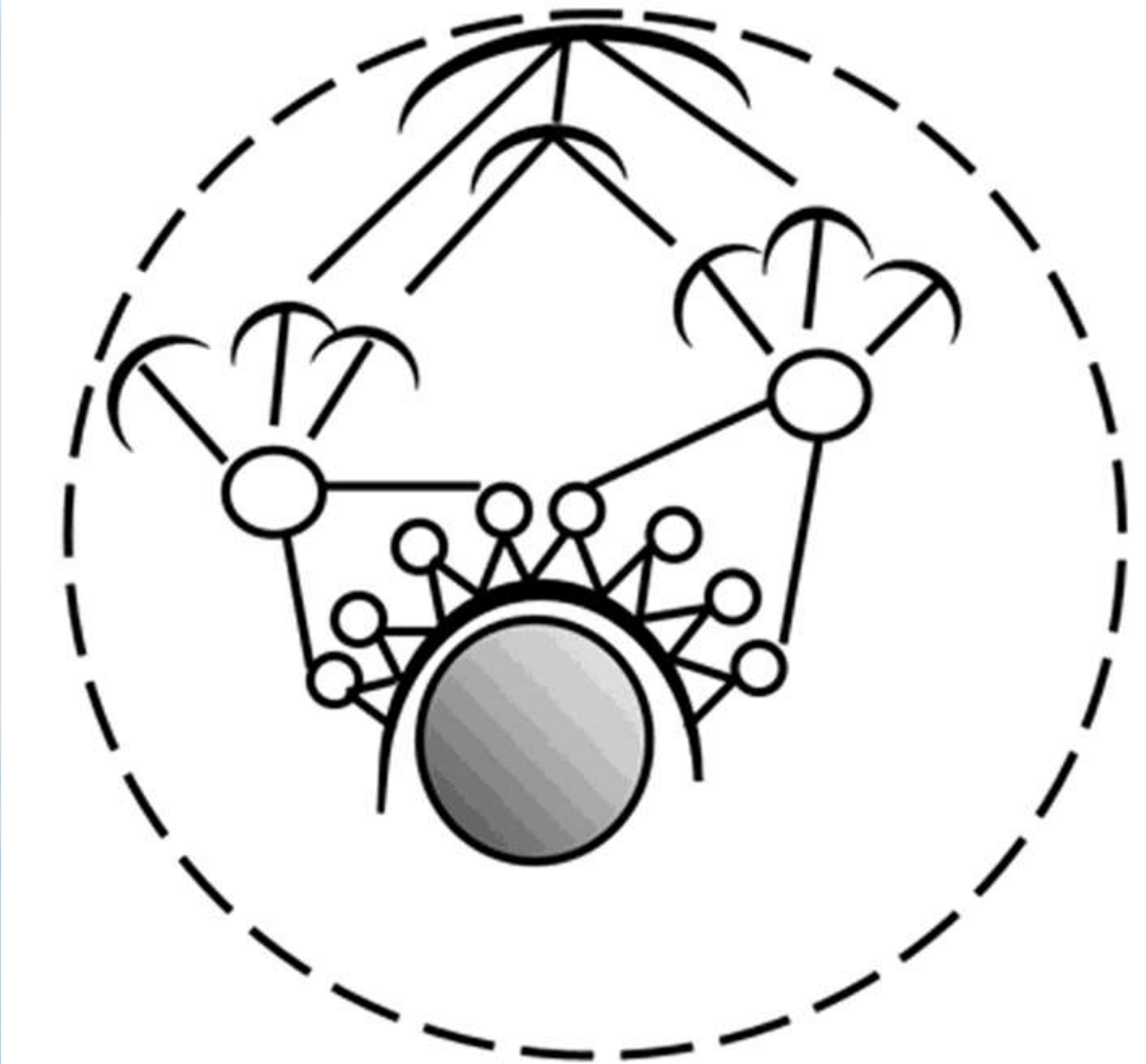


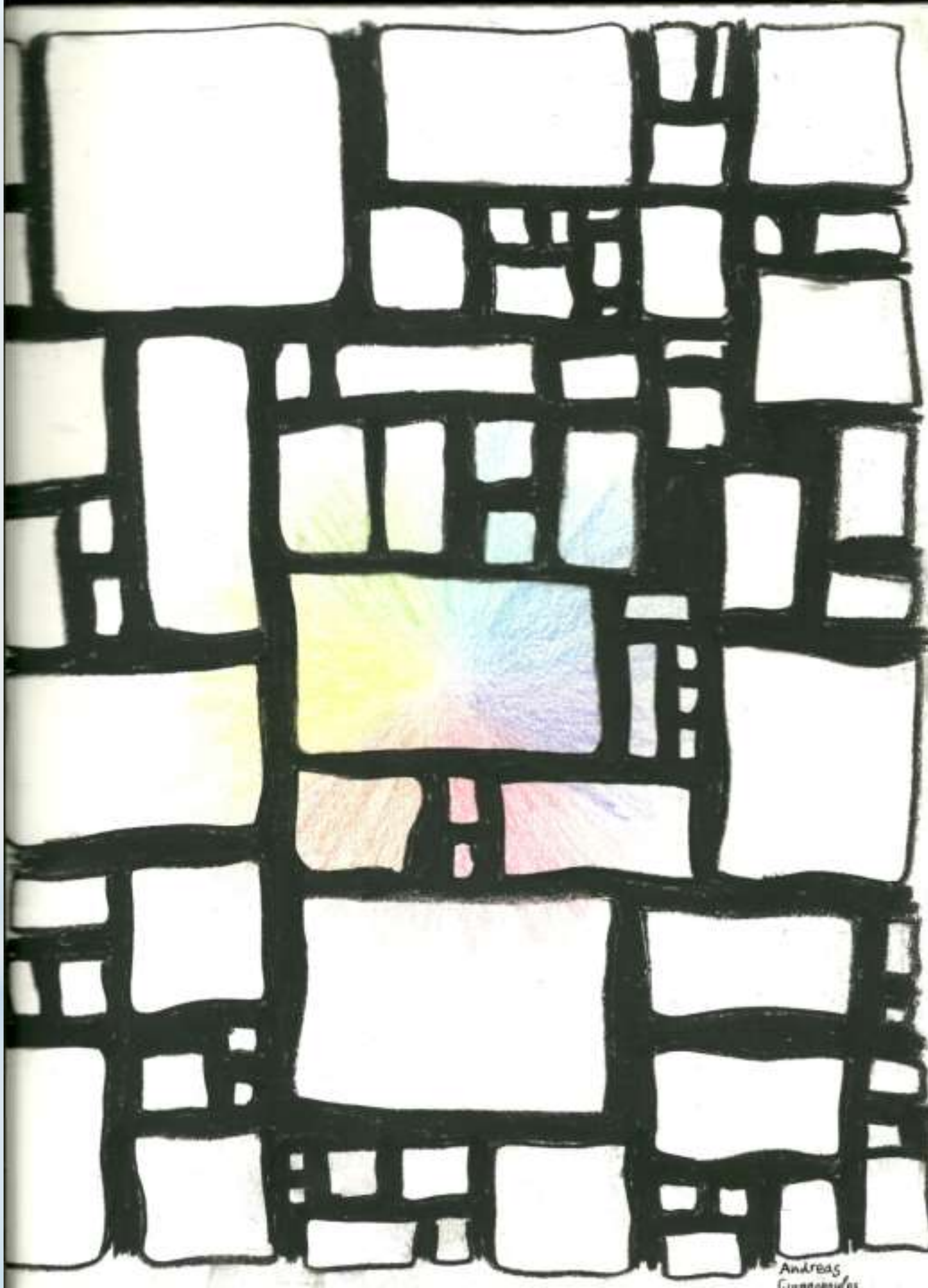
TIERS OF SCIENCE
Anand Das
?
MORE knowledge!
STRING THEORY
cookies
where is everyone?
yawn
huh?
L=T-V
this is Greek to me
E=hν
...ok
F = mg
interesting
where are the hula hoops?
sure
seen that before
cool
...attraction
attraction!
It's dark in here
coooool
wheeee

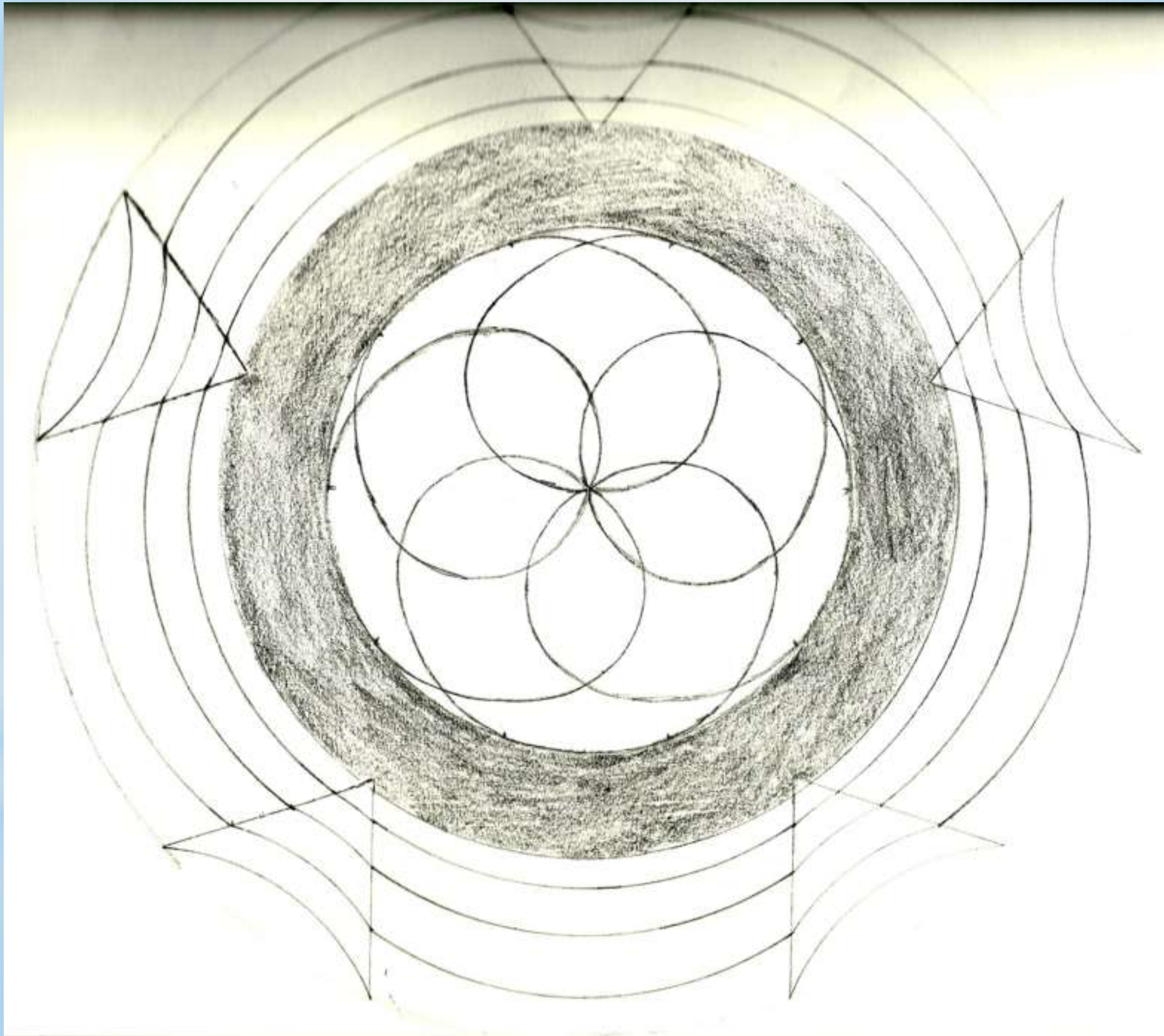

1) Everyday thinking
oh! Look at that tree
cool tree
sense experienced!
2) the concept
What's the tree made of?
How does it work?
general concepts/degrees of abstraction
3)
plant cells!
atoms!
photosynthesis
(system of the greatest conceivable unity)
primary
secondary
4) set of rules:
all plants = photosynthesize & are made of cells!
5) Relation of concept to sense experience
cool!


essential reality
= primary concept or "linking" law / statement
PRIMARY CONCEPT
PRIMARY CONCEPT
statement about reality (natural law)



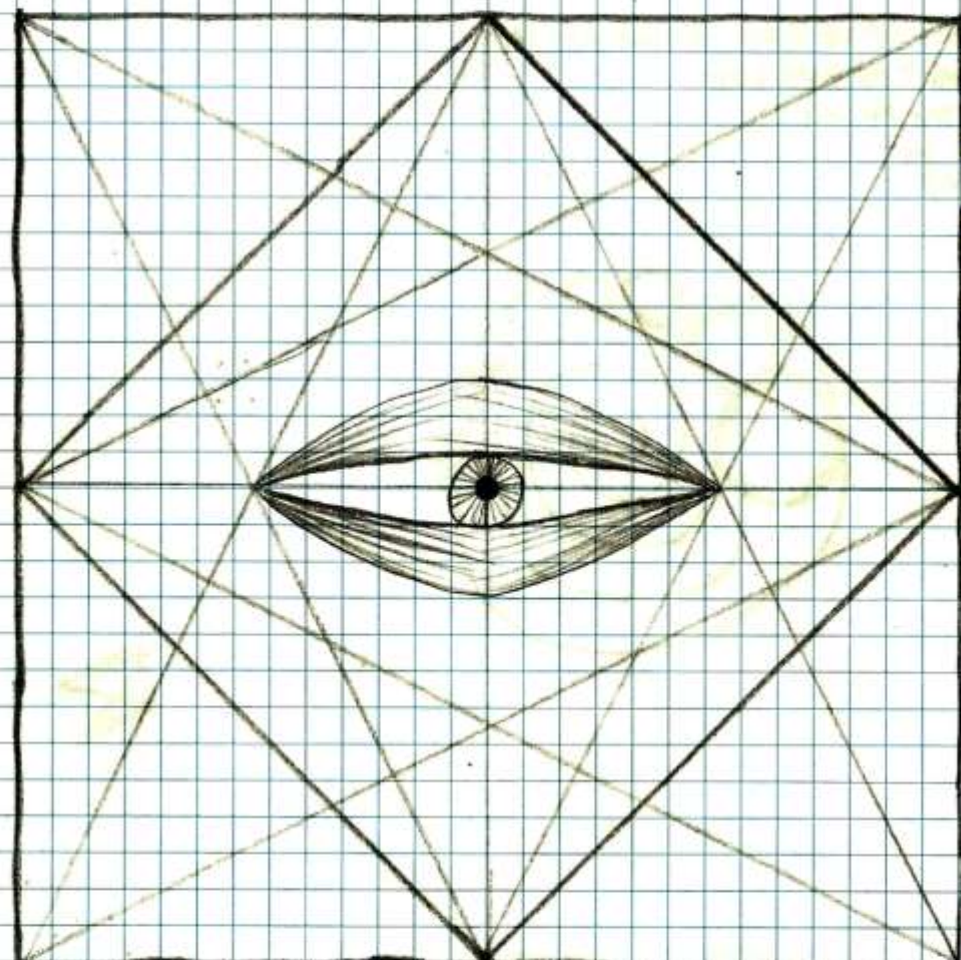


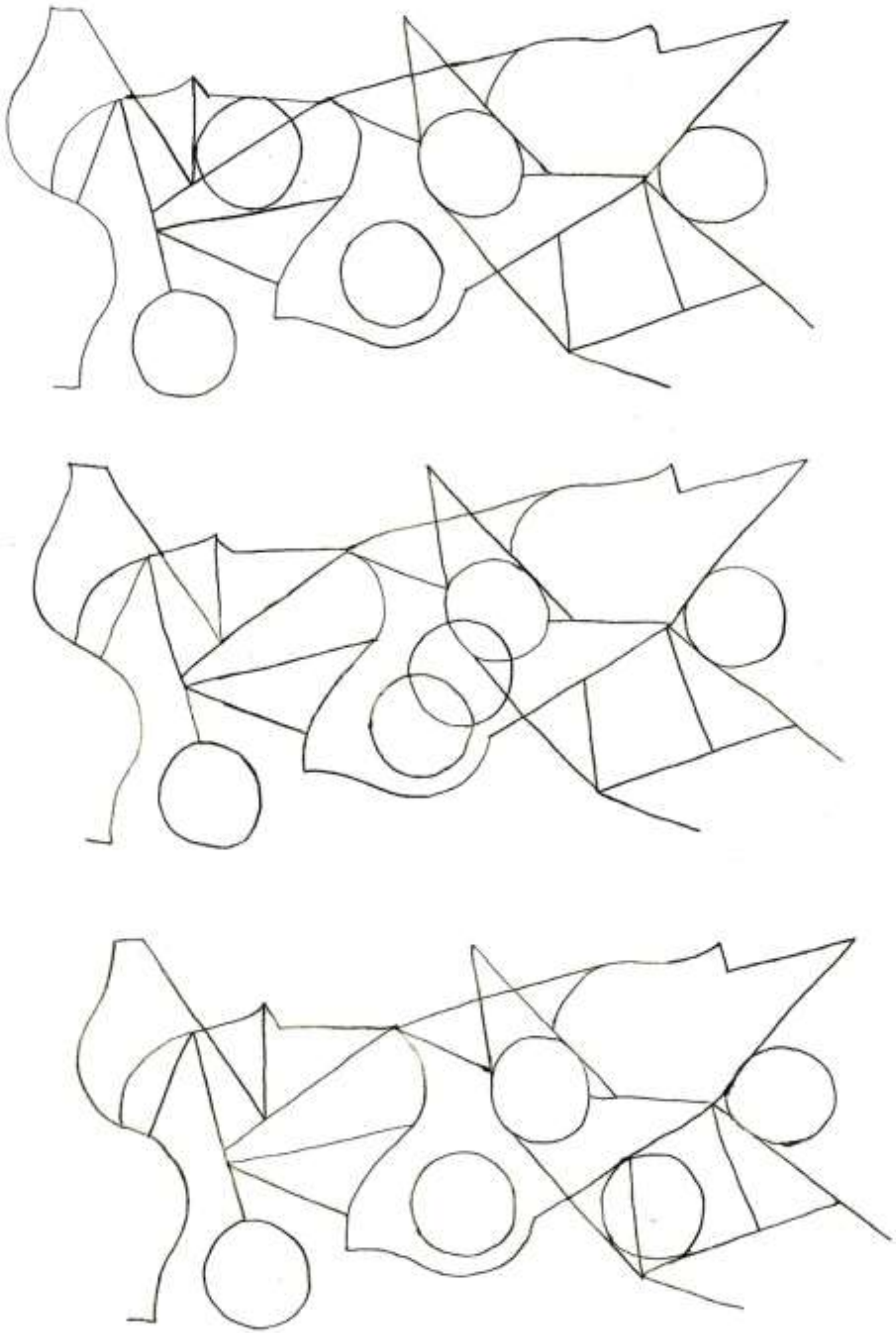

Andreas



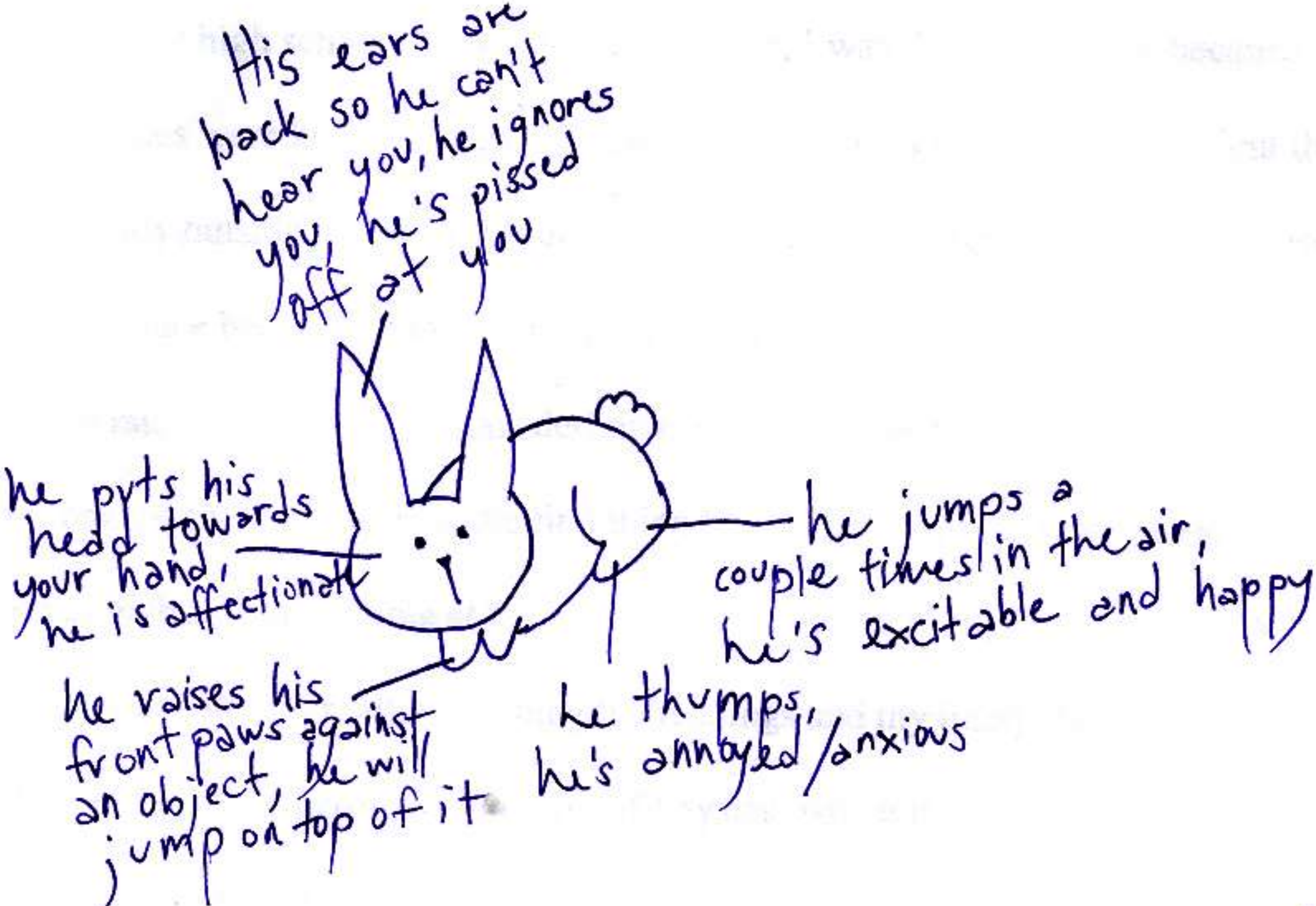
EINSTEIN'S DIMENSIONS OF REALITY

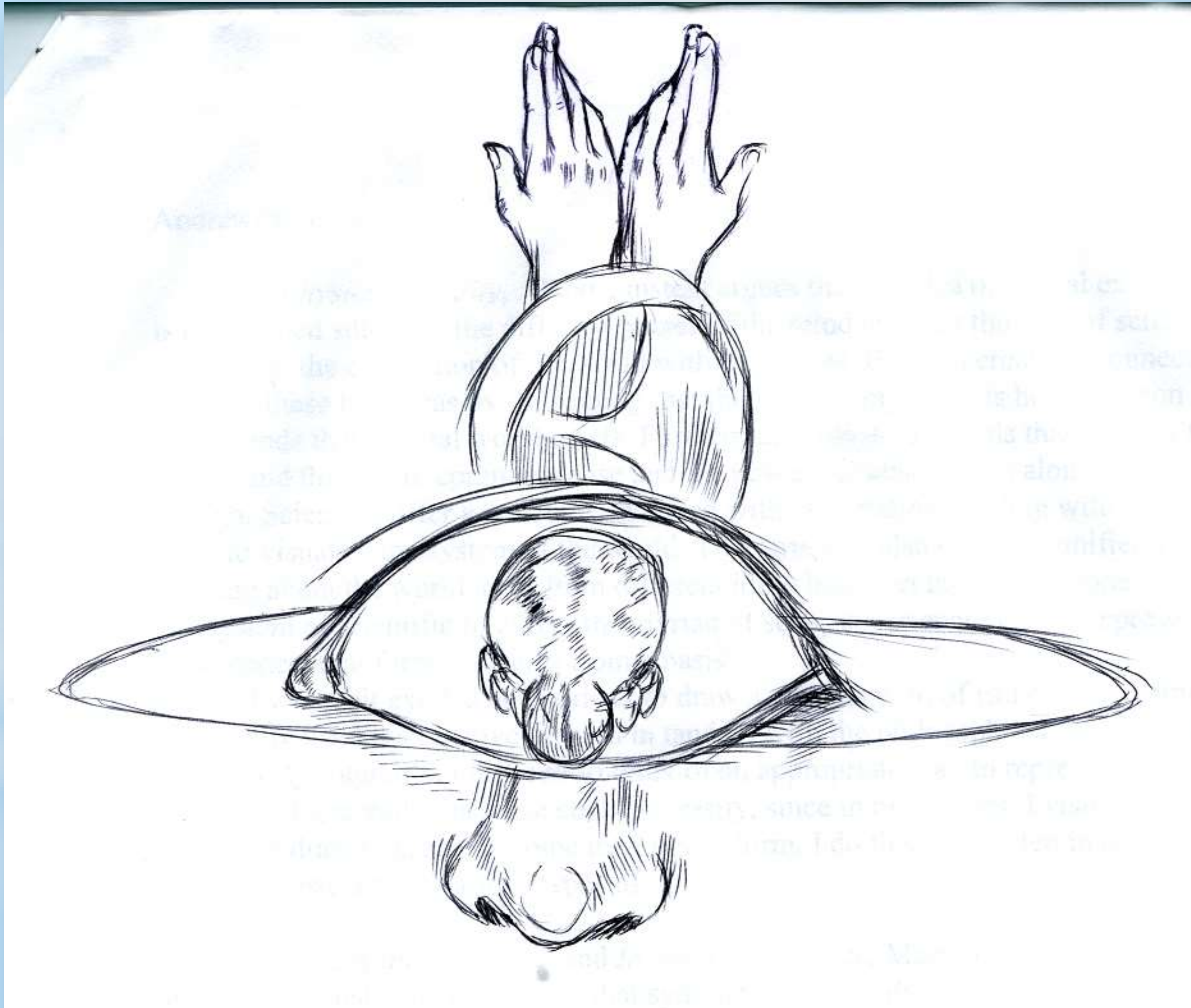
Dylan Sheehan

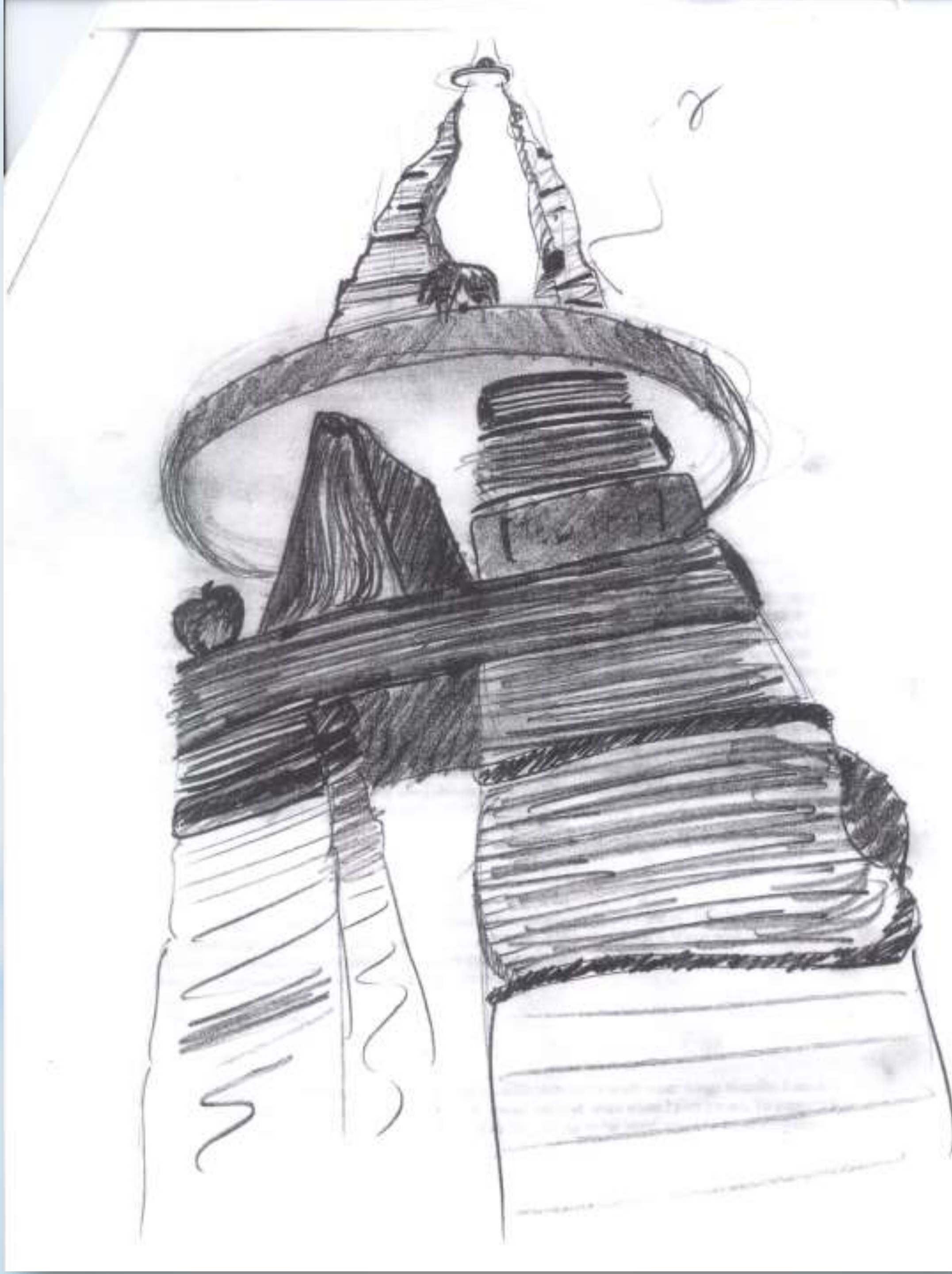




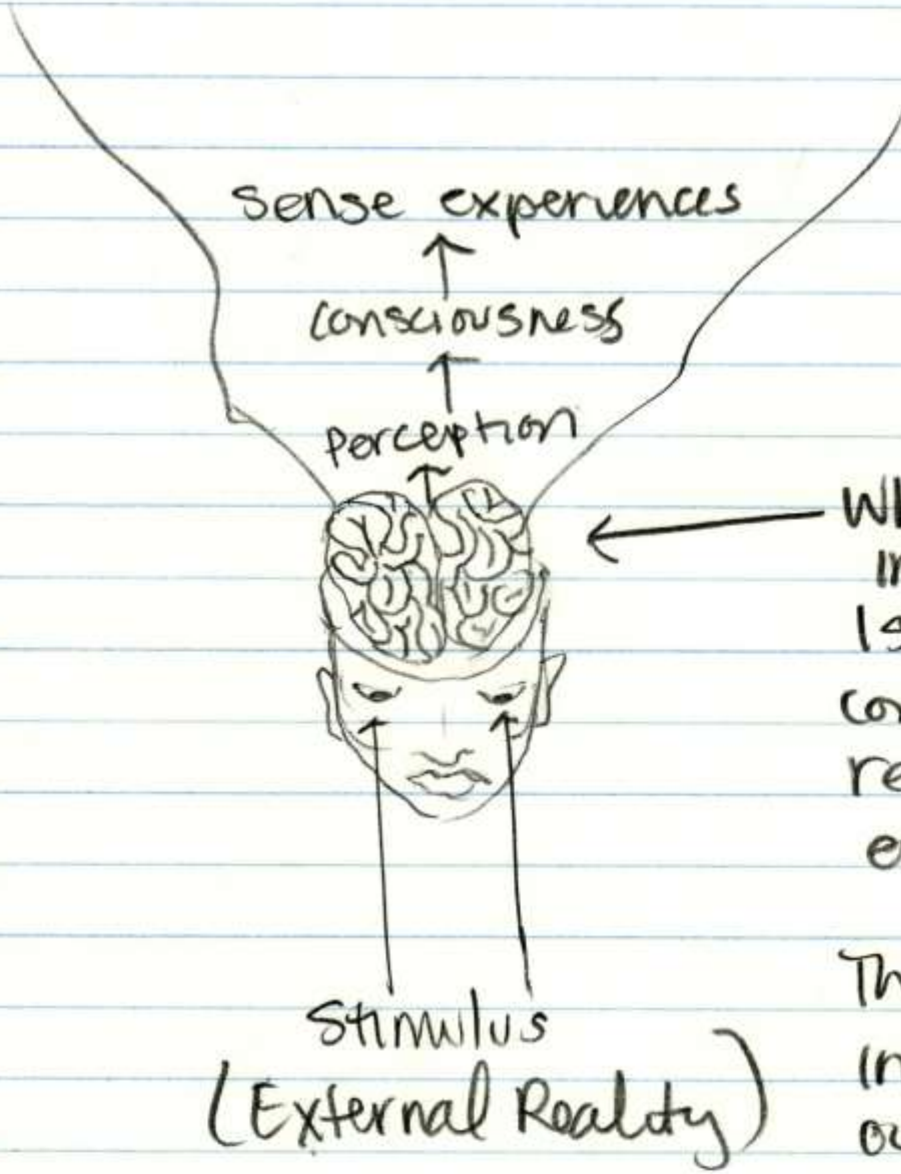

3 Different Building Blocks


His ears are back so he can't hear you, he ignores you, he's pissed off at you
he puts his head towards your hand, he is affectionate
he jumps a couple times in the air, he's excitable and happy
he raises his front paws against an object, he will jump on top of it
he thumps, he's annoyed/anxious



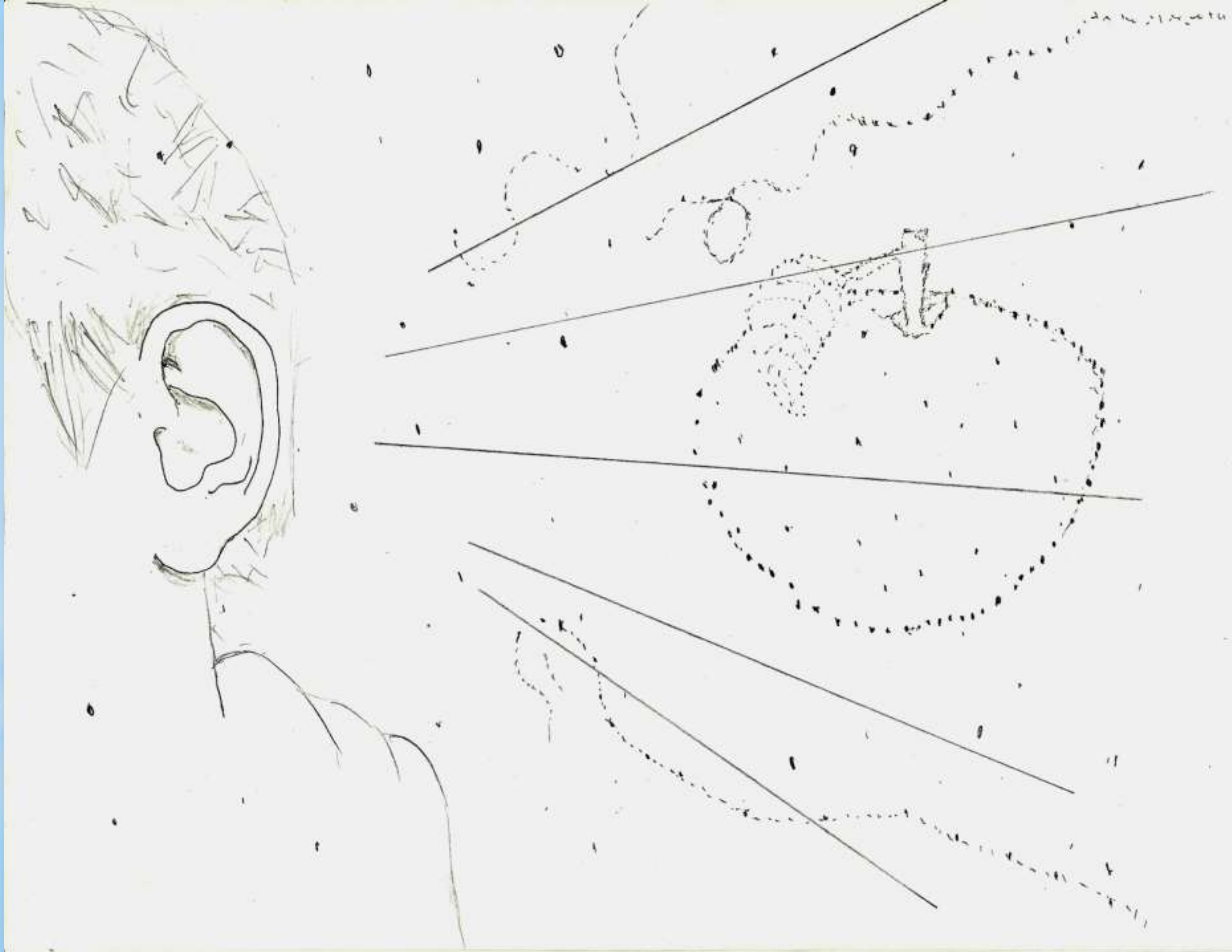


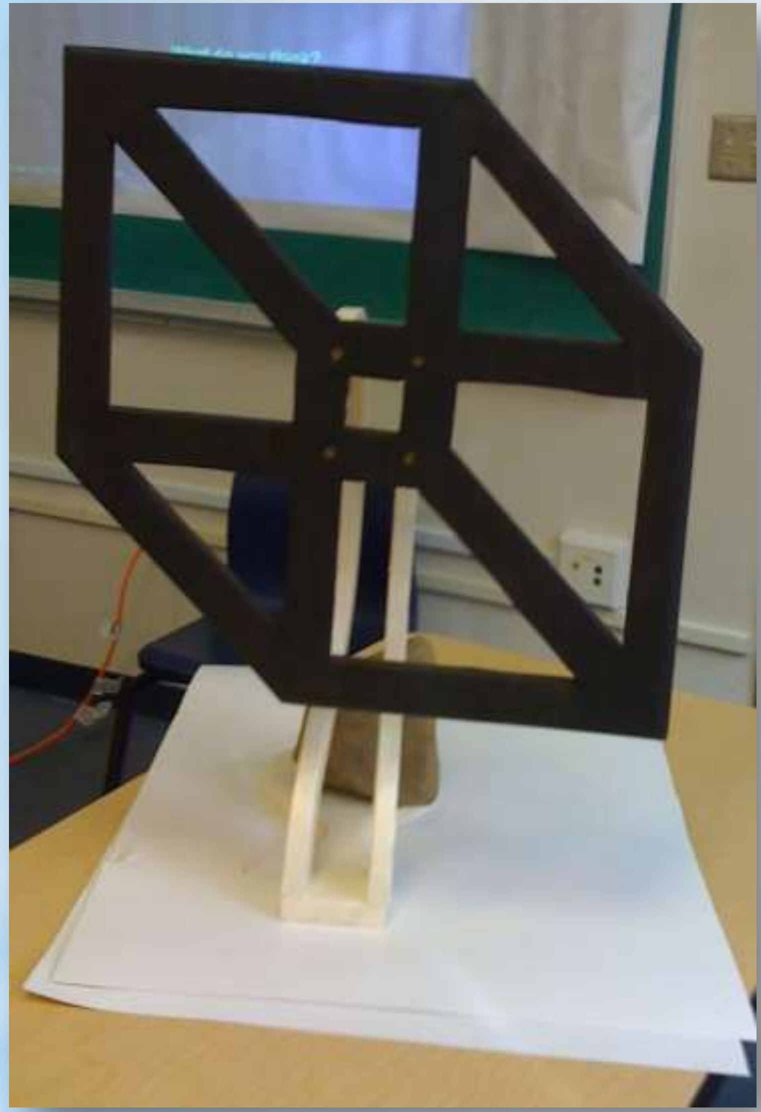

1 2 3 4 5



What is happening in here?
Is this where the connection between reality and sense experiences take place?

The solution cannot be in our minds because our consciousness distort reality.

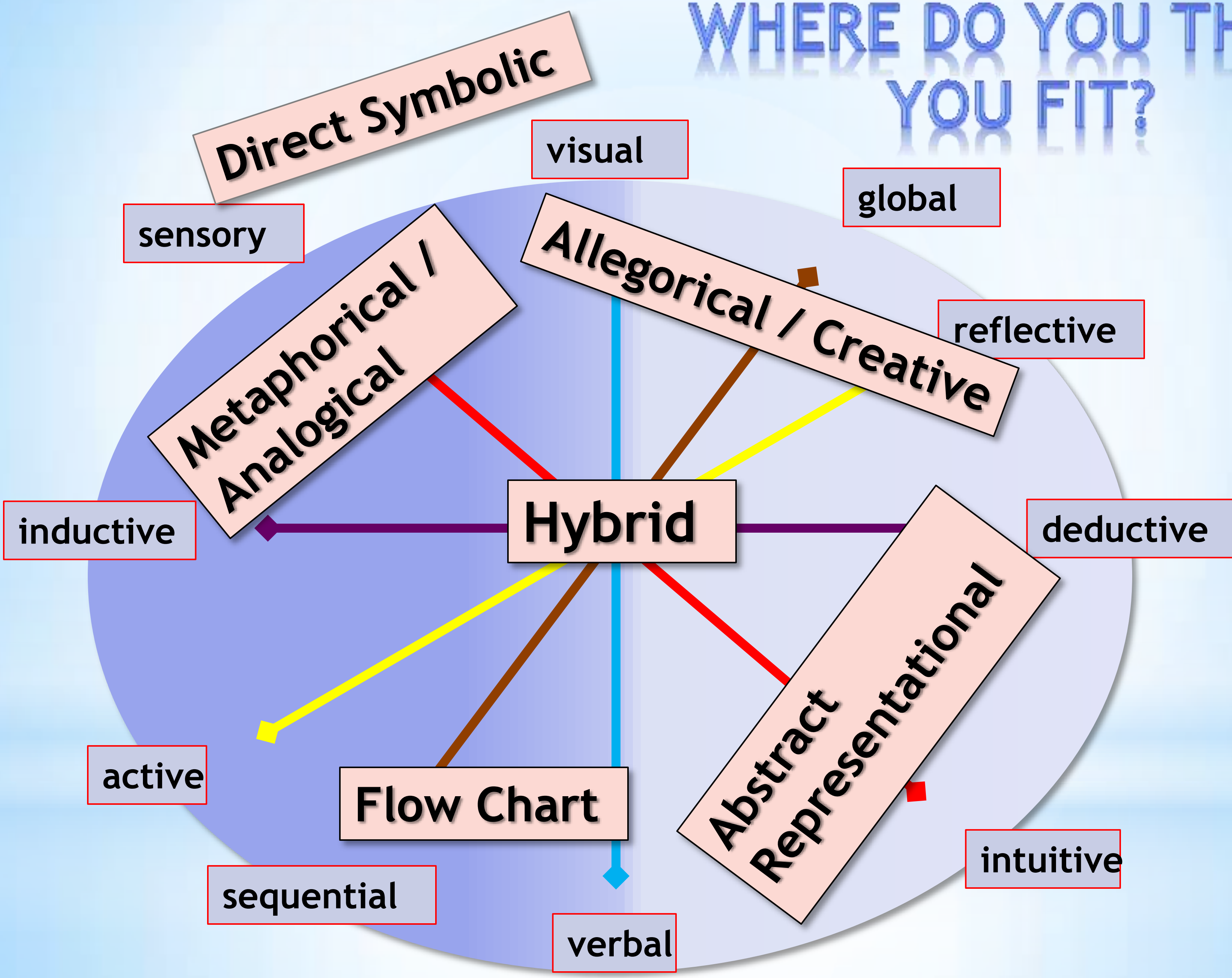






This is a
lighted version
of a world with
no eyes
Flower
Scent

WHERE DO YOU THINK YOU FIT?



Thinking about physics through the lens of symmetry

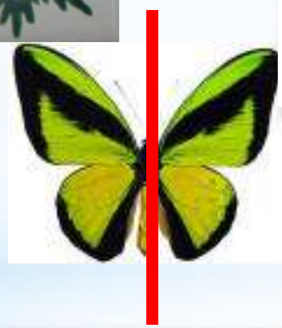
Symmetry is not thought of as a static, unchanging pattern, but as a dynamic set of operations.

A symmetry operation is when you make a change in a system (such as a rotation), and the system remains unchanged.

The set of all symmetry operations that leave a system unchanged forms a group.

rotational symmetry

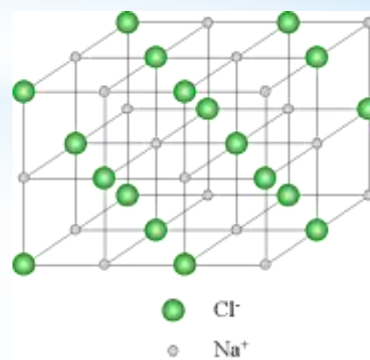
translational symmetry



reflection symmetry

screw symmetry = translation + rotation

glide symmetry = translation + reflection

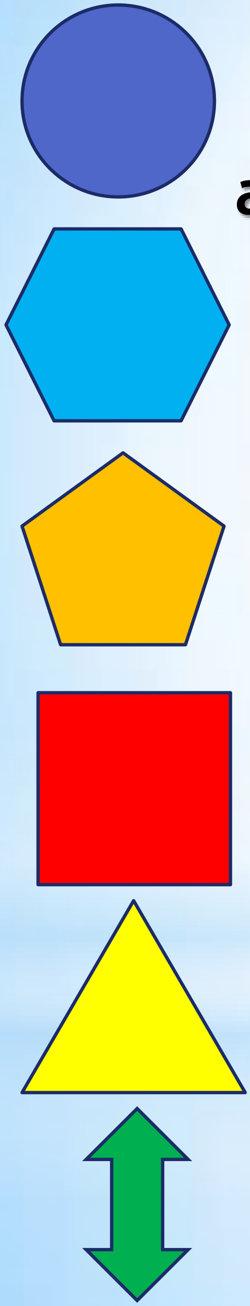


There are 3 basic types of symmetry observed in Nature, and 2 composites.

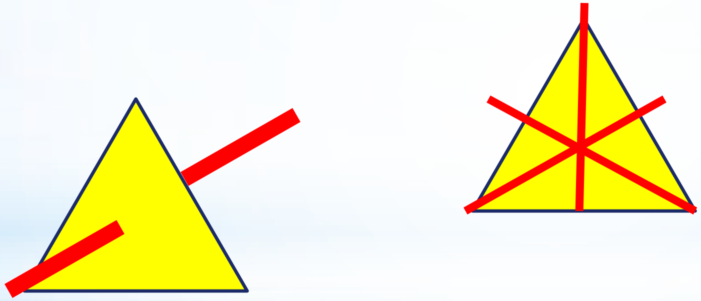
The dyptich in religious art:
reflection symmetry

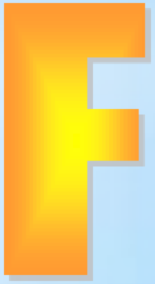
samples of Morris Wallpaper patterns – translational symmetry

“Marylin Dyptich” – Andy Worhol, 1962



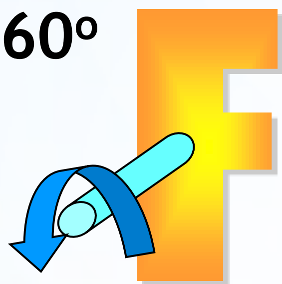
The types of symmetry transformations that leave a system unchanged tell you about the fundamental properties of that system.



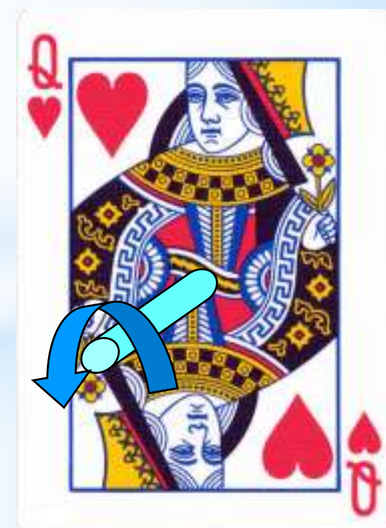


Cyclic groups have only one axis of rotation, and no reflection planes:
C_n where n = integer, have one axis of rotation, are symmetric under rotation by an angle 360/n, but have no reflection planes. Examples include:

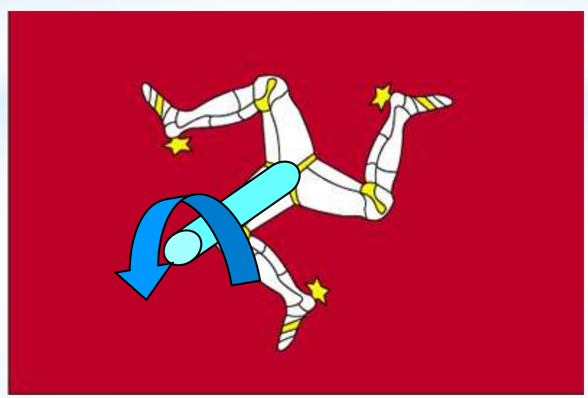
C_1 : the letter F
$\theta=360^o$



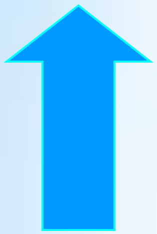
C_2 : a playing card
$\theta=360^o/2 = 180^o$



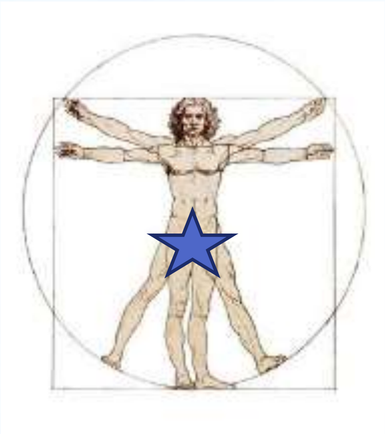
C_3 : the triskelion
$\theta=360^o/3 = 120^o$



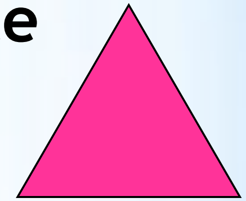
Dihedral groups: D_n where n = an integer, have rotation symmetry about a fixed axis, when rotated through an angle 360/n, and n reflection planes. Examples include:



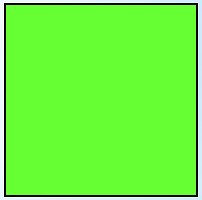
D_1
arrow, butterfly, human body



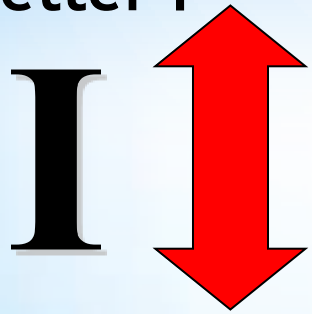
D_3 : an equilateral triangle

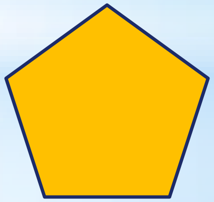


D_4 : a square



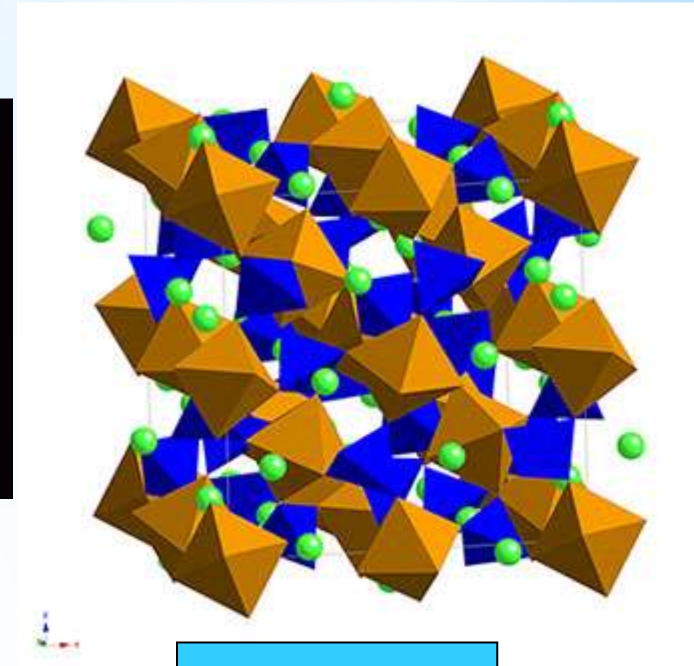
D_2 : a double sided arrow, the letter I





D_5 : a pentagon

In Nature, we do find repeating patterns of atoms in 3-D manifest as crystals.



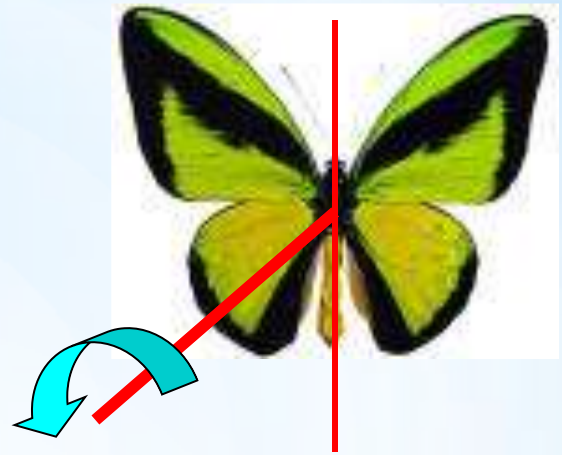
garnet structure

The set of all <u>symmetry transformations</u> of any object that leave the object unchanged forms a GROUP.

There are four properties that define a group:

1. **Closure – the group must be closed under its operations**
2. **Associativity – (a * b)*c = a*(b*c)**
3. **Identity element – leaves a member unchanged**
4. **Inverse – when combined with any member, gives the Identity element**

Let's take the example of this butterfly:
It has one rotation axis, and one reflection plane...



If I only rotate it by 90^0

or by 180^0 , it does not look the same.

I have to rotate it by 360^0 to get it to look the same.

So, there are 2 symmetry operations for the butterfly that leave it unchanged after each operation: Reflect about vertical axis, and rotate 360^0.

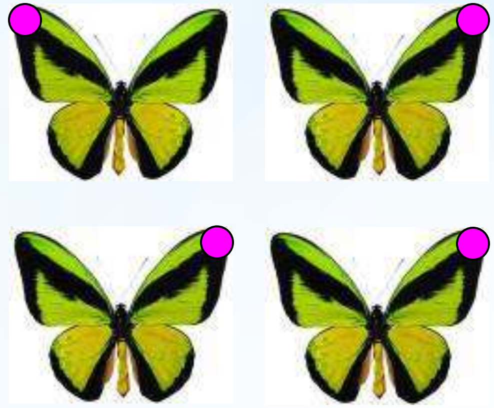
Since a symmetry operation – BY DEFINITION – leaves the system unchanged, in order to follow which operations have been performed, we have to introduce a wee bit of asymmetry.

***We can do this by putting a dot on one wing.
Then we can see which symmetry operations have been performed, and we can test to see if these operations are commutative.***

Rotation leaves the butterfly unchanged.

Reflection (vertical flip) produces a mirror image. Since the butterfly is bilaterally symmetric, the only way we can tell that a reflection has been performed is by the right-left reversal of the dot on its wing.

Are vertical flips and 360^0 rotations commutative?



First rotate, then flip

First flip, then rotate

$$F \circ R = F \qquad R \circ F = F$$

R x F = F

F x R = F

In the Butterfly group, flips and rotations are commutative. We call this type of group ABELIAN, after the mathematician Niels Henrick Abel

R x R = I

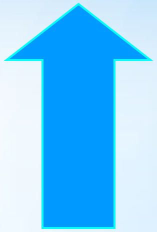
F x F = I

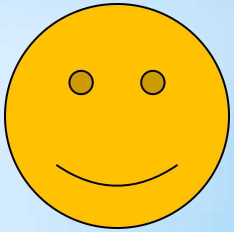
Two flips or two rotations always gives the identity element – that is, they get you back to the same place from which you started.

We can arrange these operations in a table like this:

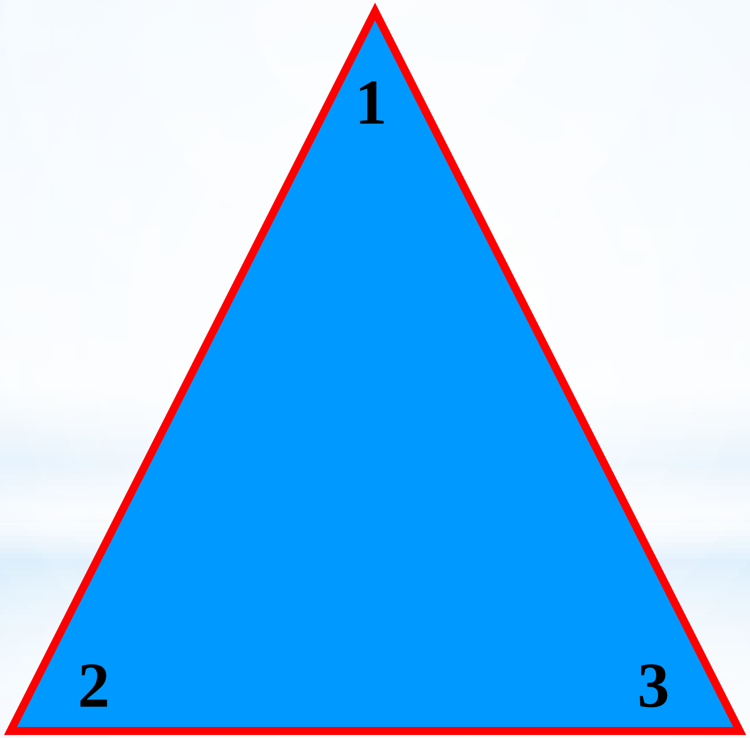
o	F	R
F	I	F
R	F	I

All objects which remain unchanged under this set of symmetry transformations belong to the same symmetry group, which has been given the name D_1.



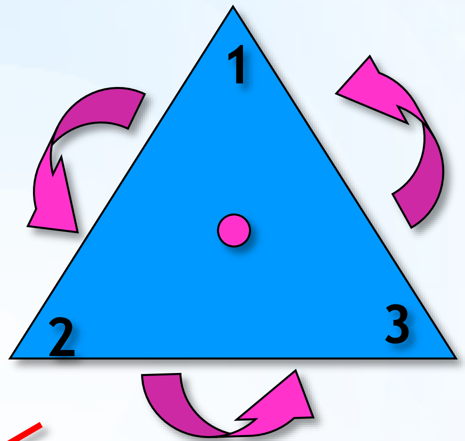


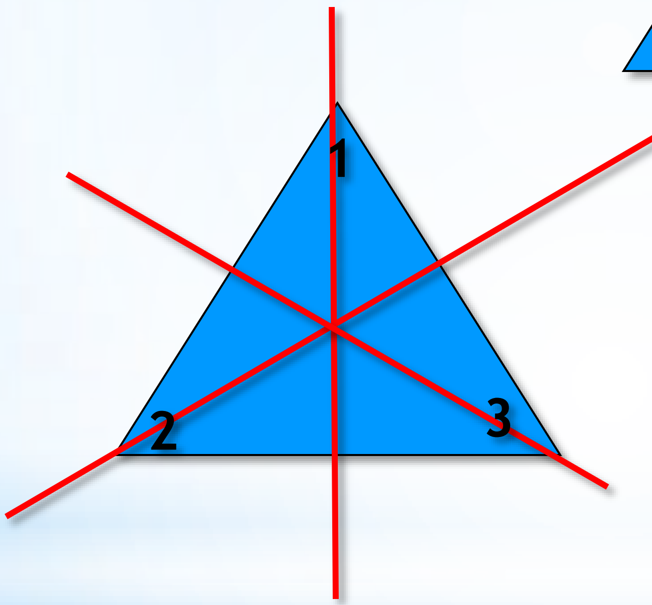
Derivation of the symmetry group D_3



Example: There are six different unique operations you can perform on this triangle that leave it unchanged:

three rotations which are multiples of 120^0 …

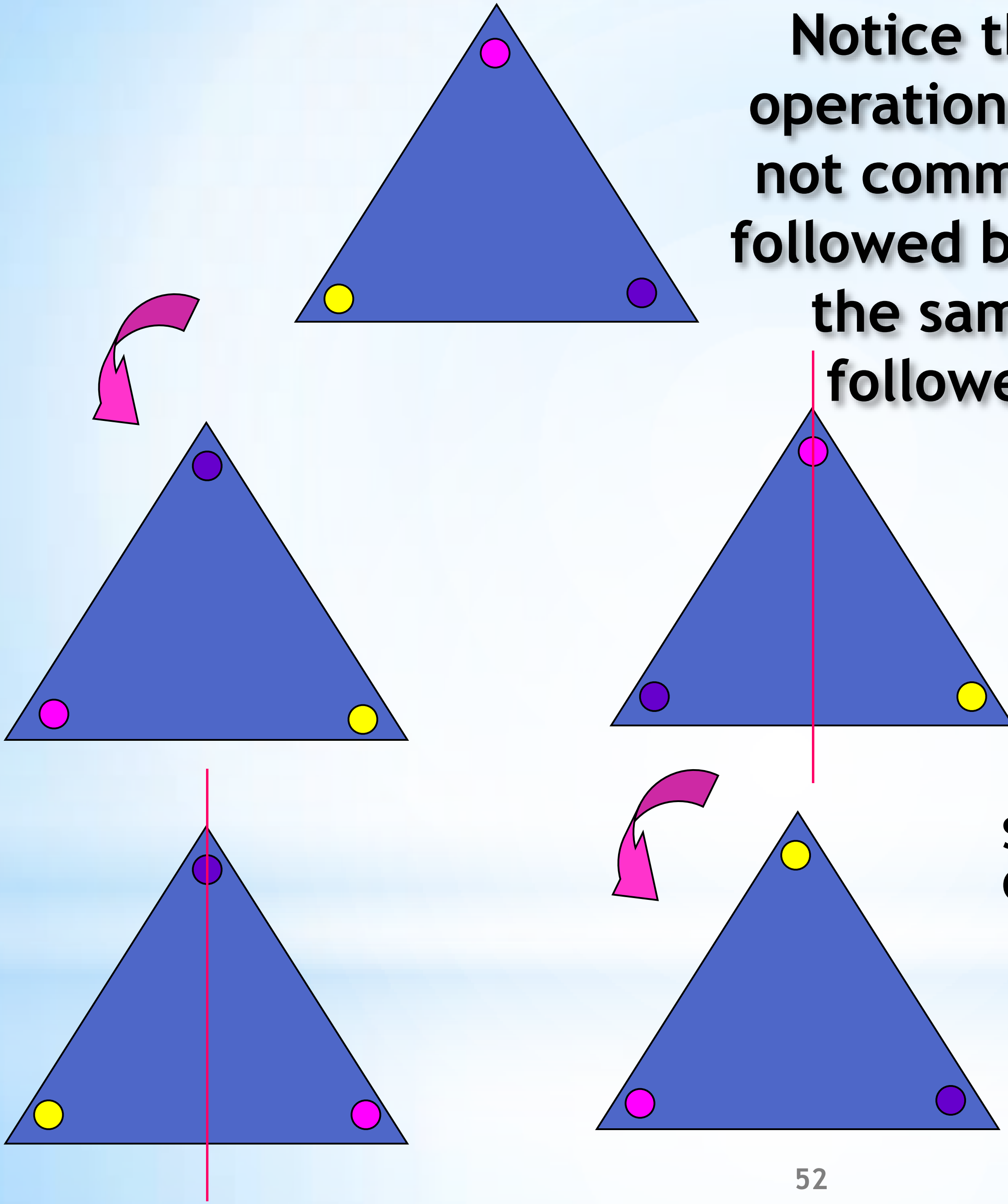




...and three reflections about axes through each of the three vertices.

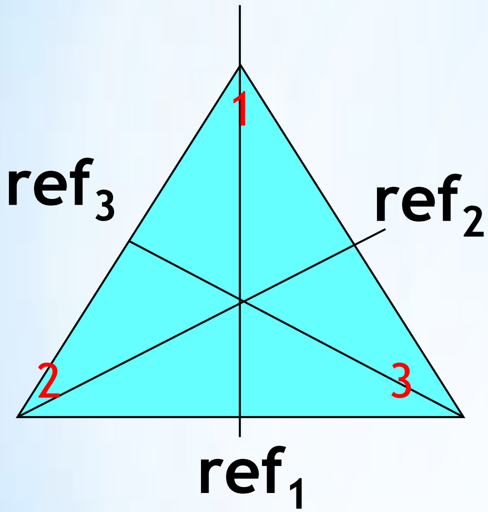
A rotation of 360^0 is called the IDENTITY ELEMENT of the group.

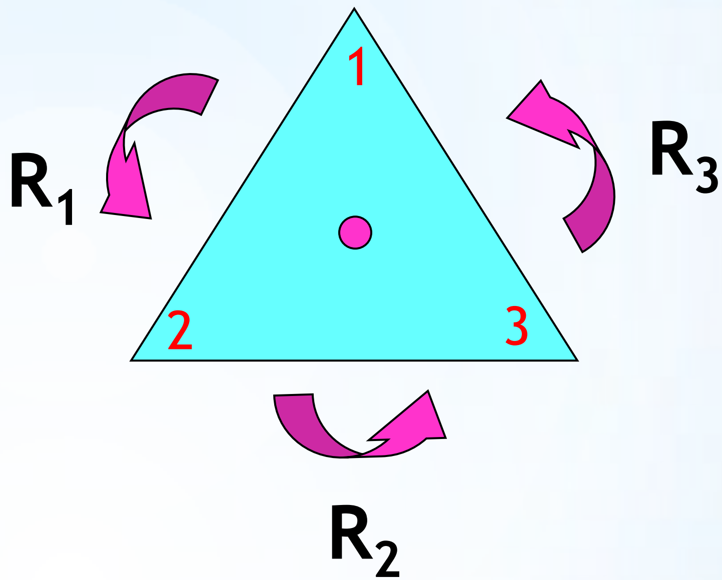
Notice that the symmetry operations for this group are not commutative: a rotation followed by a reflection is not the same as a reflection followed by a rotation.



So D_3 is a non- Abelian Group.

Notice that the group of reflections and rotations of the equilateral triangle is equivalent to the set of permutations of the numbers 1,2,3. (only true for triangle!)





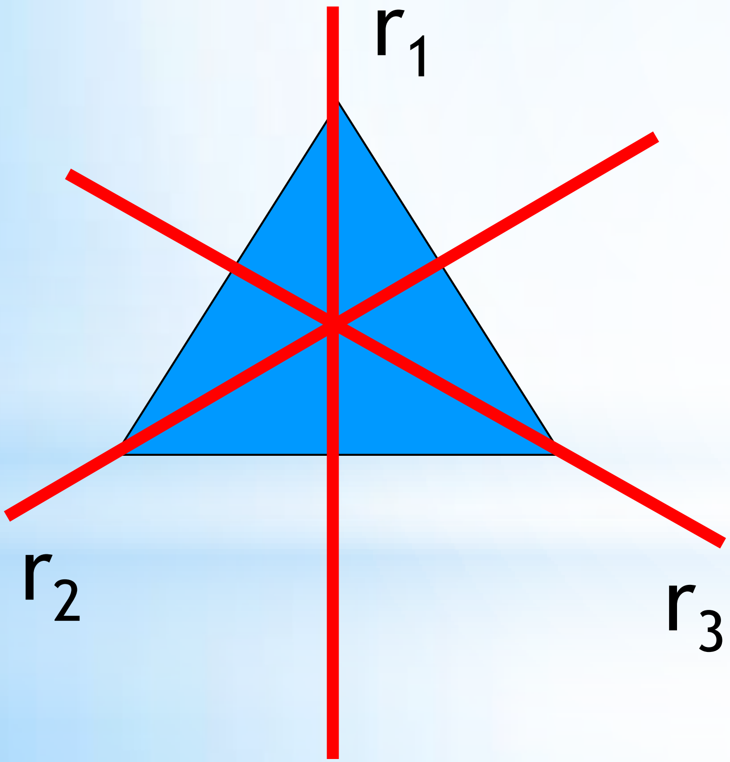
reflections	
ref$_1$	1-3-2
ref$_2$	3-2-1
ref$_3$	2-1-3

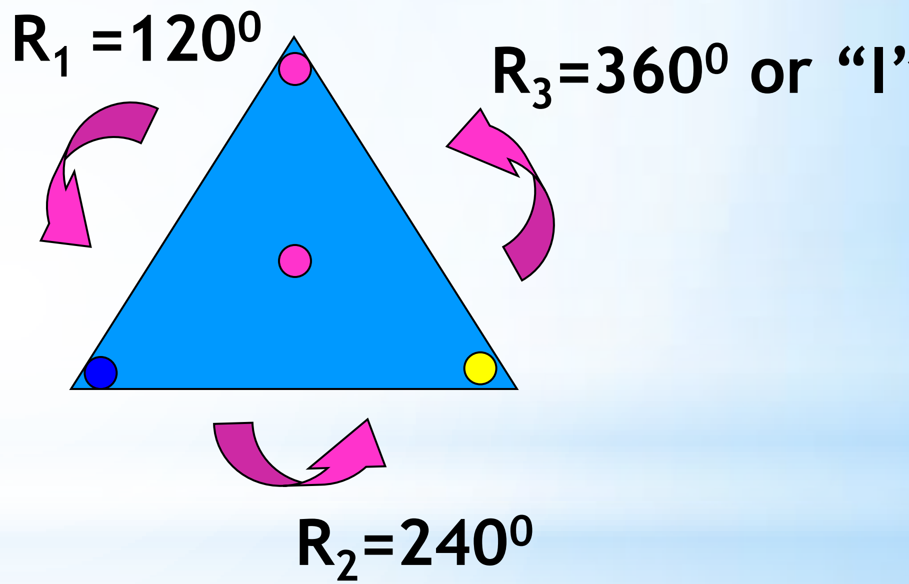
ROTATIONS		
R$_1$	120^0	3-1-2
R$_2$	240^0	2-3-1
R$_3$	360^0	1-2-3

= Identity

Work with a partner:

Show that any two operations done in sequence always yields a third operation - i.e., the group "equilateral triangle" is closed under rotations and reflections, has an identity element, and every element has an inverse. Next slides show you how:

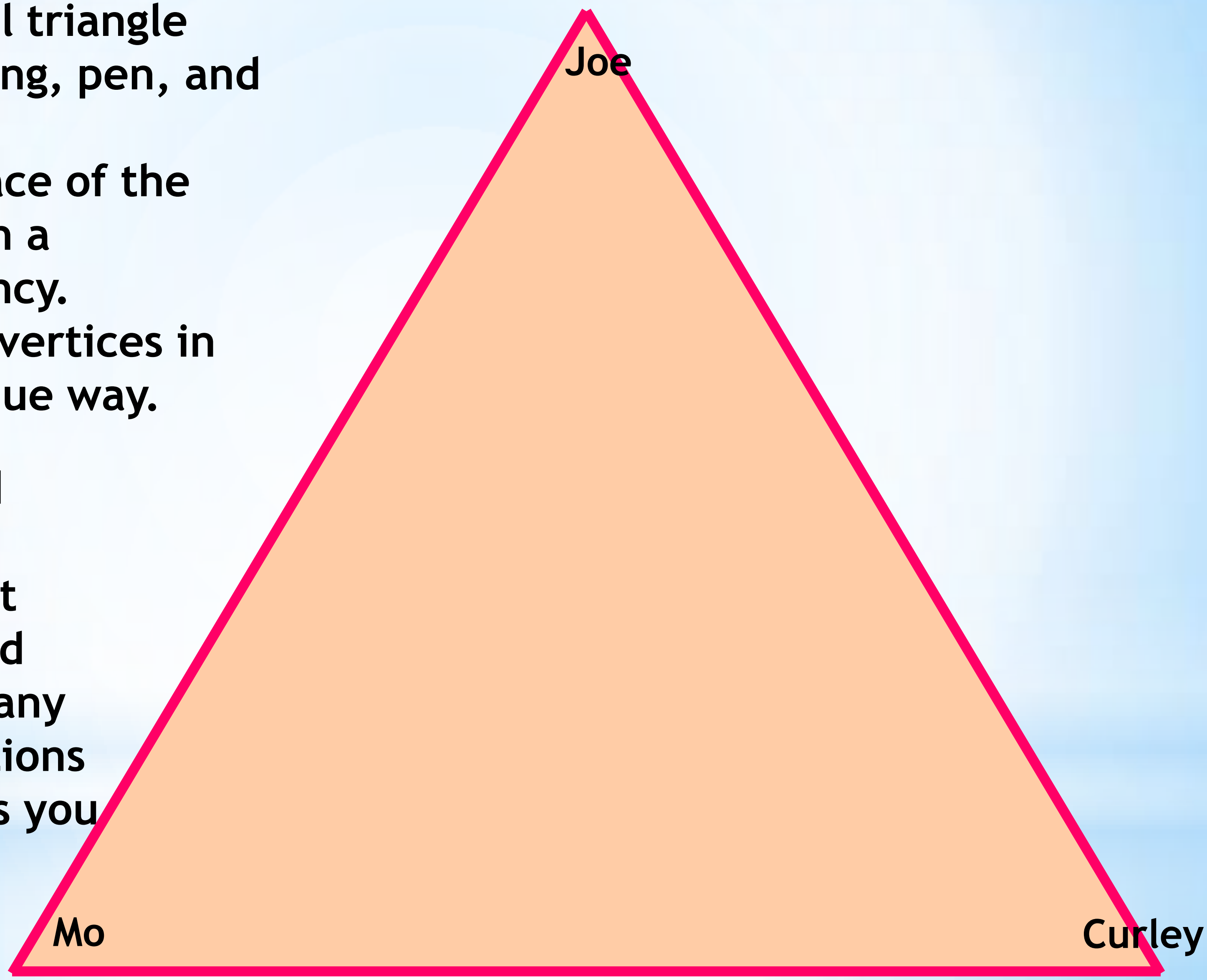




Note that rotate by 360 = don't rotate, so we call it the Identity, or I

1. Construct an equilateral triangle with a string, pen, and ruler.
2. Make a trace of the triangle on a transparency.
3. Label the vertices in some unique way.

4. Rotate and reflect the transparent triangle and show that any two operations always gets you a third operation.



Do in groups of 3 with people you have not worked with yet.

DO THIS OPERATION FIRST

DO THIS OPERATION SECOND

	R1	R2	R3	r1	r2	r3
R3	R1					
R2						
R1						
r1						
r2						
r3						

Answer key

$\circ$	rot_1	rot_2	rot_3	Ref_1	Ref_2	Ref_3
rot_1	rot_2	rot_3	rot_1	Ref_3	Ref_1	Ref_2
rot_2	rot_3	rot_1	rot_2	Ref_2	Ref_3	Ref_1
rot_3	rot_1	rot_2	rot_3	Ref_1	Ref_2	Ref_3
Ref_1	Ref_2	Ref_3	Ref_1	rot_3	rot_1	rot_2
Ref_2	Ref_3	Ref_1	Ref_2	rot_2	rot_3	rot_1
Ref_3	Ref_1	Ref_2	Ref_3	rot_1	rot_2	rot_3

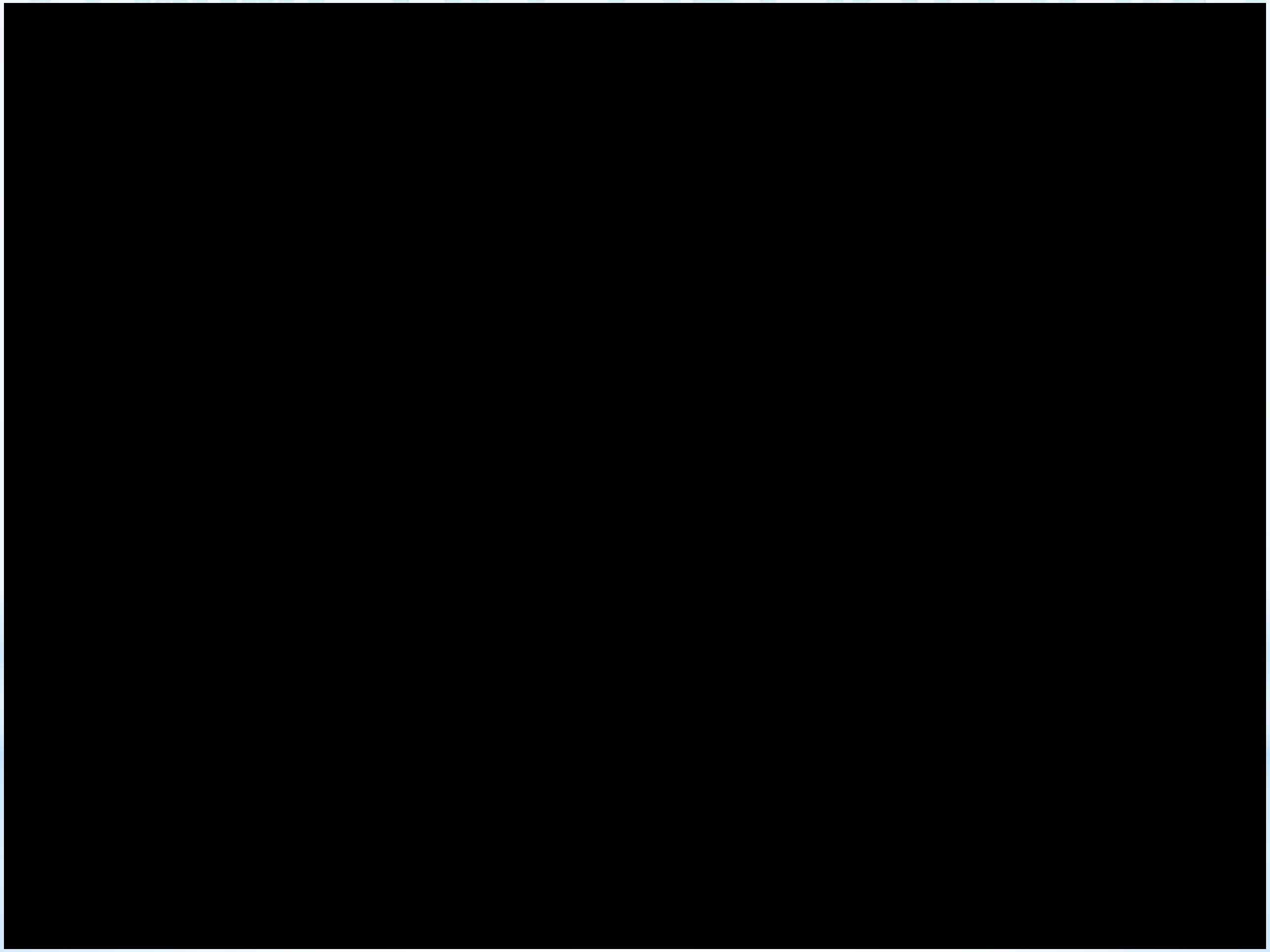
What do you notice?

Answers:

- *Two rotations or two reflections always yield a rotation.*
- *A rotation followed by a reflection and a reflection followed by a rotation always yields a reflection.*
- *Each operation appears only once in any row or column within each quadrant.*

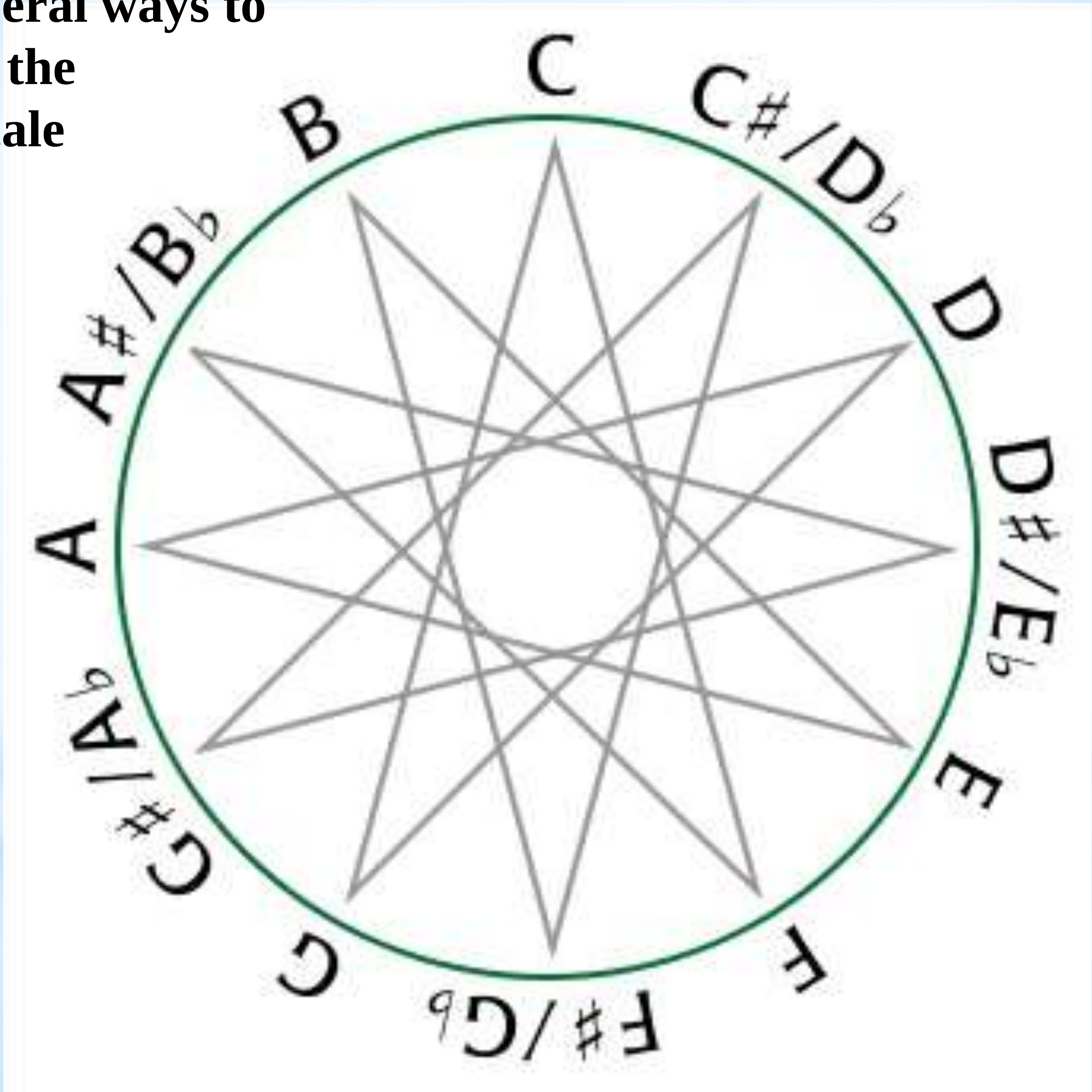
$\circ$	rot_1	rot_2	rot_3	Ref_1	Ref_2	Ref_3
rot_1	rot_2	rot_3	rot_1	Ref_3	Ref_1	Ref_2
rot_2	rot_3	rot_1	rot_2	Ref_2	Ref_3	Ref_1
rot_3	rot_1	rot_2	rot_3	Ref_1	Ref_2	Ref_3
Ref_1	Ref_2	Ref_3	Ref_1	rot_3	rot_1	rot_2
Ref_2	Ref_3	Ref_1	Ref_2	rot_2	rot_3	rot_1
Ref_3	Ref_1	Ref_2	Ref_3	rot_1	rot_2	rot_3

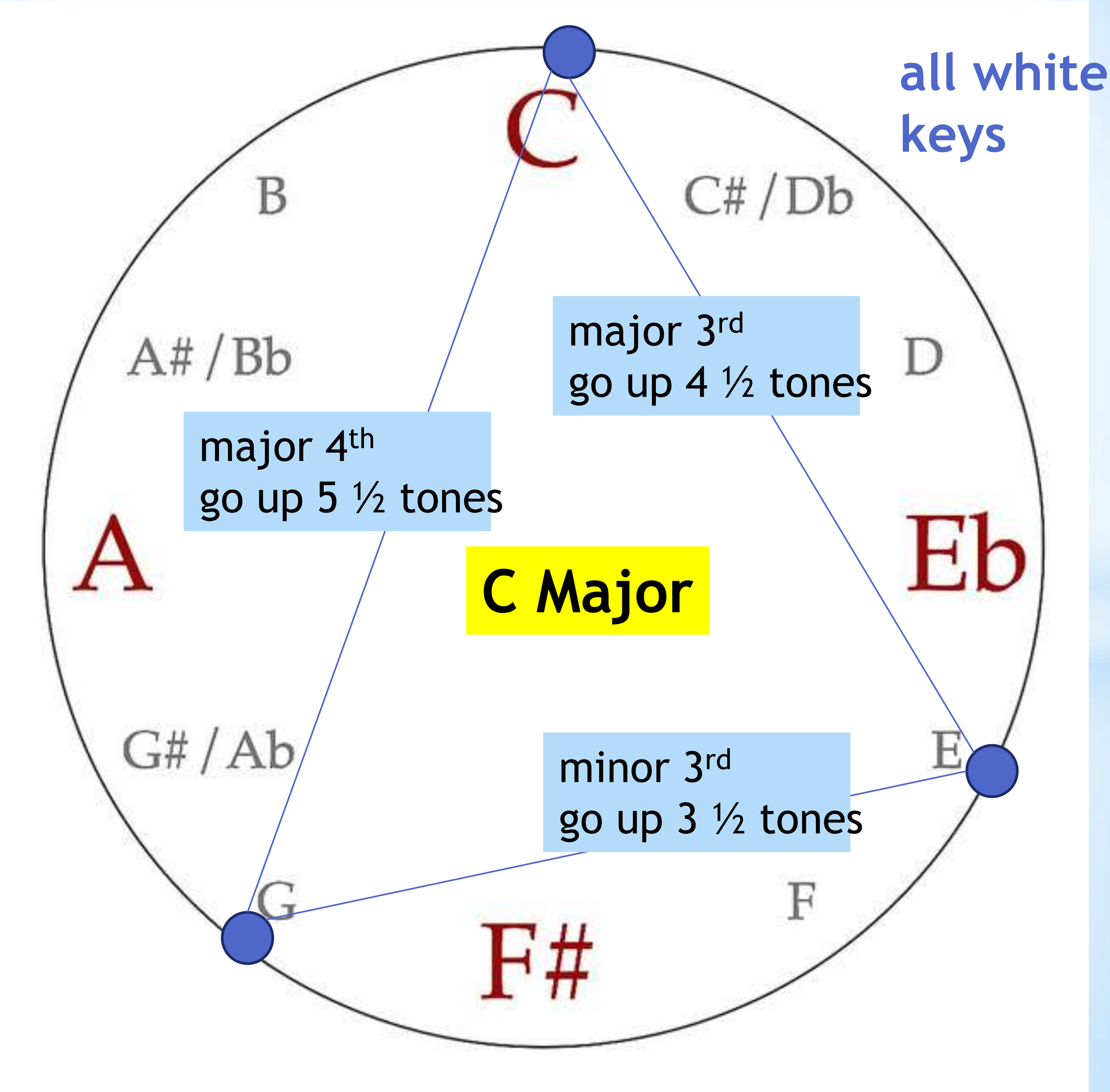
CULTURAL EXPRESSIONS OF SYMMETRY OPERATIONS

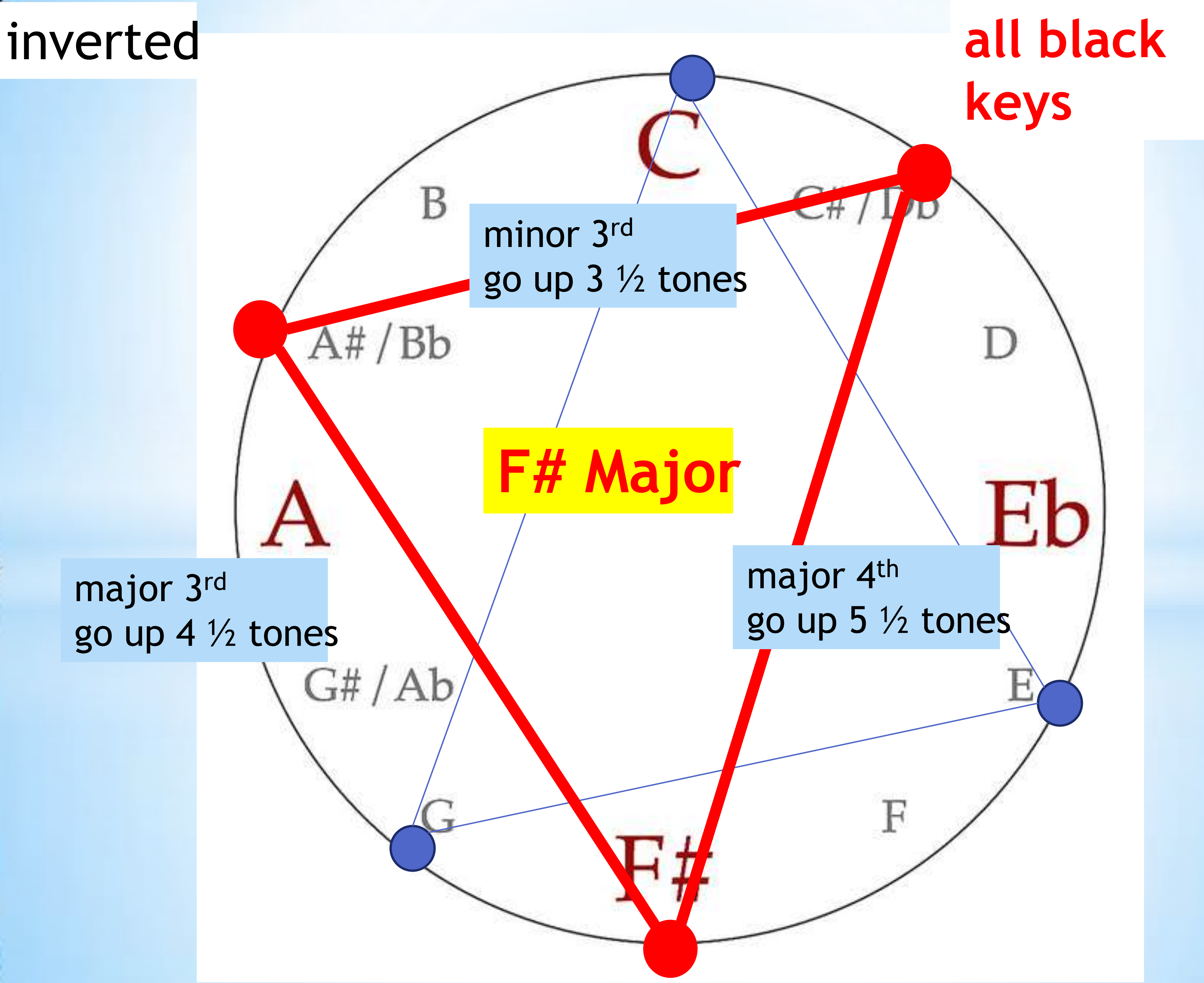


J.S. Bach: quintessential example of symmetry in western music

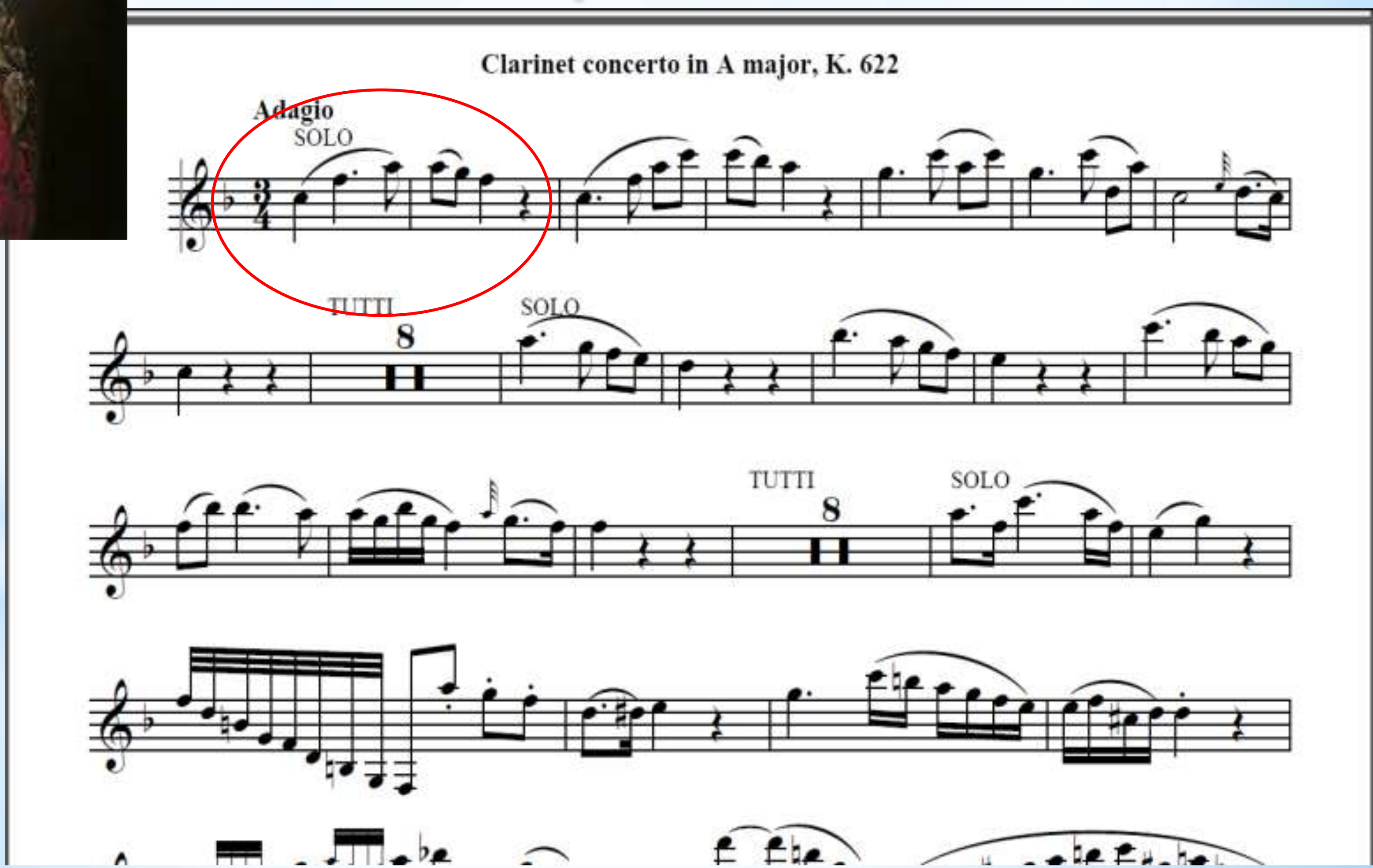
one of several ways to represent the 12 tone scale on a circle



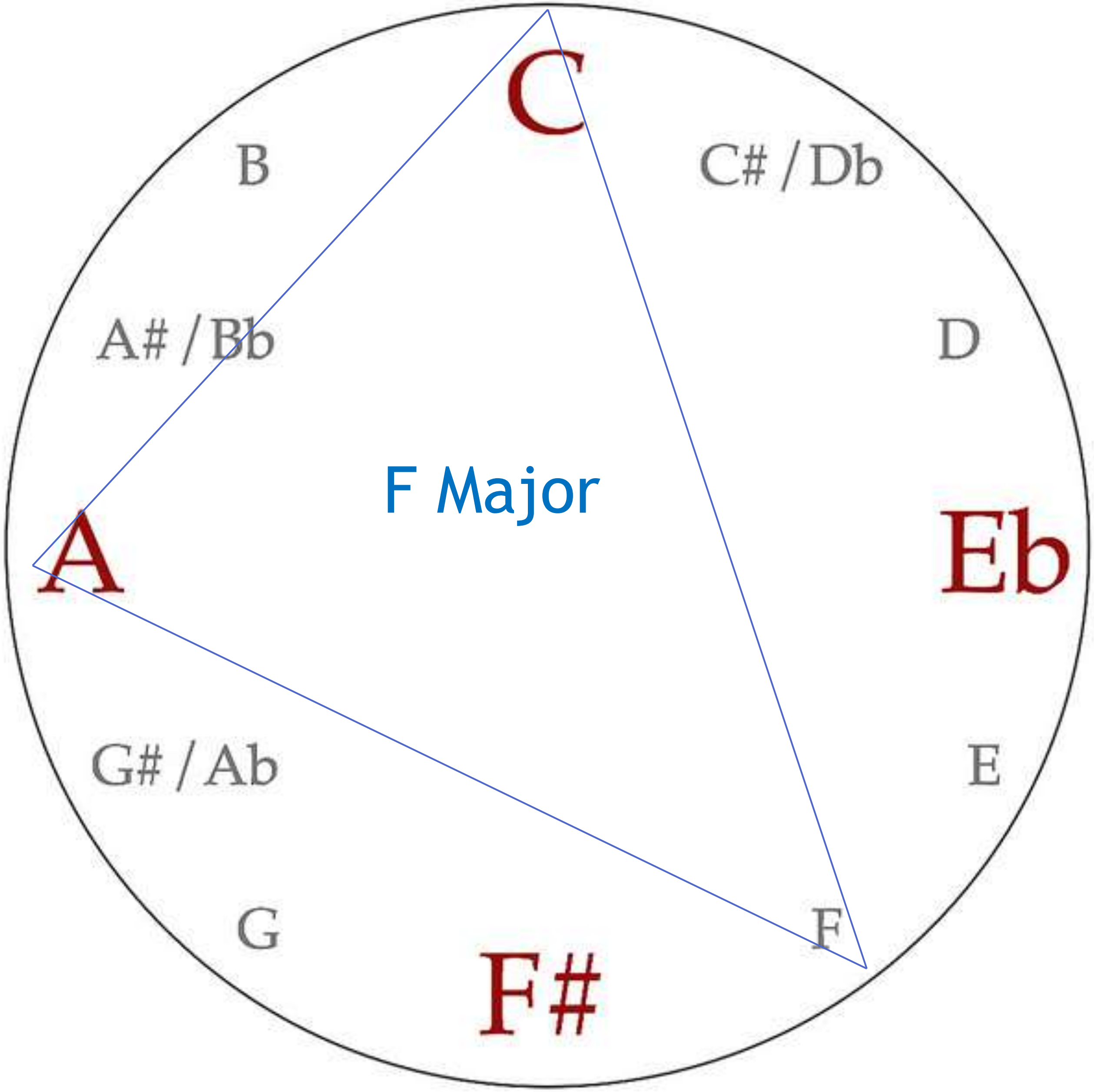

all white keys
C
C# / Db
D
Eb
E
F
F#
G
G# / Ab
A
A# / Bb
B
major 3rd
go up 4 ½ tones
major 4th
go up 5 ½ tones
C Major
minor 3rd
go up 3 ½ tones


inverted
all black keys
C
B
C# / Db
minor 3rd
go up 3 ½ tones
A# / Bb
D
F# Major
A
Eb
major 3rd
go up 4 ½ tones
major 4th
go up 5 ½ tones
G# / Ab
E
G
F
F#

Mozart clarinet concerto composed in 1791



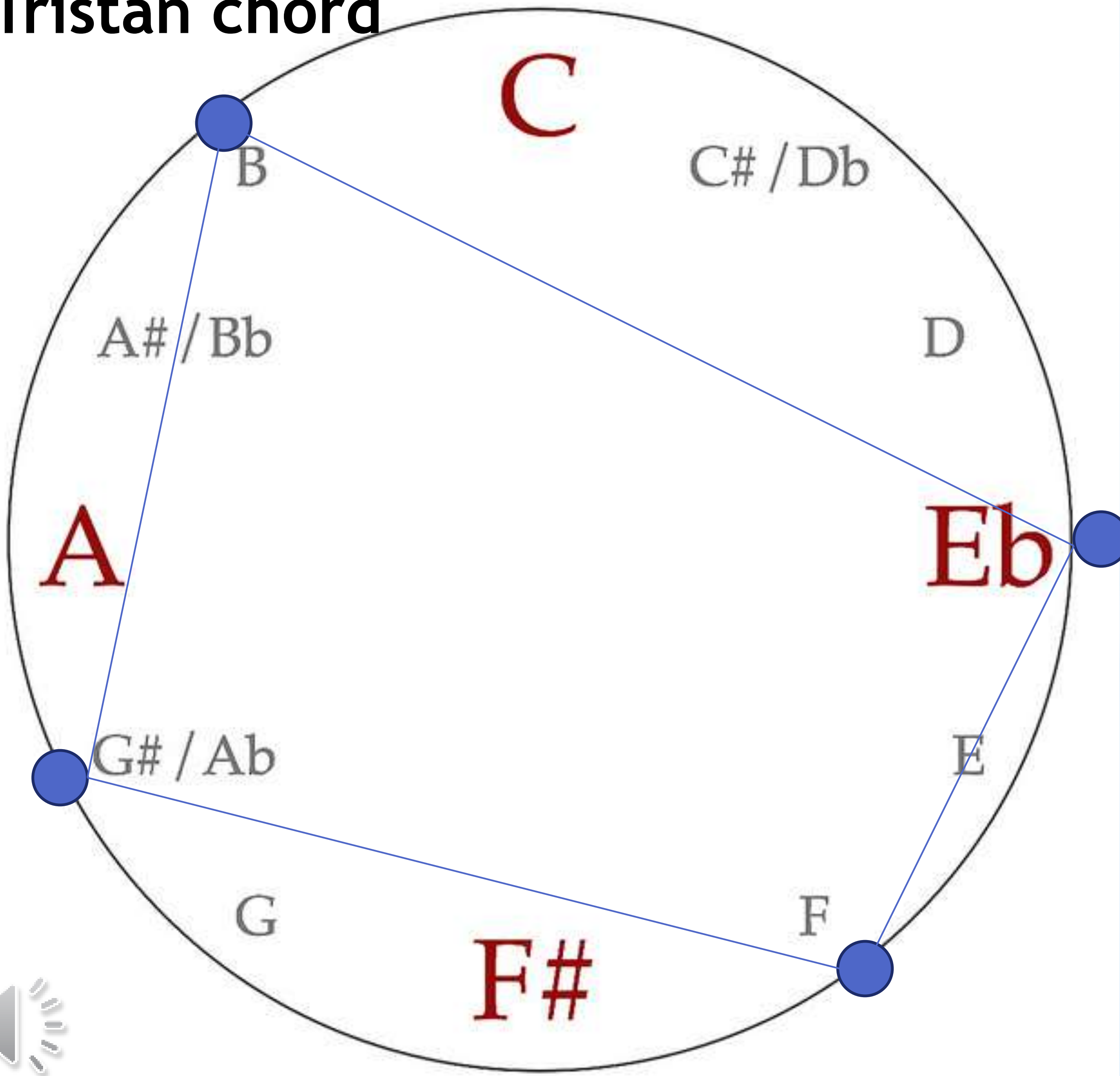
First 3 notes are F major chord starting on C



The Tristan Chord from Wagner's Tristan und Isolde

What distinguishes the chord is its unusual relationship to the implied key of its surroundings. When *Tristan und Isolde* was first heard in 1865, the chord was considered innovative, disorienting, and daring. Musicians of the twentieth century often identify the chord as a starting point for the modernist disintegration of tonality. (Wikipedia)

Tristan chord

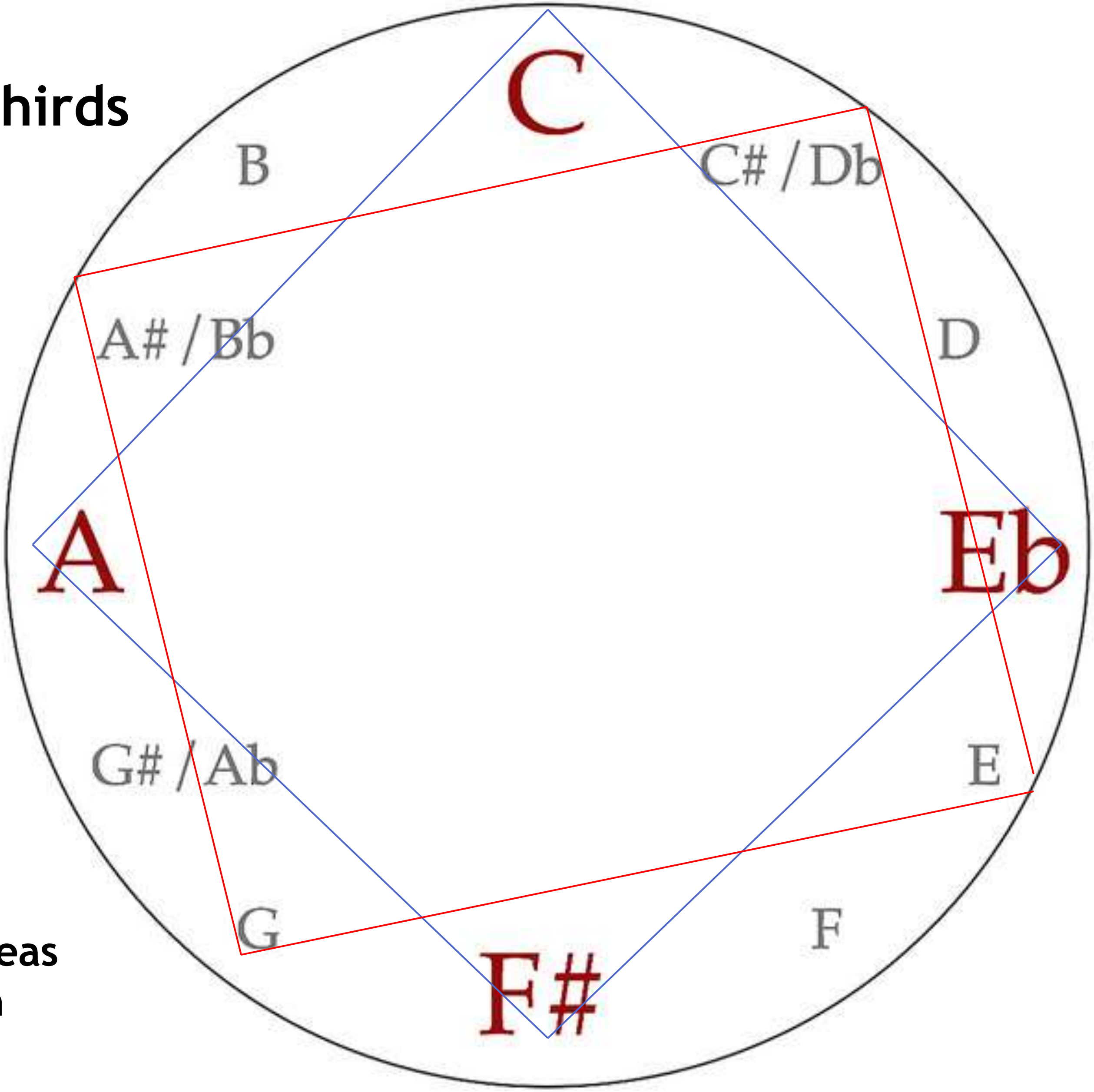


midi

orchestra

making musical shapes, playing with symmetry operations

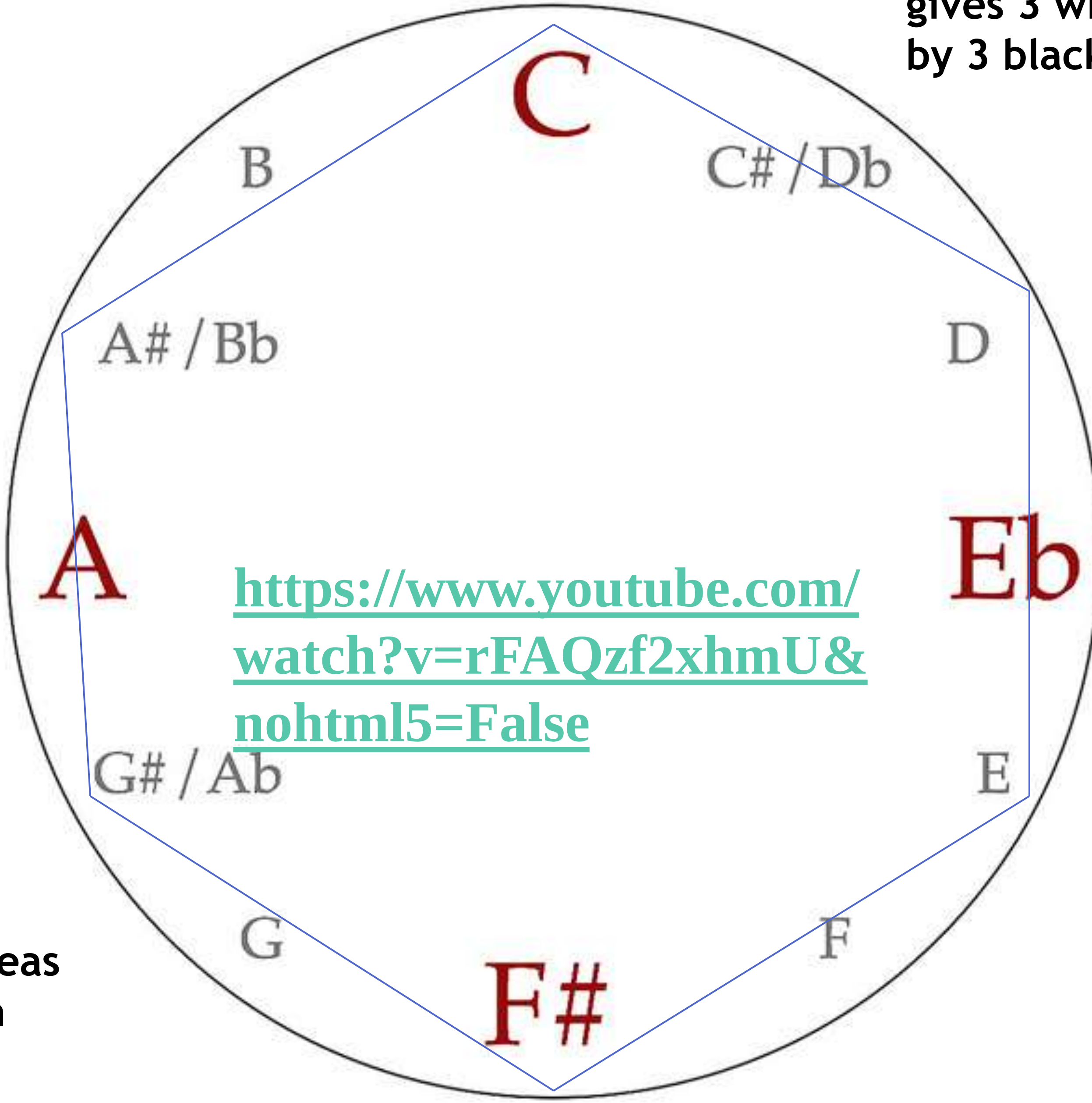
square =
all minor thirds



(potential ideas for your own symmetry demos)

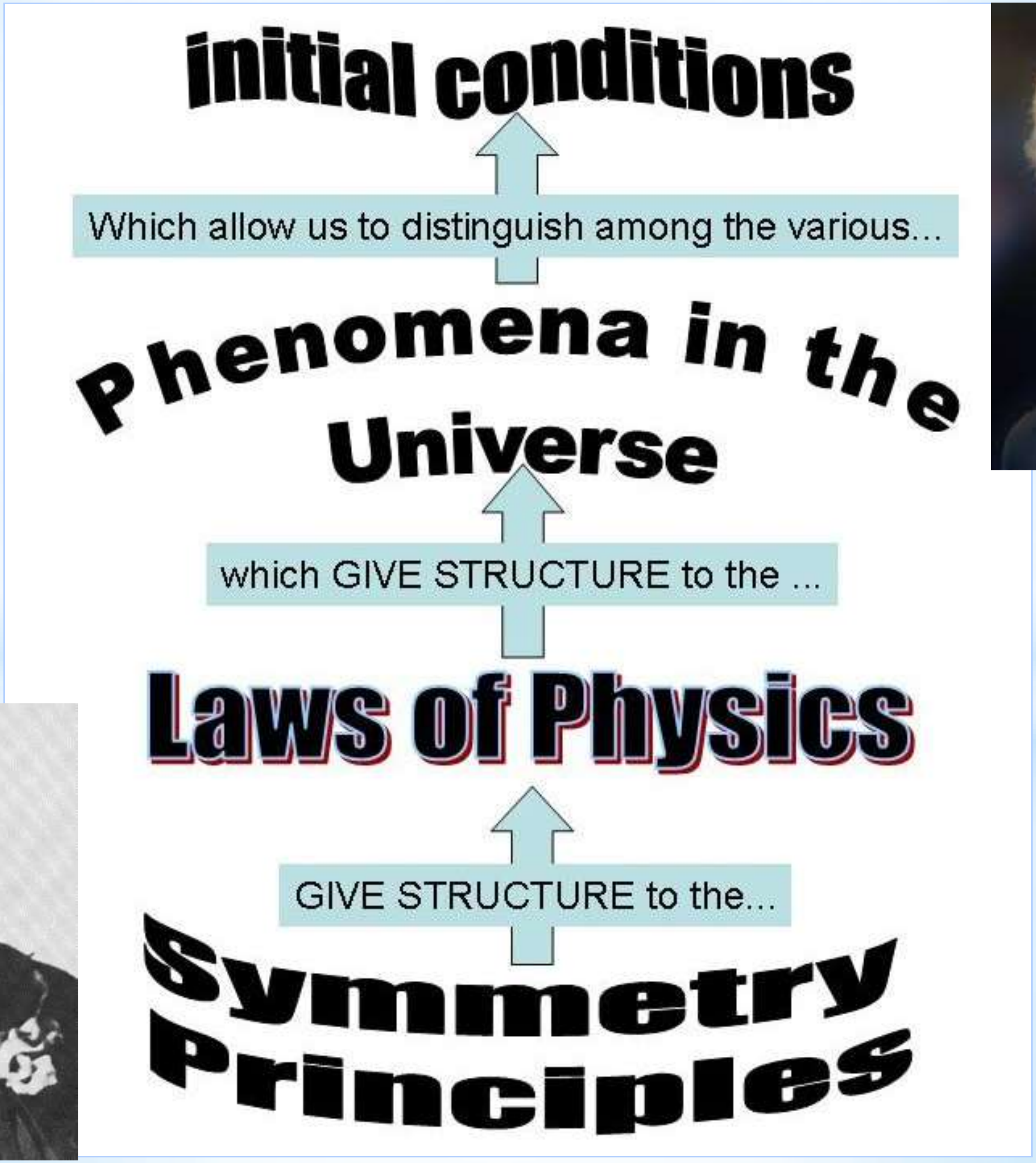
sound of a hexagon

hexagon starting on C gives 3 white followed by 3 black notes

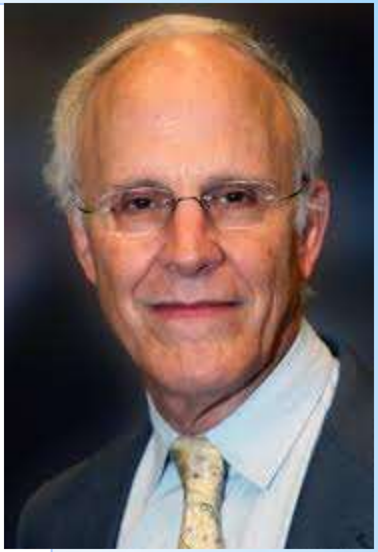


https://www.youtube.com/watch?v=rFAQzf2xhmU&nohtml5=False

(potential ideas for your own symmetry demos)

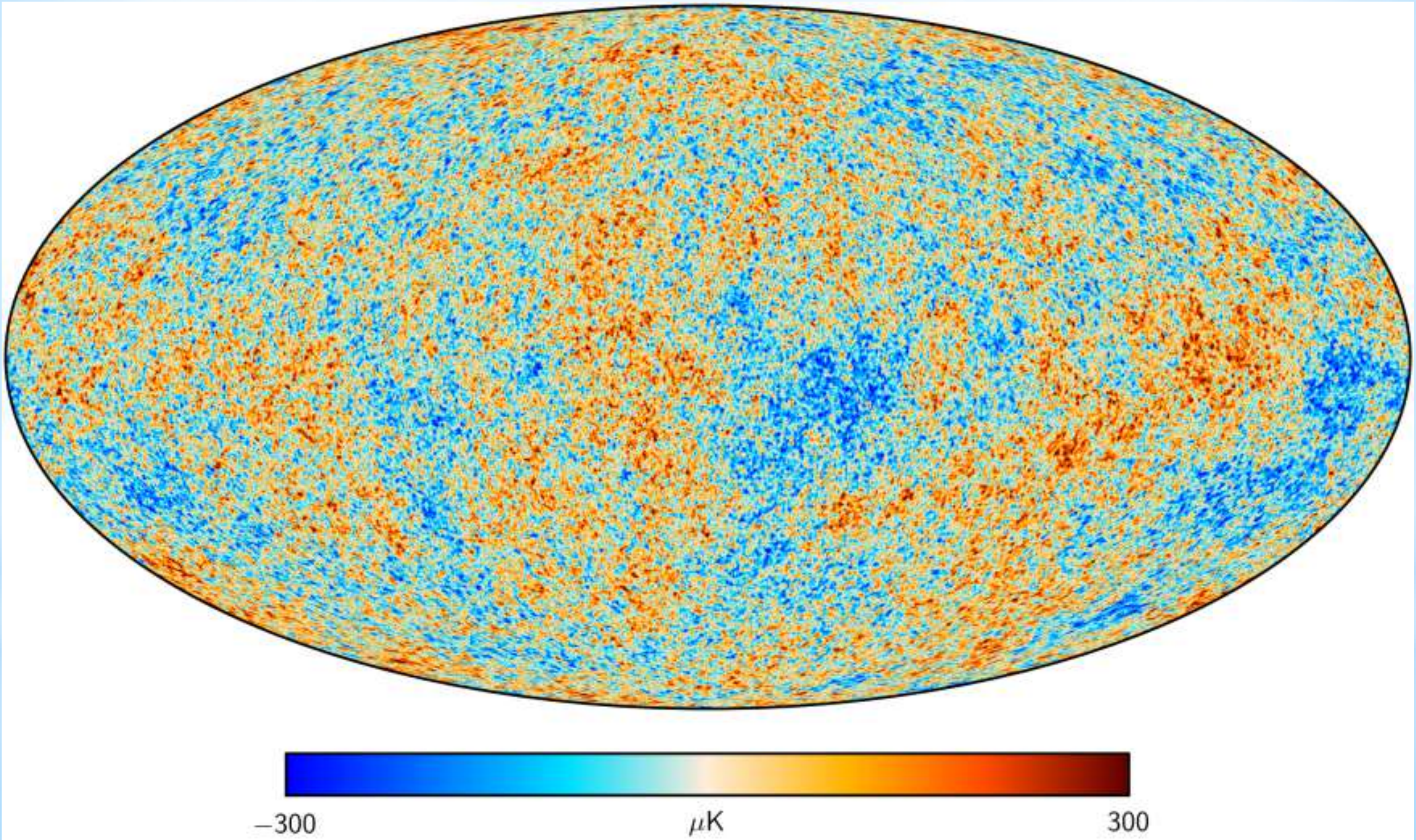


Noether's Principle



quote from Professor David Gross, 2004 Nobel Laureate and Director Emeritus of the KITP at UCSB

Sound waves from the very early universe, up to around 380,000 years old, manifest as patterns in the structure of the universe today.



What is the CMB?

The Cosmic Microwave Background (CMB) is the oldest light we can detect.

It comes from a time ~ 380,000 years after the Big Bang, approximately 13.7 billion years ago.

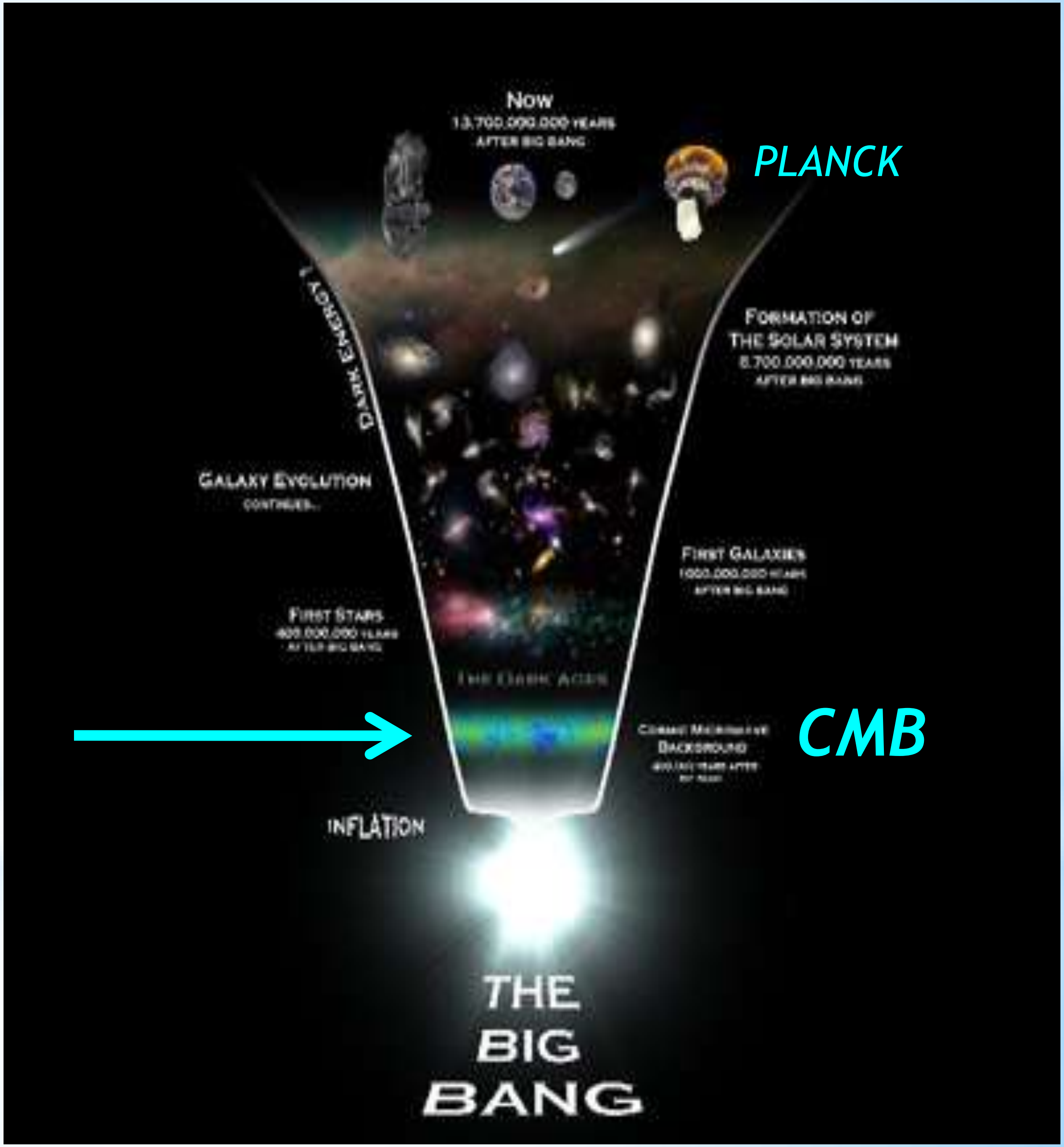
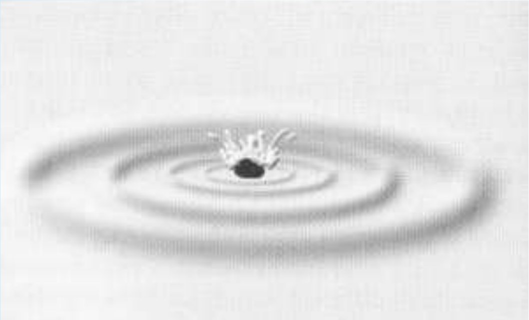
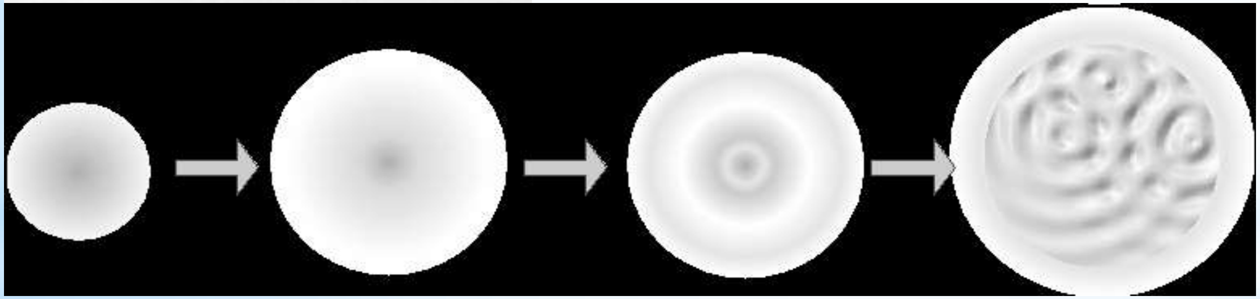
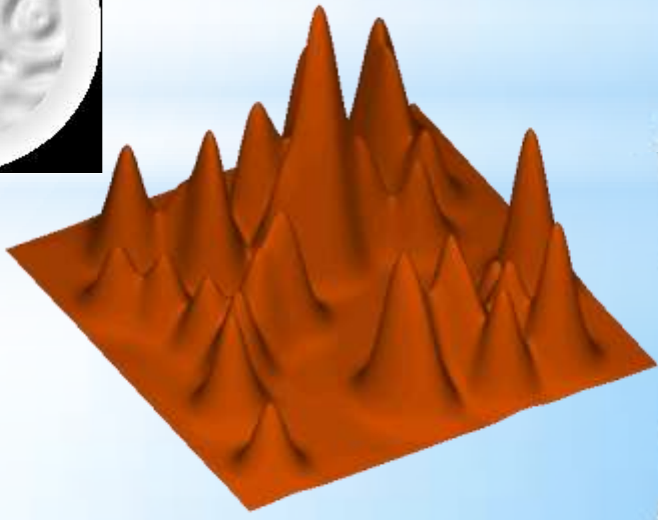


Image credit: Rhys Taylor, Cardiff University, Planck Collaboration

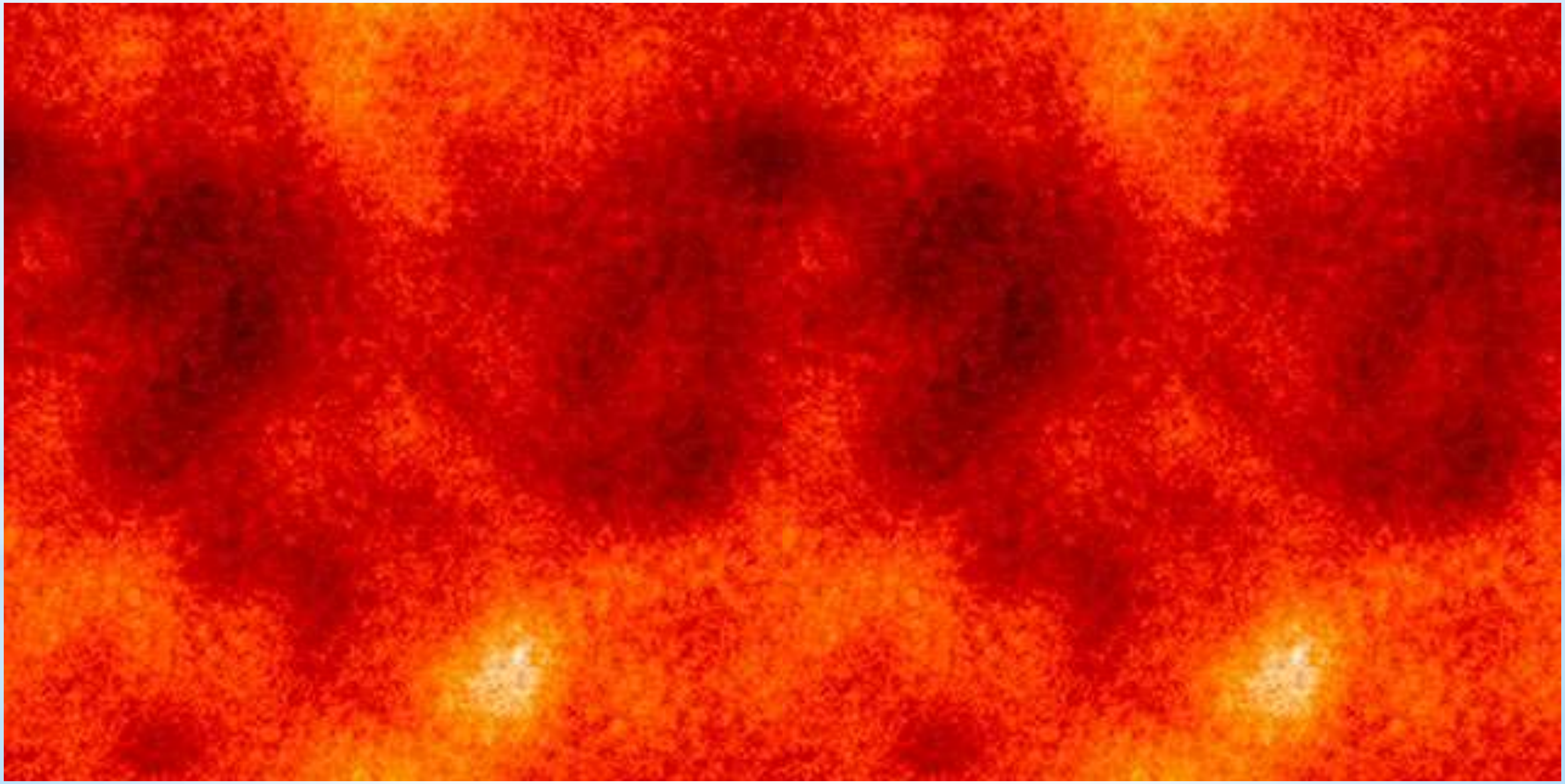
As we understand today, for the first 380,000 years of its existence the young expanding universe was filled with a plasma of tightly coupled photons and charged particles. Dark matter, which does not interact electromagnetically, collected first in pockets, and is believed to have initiated the growth of acoustic waves in the photon-baryon fluid.







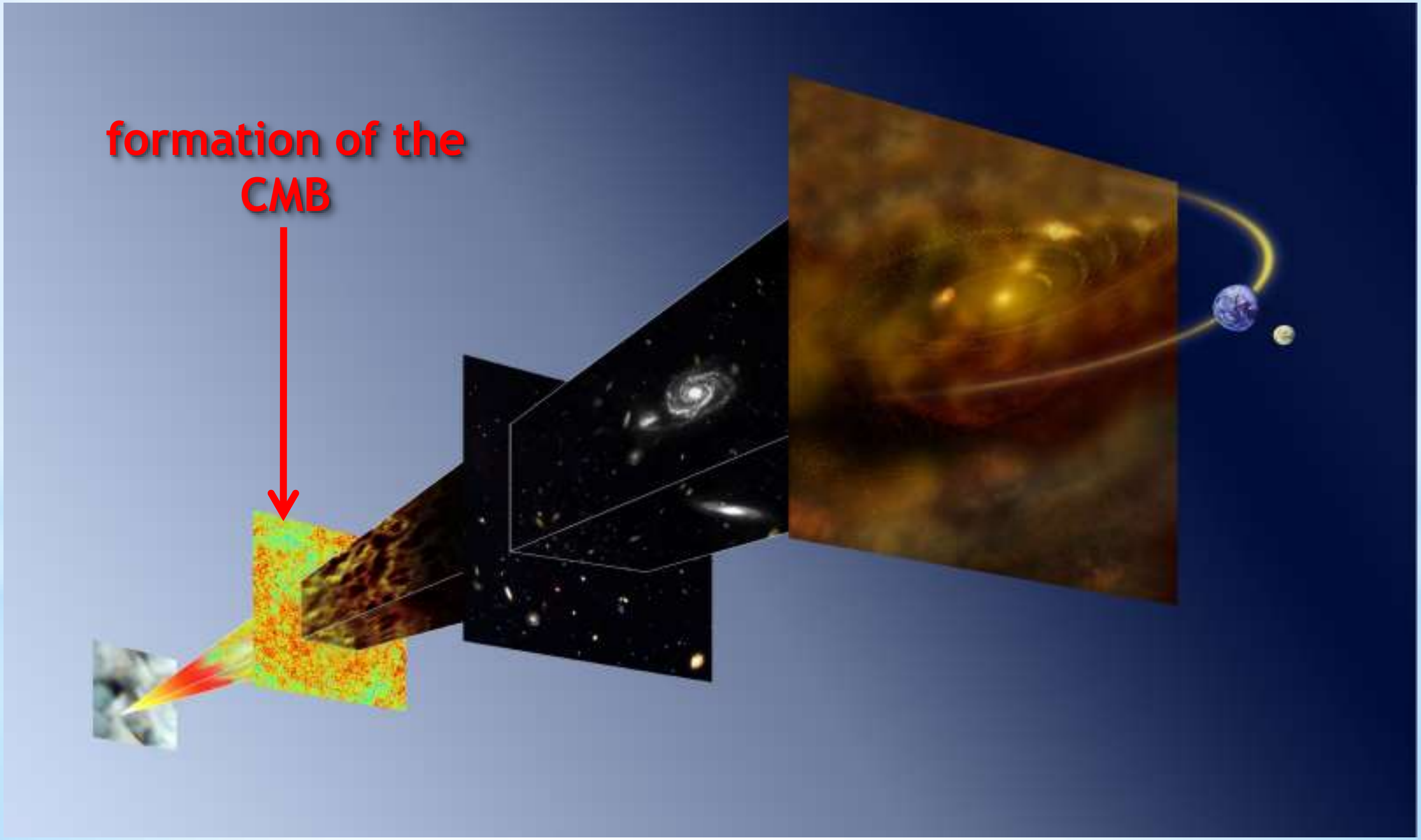
At around 380,000 years, the temperature of the expanding universe cooled to approximately 3000 Kelvin, the temperature below which hydrogen cannot remain ionized. Over a relatively short period, electrons combined with protons to form neutral hydrogen, and light started to travel freely.



This event in the history of the universe is often referred to as recombination. It is from this event in the history of the universe that we observe the

Cosmic Microwave Background

as the light that scattered off the primordial sound waves for the last time.



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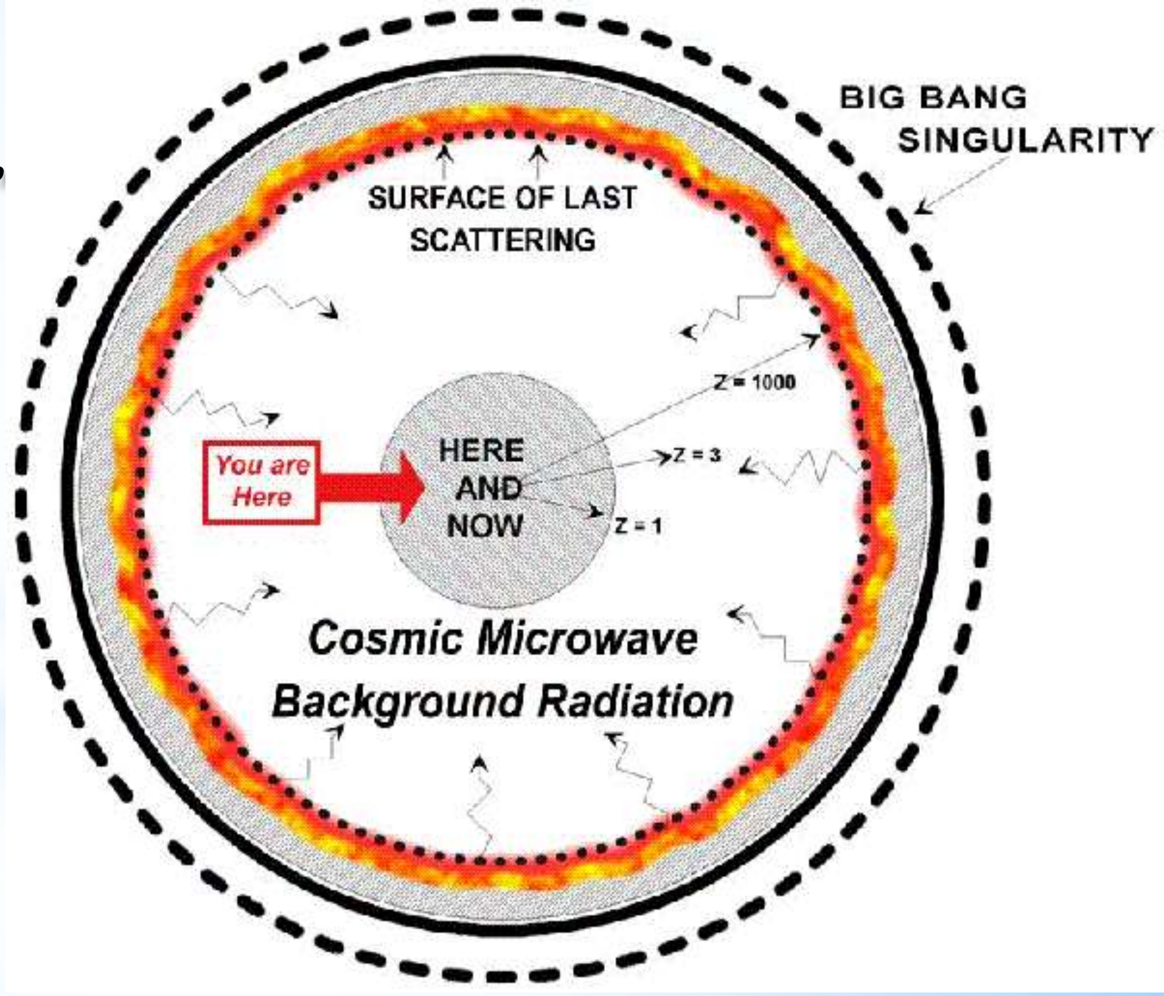
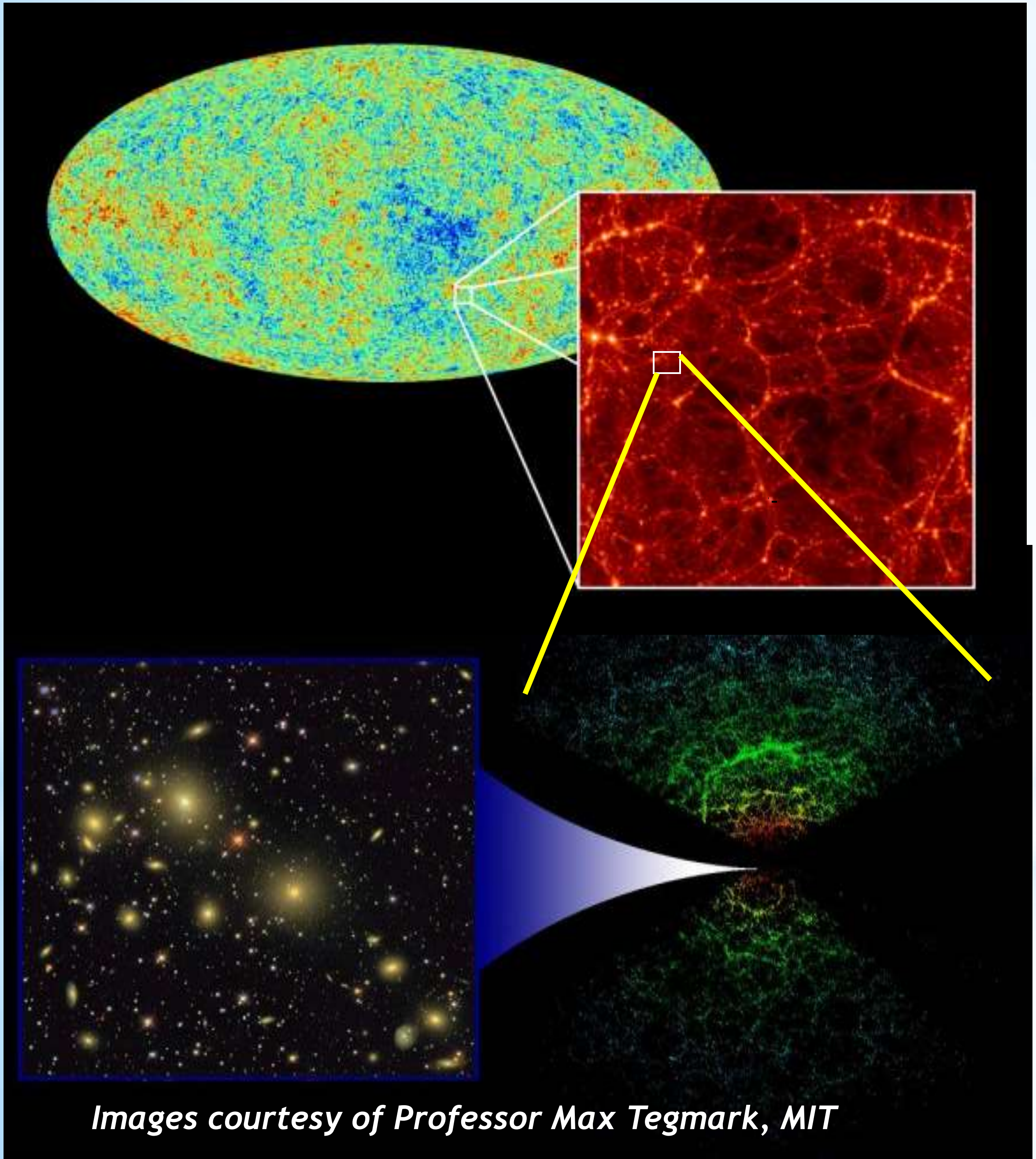
The small-amplitude fluctuations in the CMB are the “light echoes” of the primordial sound waves that permeated the very early universe.

Because the universe has expanded by a factor of ~ 1100 since that time, so have the wavelengths of this light, which we now see as microwaves rather than visible light.



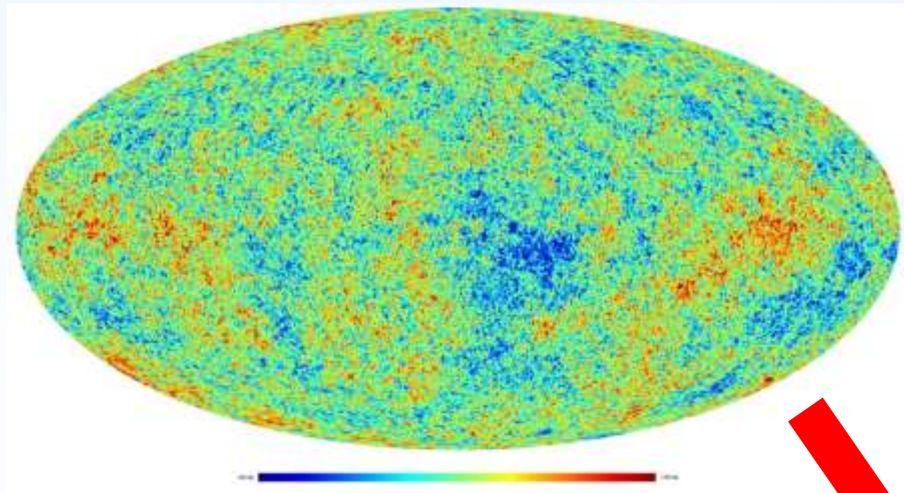


The variations in density left by these primordial acoustic waves were sufficient to eventually initiate the growth of structure in the universe.

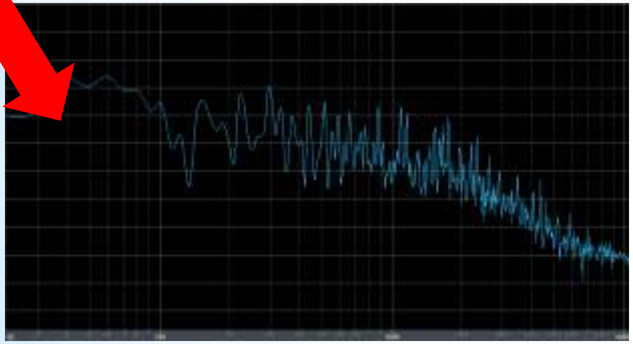
Thus, they are of great interest in figuring out the history of the universe.

Understanding the Power Spectrum of the CMB

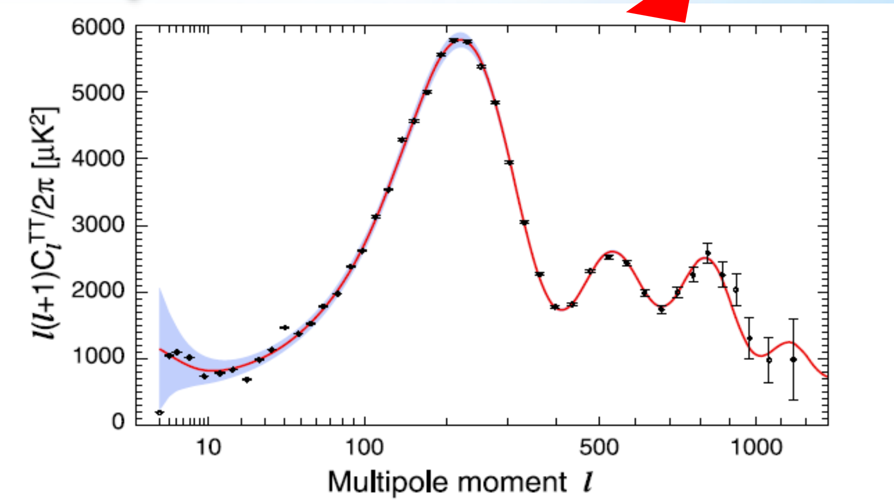
Just as the power spectrum of an instrument is determined by the physical properties of that instrument, the power spectrum of the temperature anisotropies of the CMB is controlled by more than 20 cosmological parameters.



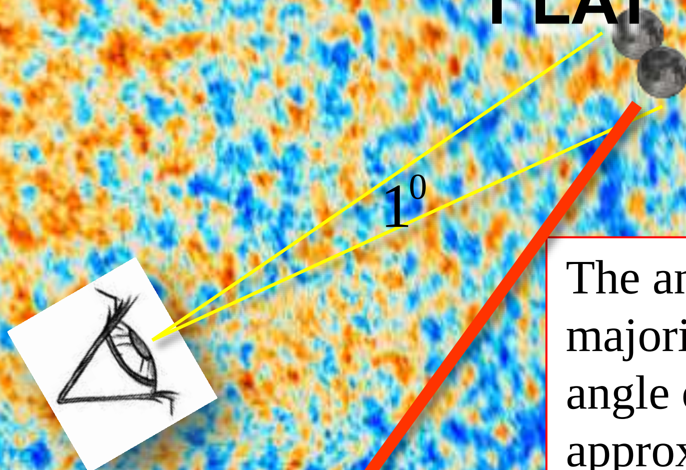
all frequencies over a certain Δt:



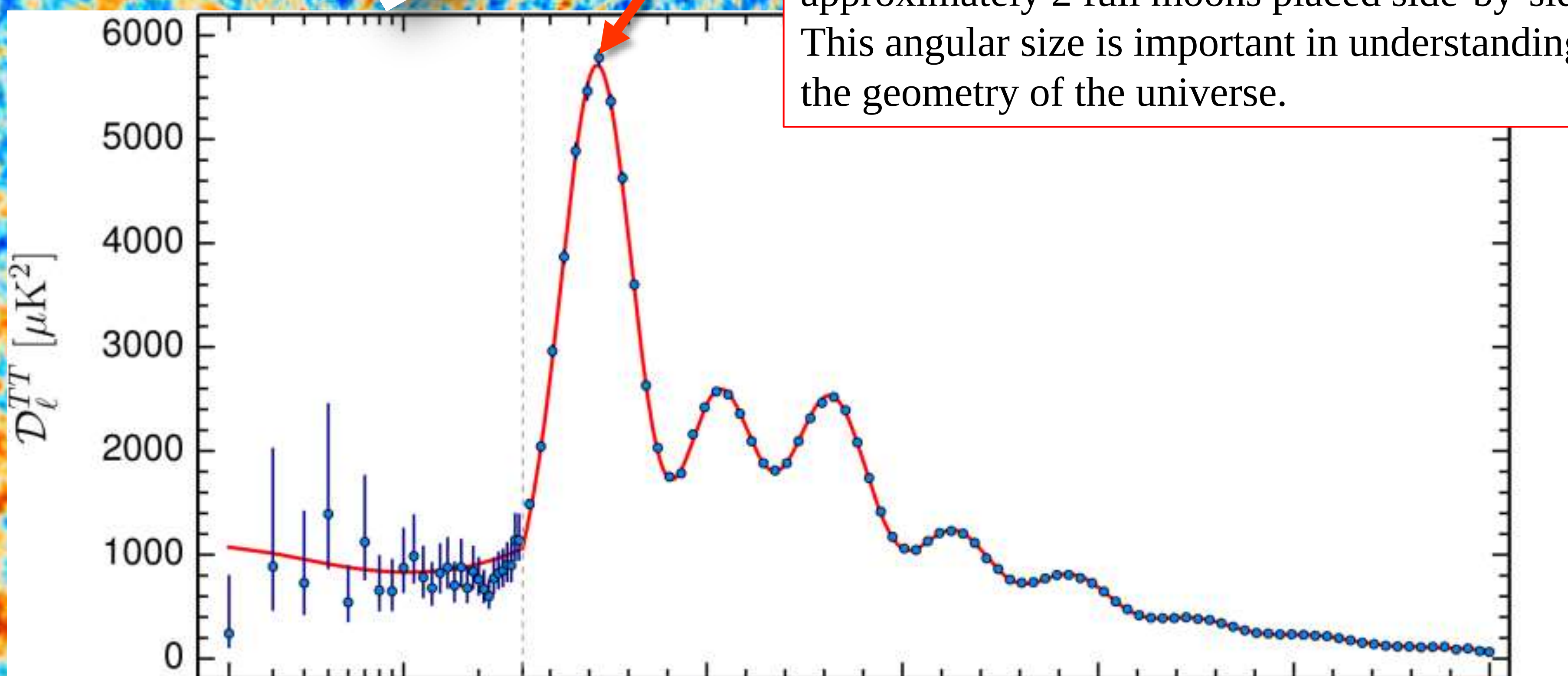
all space at one δt:



Planck confirms that the geometry of the Universe is FLAT



The angular scale of first peak shows that the majority of temperature fluctuations subtend an angle of ~1^0 on the sky today, which is approximately 2 full moons placed side-by-side. This angular size is important in understanding the geometry of the universe.



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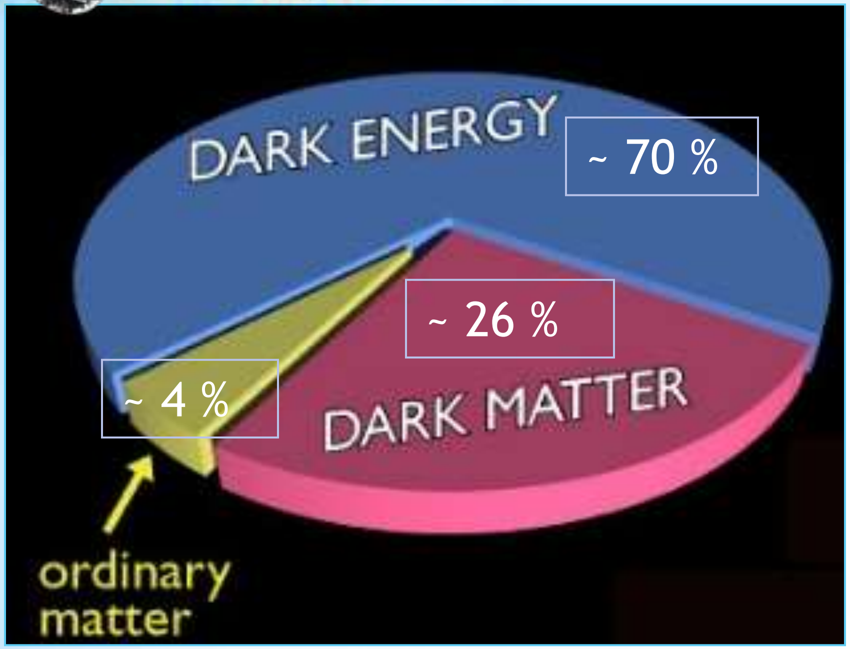
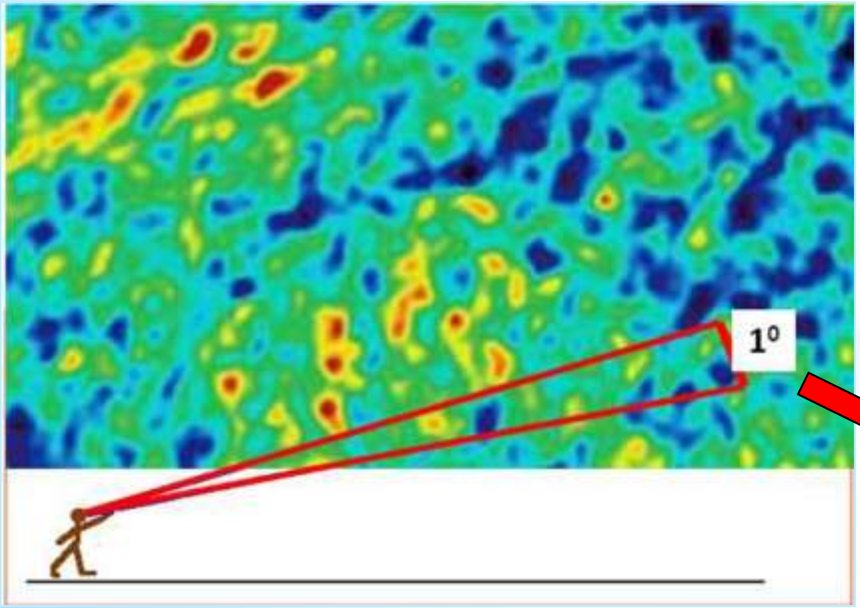
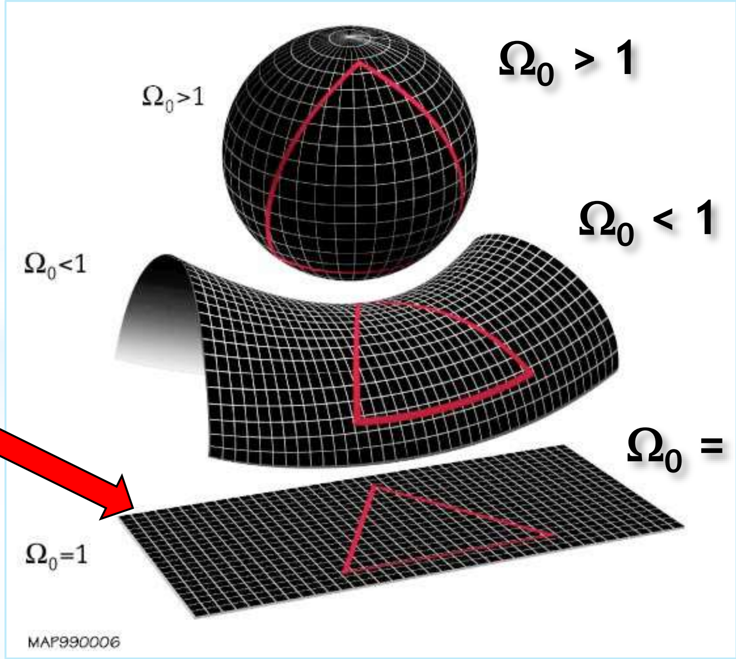


The wavelength of the fundamental at recombination provides a cosmic meter stick with which to measure the curvature of the universe.

$$\Omega_0 = \Omega_b + \Omega_\Lambda + \Omega_{cdm}$$
$$\Omega_0 = (\rho_b + \rho_\Lambda + \rho_{cdm}) / \rho_{critical}$$





The power spectrum of the temperature map represents the angular power present at all spatial wavelengths in the CMB in our observable universe 'today,' expressed as a spherical harmonic expansion.

$$T(\theta, \varphi) = \Sigma a_{l,m} Y_l^m(\theta, \varphi)$$

$-90^0 < \theta < 90^0$
$0^0 < \varphi < 360^0$

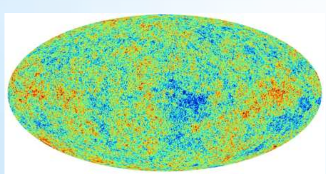
$$Y_l^m(\theta, \phi) \equiv \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} P_l^m(\cos\theta) e^{i m \phi},$$

...

$a_{\ell,m}$ are the expansion coefficients

$\mathcal{P}_\ell^m$ are the associated Legendre Polynomials

m varies from $-\ell$ to $+\ell$, and the sum is taken over all possible m's



$C_\ell = <|a_{\ell,m}|^2>$ = squared avg. of expansion coeff.'s for any ℓ, averaged over all m's

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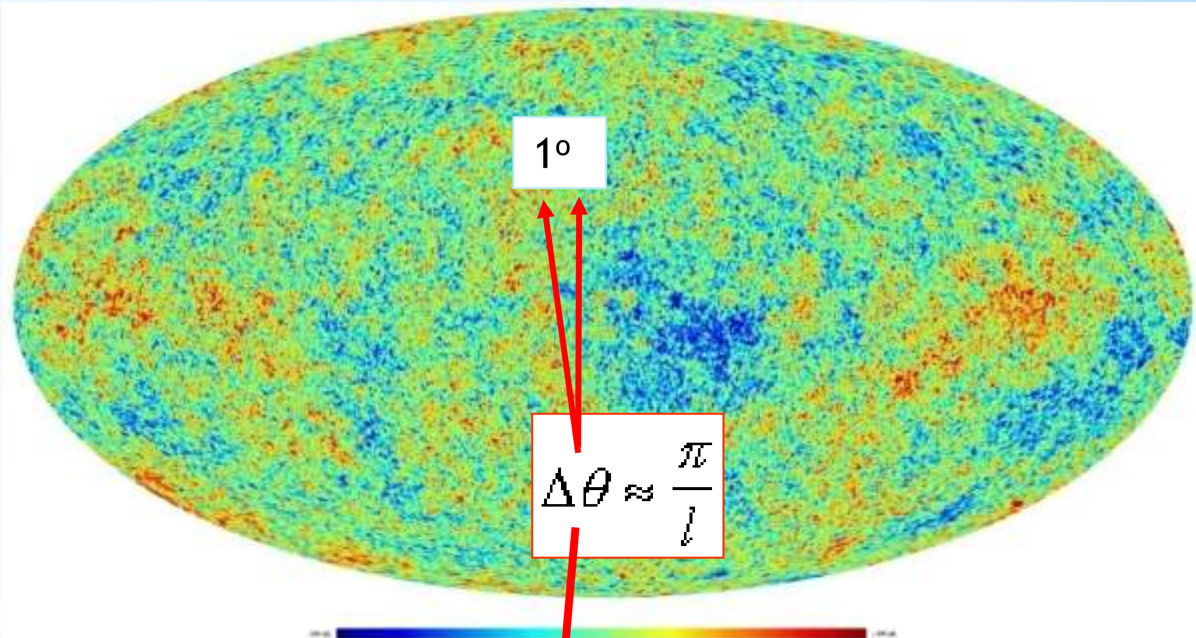
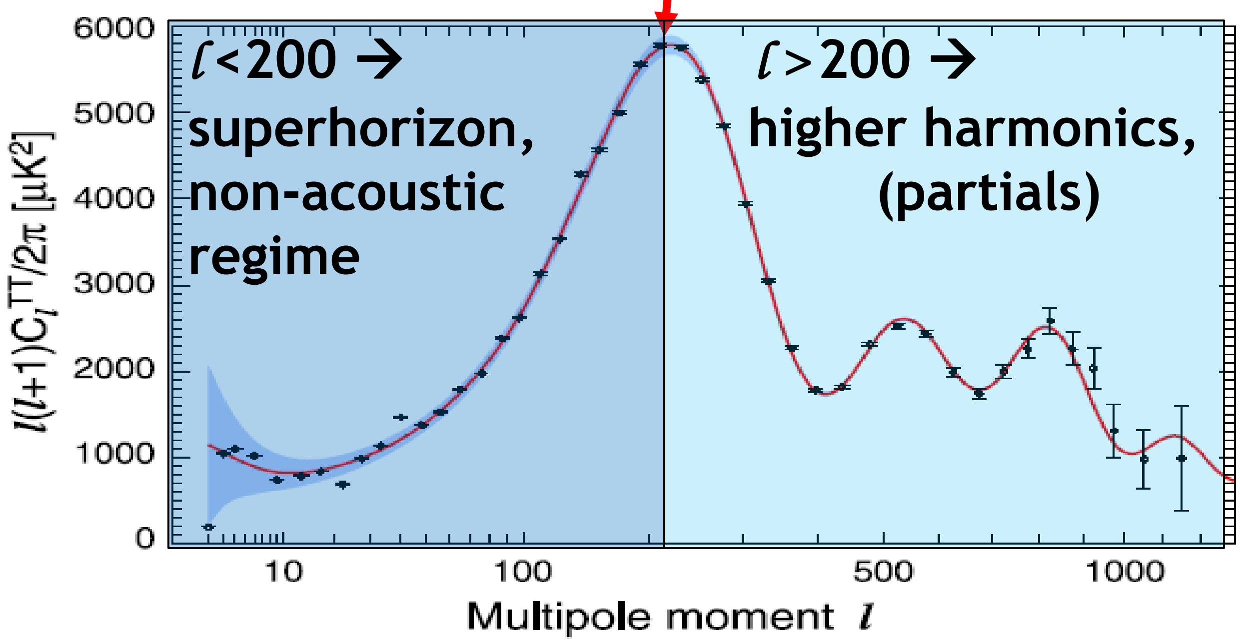
Understanding the Power Spectrum of the CMB

ℓ ≅ *200* corresponds to 1^0 on the sky "today."

This angular scale represents the *sound horizon* at t = 380,000 years.

The first peak in the power spectrum at *ℓ* ≅ *200* represents the fundamental tone of the universe.

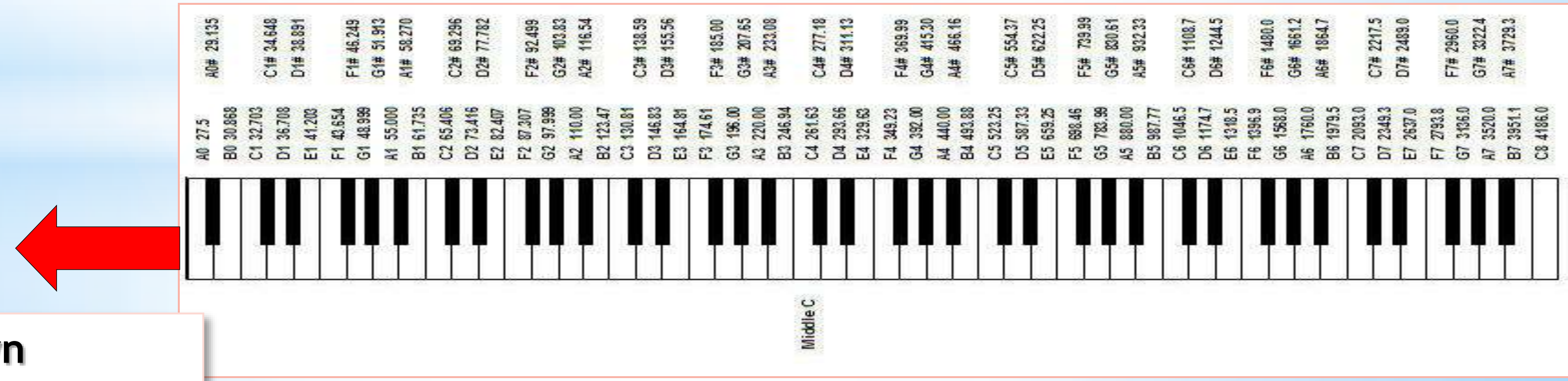




The fundamental is the longest wave that began sometime in the beginning of the universe, and was just reaching the edge of its observable universe at recombination. It is thus an important measurement for understanding the geometry of the universe.

Distance the longest wave could have crossed at recombination = v_{sound} t = (.6c) (380,000 years) = 2 x 10^{21} meters, or ~ 228,000 light years = half a wavelength

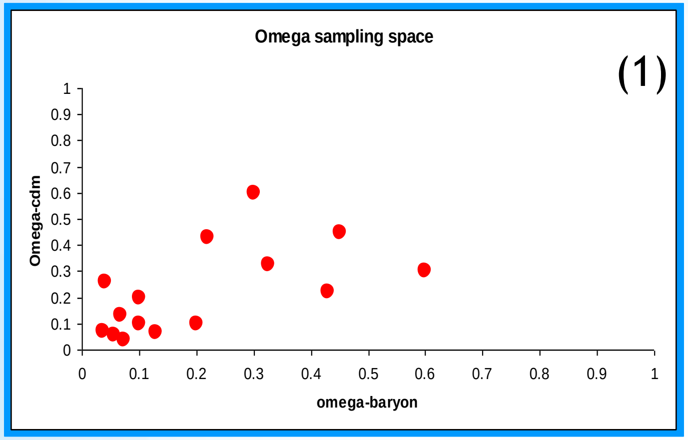
⇒ Fundamental wavelength was ~ 450,000 light years, which corresponds to a frequency of ~ 7 x 10^{-14} Hz, or slightly more than 49 octaves below the lowest note on the piano (27 Hz).

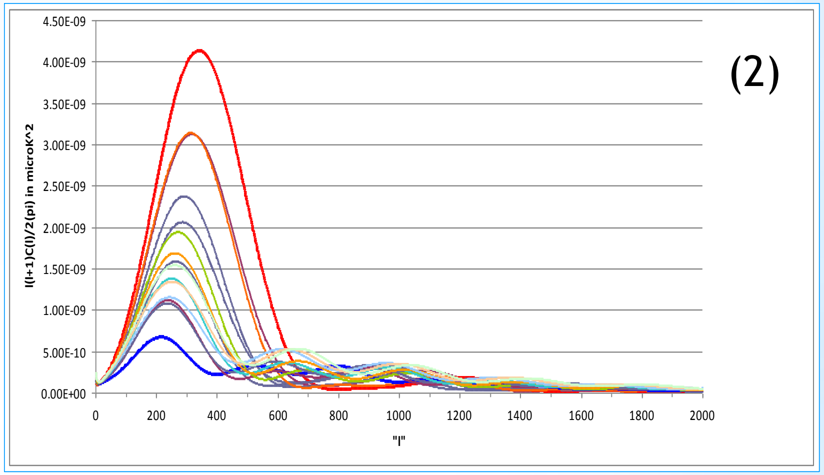


Go down
49 more octaves

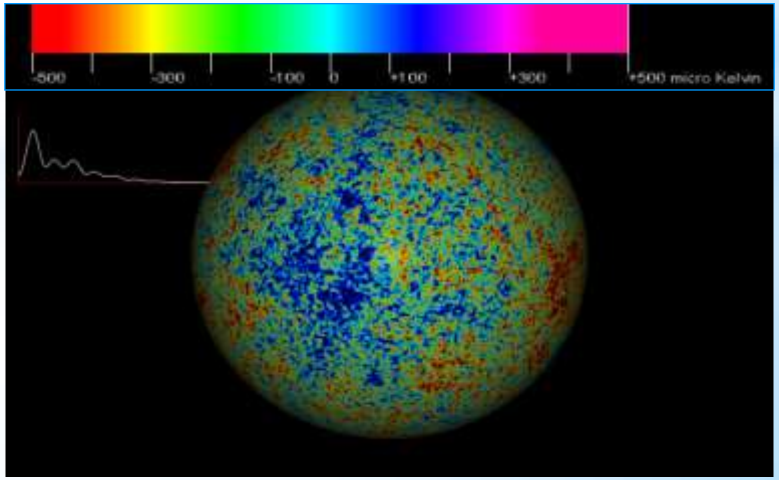
Creating the Interactive Visualization

We used the software package CAMB to create 15 model universes, keeping the total density parameter $\Omega_0 = 1$ for a flat universe, varying the relative proportions of baryons, dark matter, and dark energy, and keeping the other cosmological parameters at the current best estimates. We used the HEALPIX algorithm to create the maps.

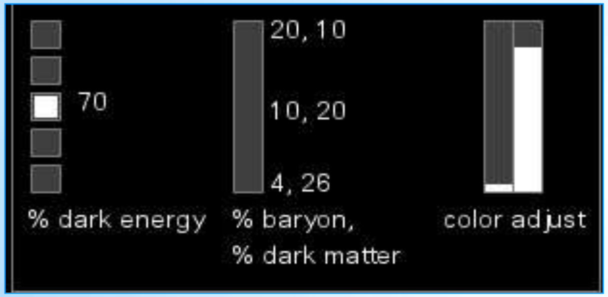




(1): Omega sampling space;
(2): 15 models on one plot;
(3): mapping the CMB on a sphere;
(4): choose model, set color-temp.



(3)

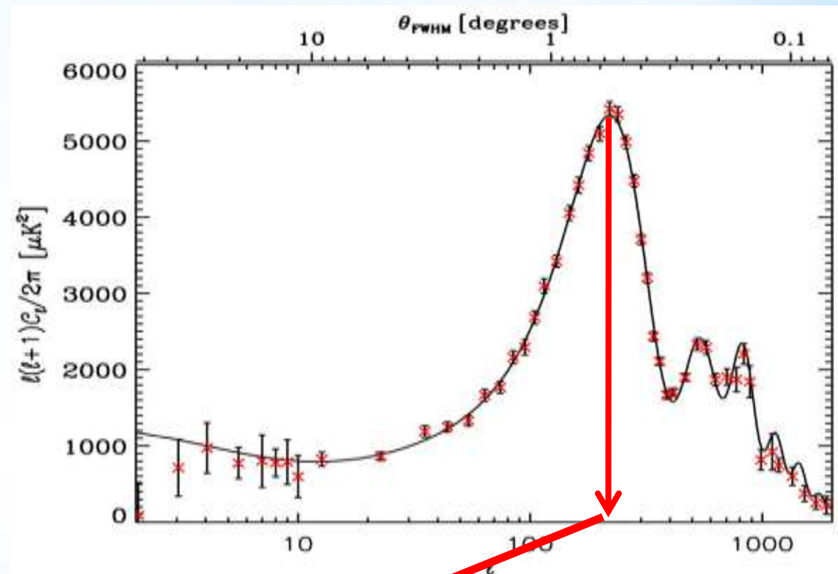


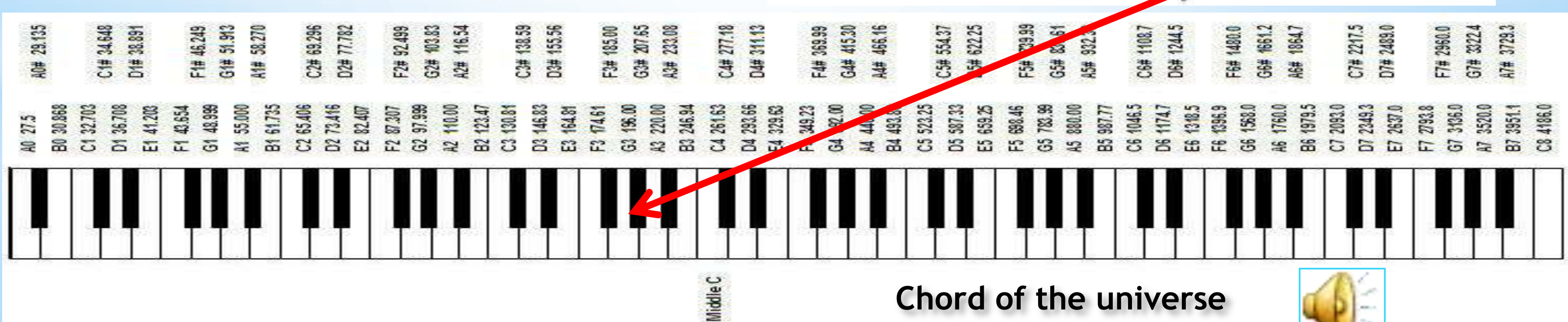
(4)

Creating the Sonification

1. We map angular wave number to Hertz, so the fundamental at *l* ~200 maps to a frequency of ~200 Hz. The higher harmonics damp out at approximately 0.08 degree, corresponding to *l* = 3000, which maps to 3 kHz in our scheme.

Conceptually, this mapping amounts to scaling up the primordial sound waves by approximately 52 octaves.





Chord of the universe played on a synthesizer

Symmetries in the complex plane – "hidden" internal mathematical symmetries - manifest as physical vibrations in space and time through the medium of sound.

examples of physical manifestation of sound waves on a flat plate

example of physical manifestation of sound waves on a spherical water droplet

So…

What do you take away from all this about symmetry, aesthetics, thinking, physics, and reality?

For next week:

Readings:

Jim Hartle – Theories of Everything

Reading reflection:
Reflect on this article, and anything else you wish related to what we've discussed so far.

https://www.youtube.com/watch?v=UTby_e4-Rhg