

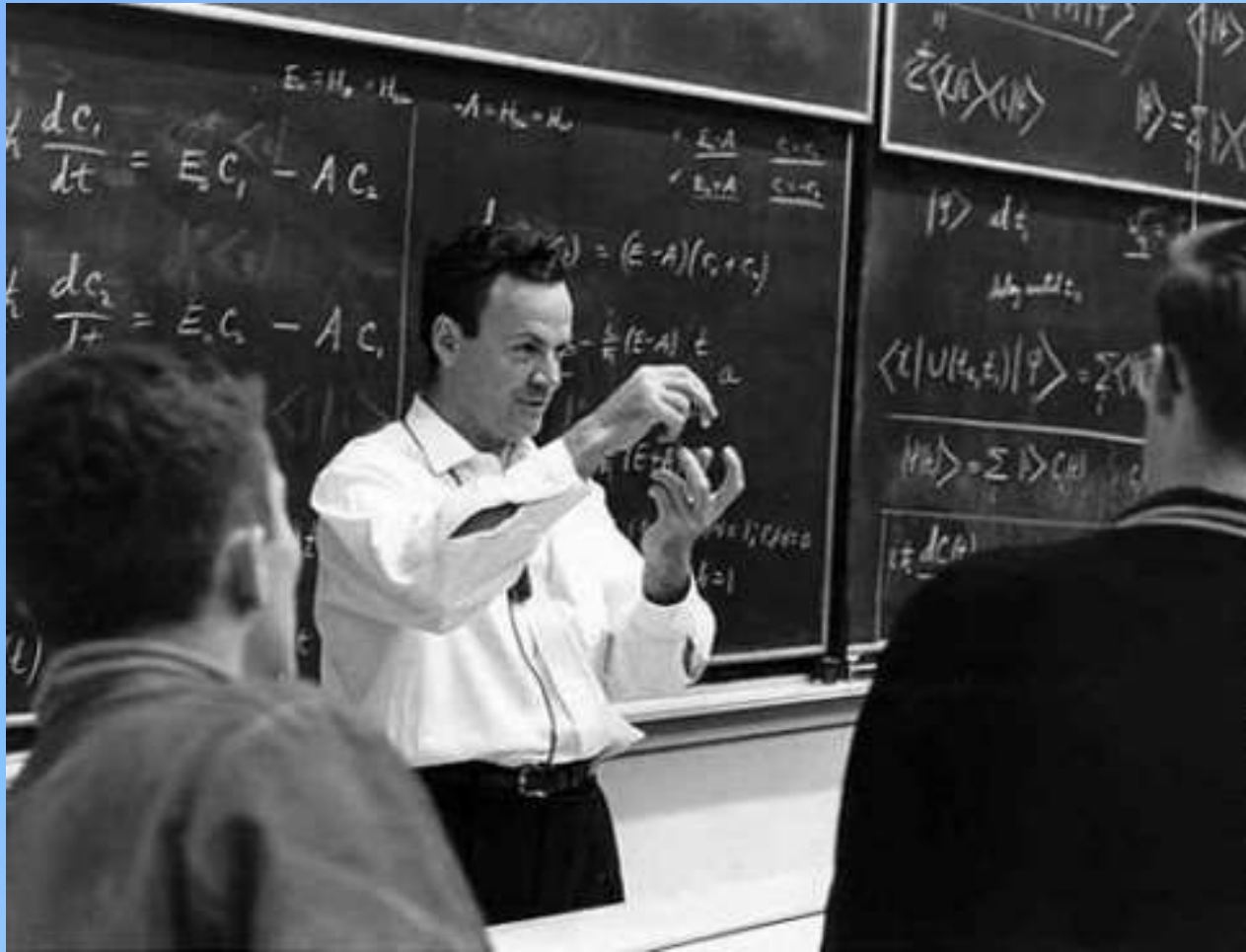
Symmetry and Aesthetics in Contemporary Physics

CS-10, Spring, 2016

Dr. Jatila van der Veen

**CLASS 5:
SYMMETRY IN PHYSICAL LAWS**

Introducing Richard Feynman: A curious character!



https://www.youtube.com/watch?v=QkhBcLk_8f0

Feynman: Symmetry in Physical Laws

Questions? Comments?

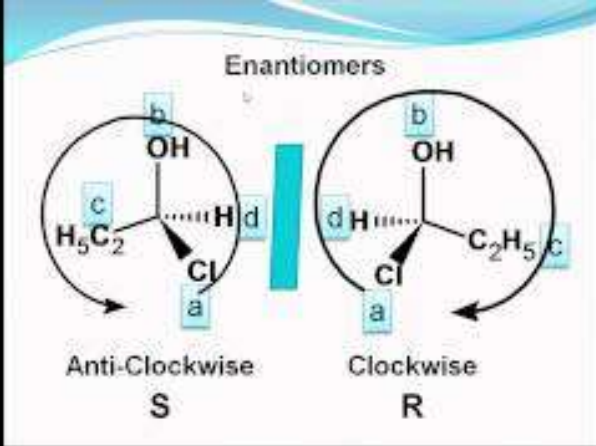
What parts did you find most interesting?

What parts did you perhaps not understand?

What parts do you perhaps not agree with?

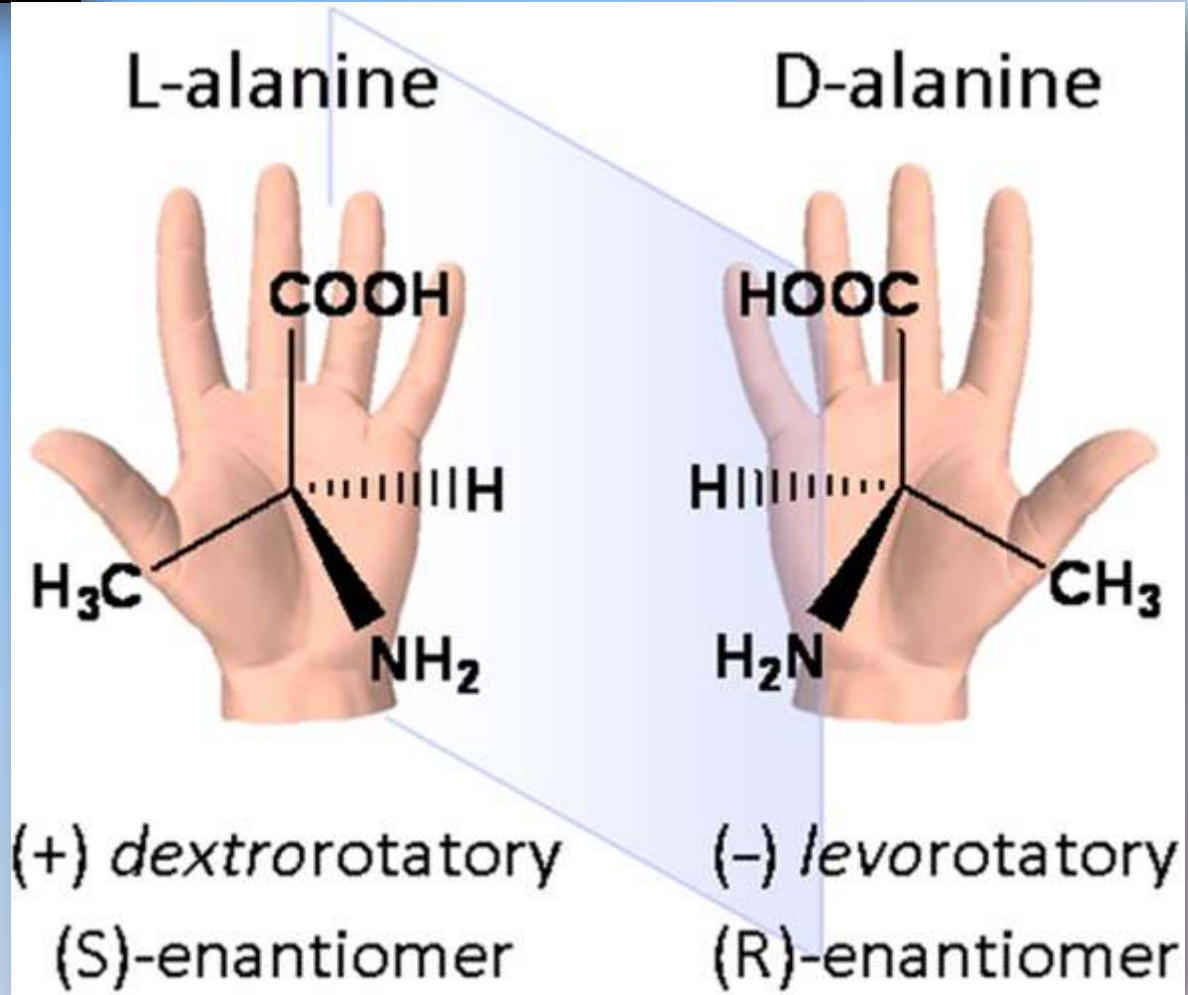
What do you think of his ending?





right-handed and left-handed amino acids are utilized differently in living things

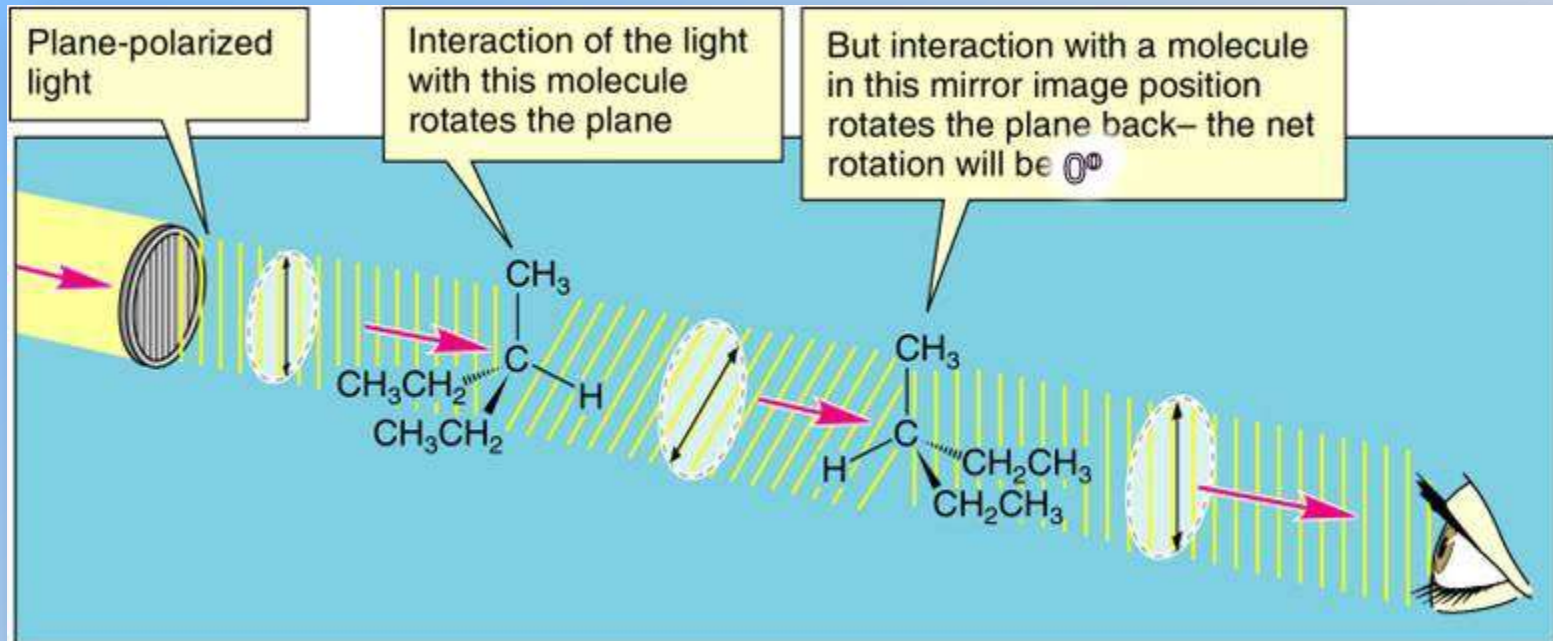
broken symmetry
or
chance of
initial conditions?

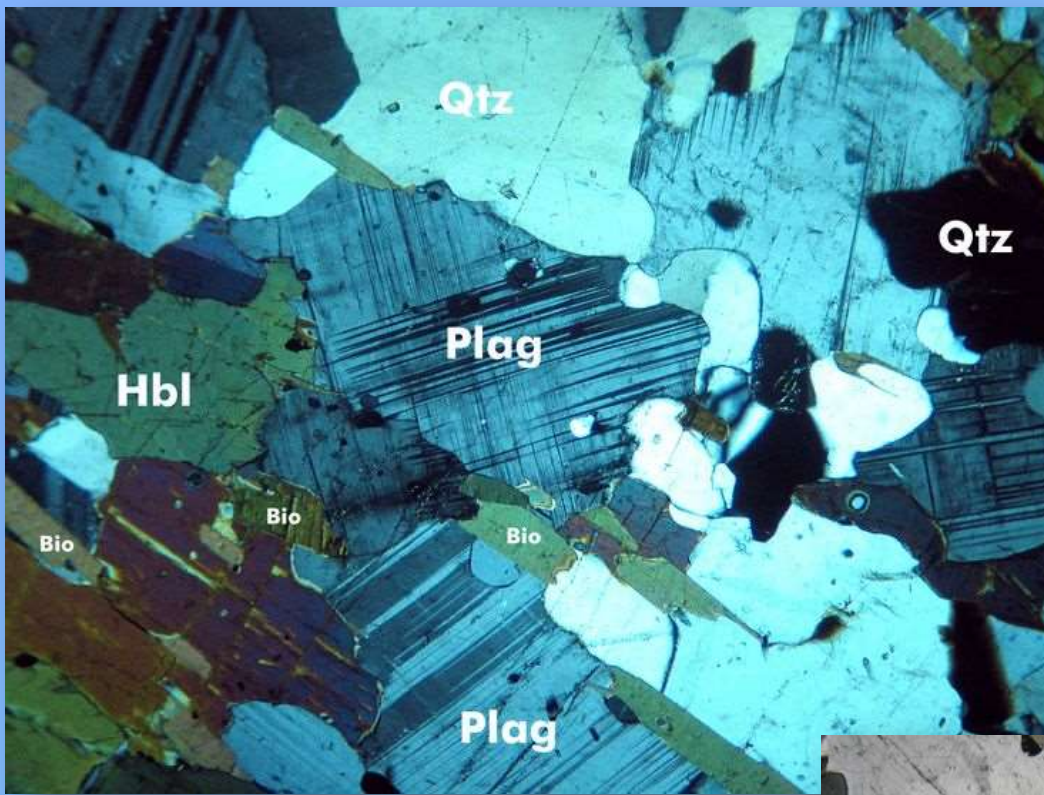




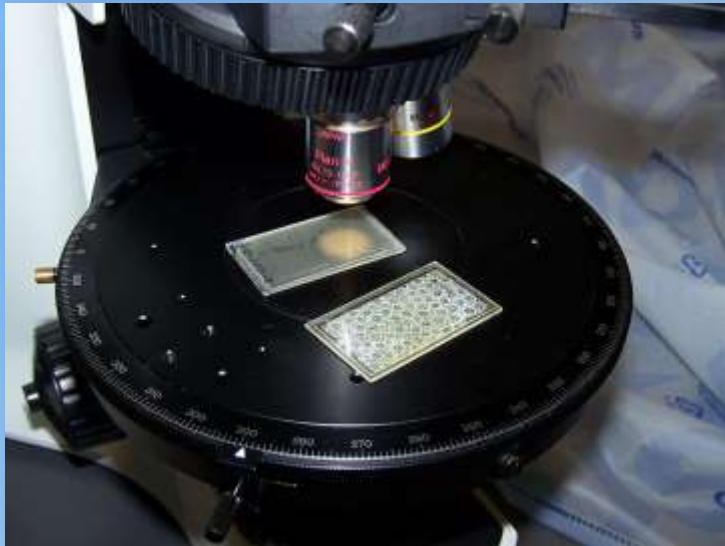
natural
sugar solutions
rotate polarized light
to the right (demo)

p. 68
in reader
(52-5,
Feynman)





In thin section, certain minerals rotate plane-polarized light in characteristic patterns and colors



Professor Chien-Shiung Wu
Wú Jiànxíóng

*mirror symmetry is not fundamental
without reversing the charge!*

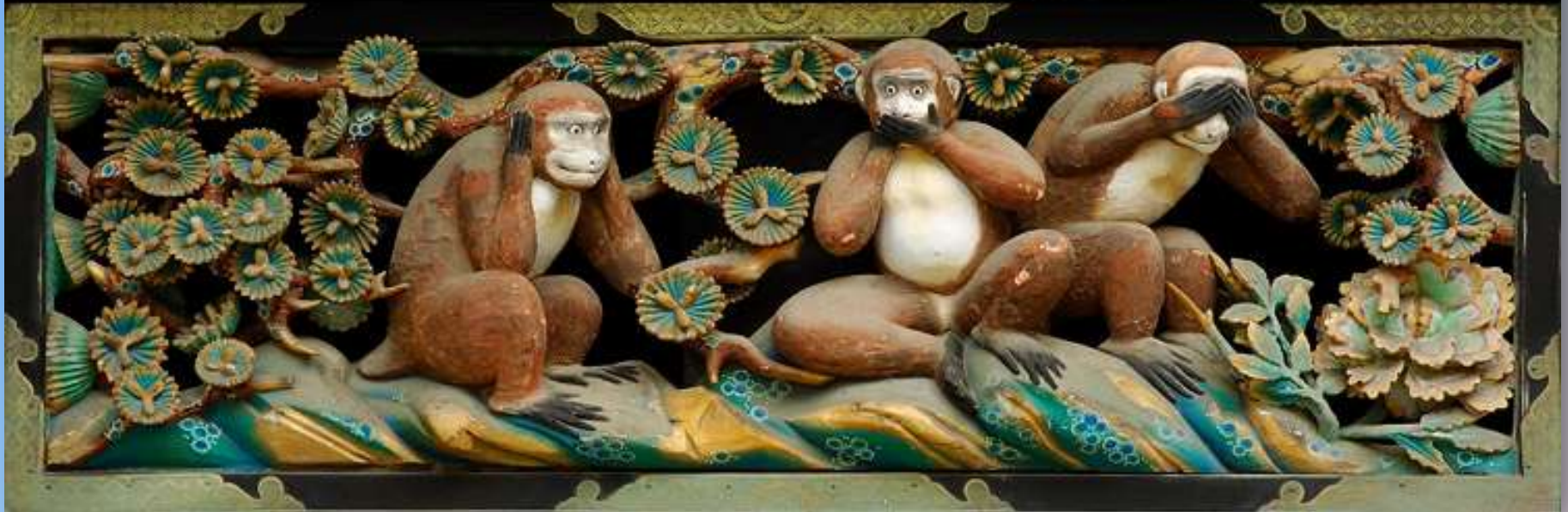


CP symmetry violation:
left-handed matter behaves
like right-handed antimatter





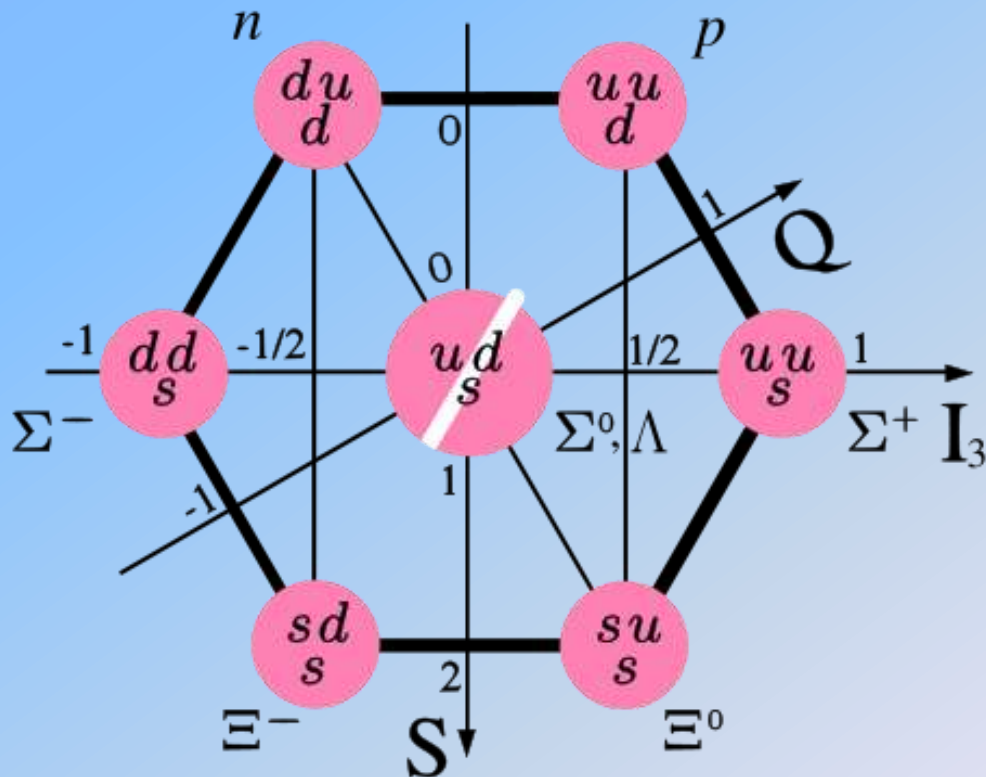
NIKKO TOSHOGU SHRINE



Almost symmetry: protons and neutrons

Particles that are affected equally by the strong force but have different charges can be treated as being different states of the same particle with isospin values related to the number of charge states.

Thus, in some internal “isospin space” protons and neutrons can be interchanged.



Rotations in a plane are a representation of the group $SO(2)$: Special Orthogonal group of order 2 which describes rotations in the Real plane.

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- * The group consists of rotations described by a matrix of sines and cosines.**
- * The group $SO(2)$ is closed under matrix multiplication.**
- * We found the identity element which “does nothing” to an object (we used a vector, or line segment).**
- Each element (rotation by θ) has an inverse (rotation by $360 - \theta$),**
- such that $r \otimes r^{-1} = I$.**

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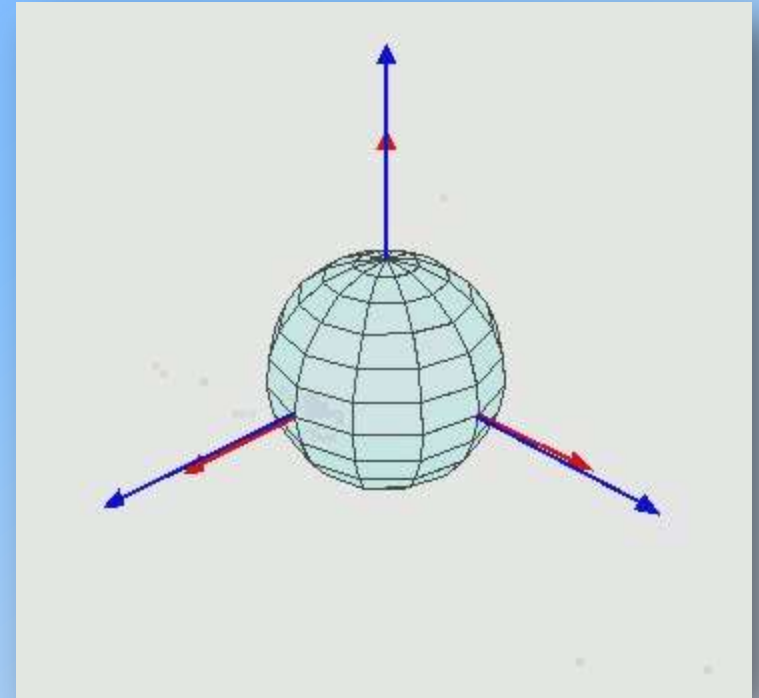
• **Orthogonal: Determinant = 1** $\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$ **det = $\cos^2\theta + \sin^2\theta = 1$**

•Generalize to a sphere: SO(3)

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}$$

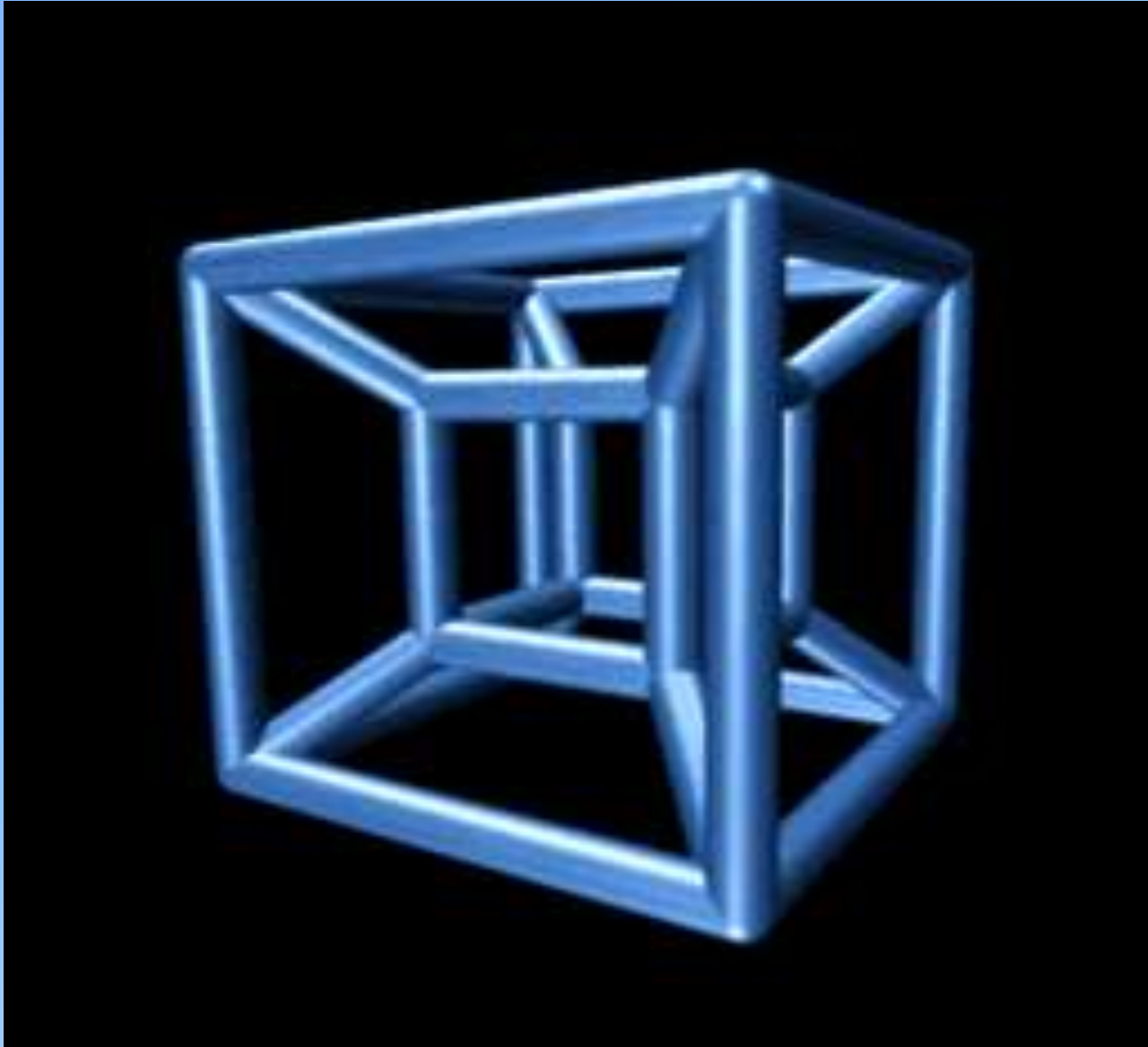
$$R_y(\theta) = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

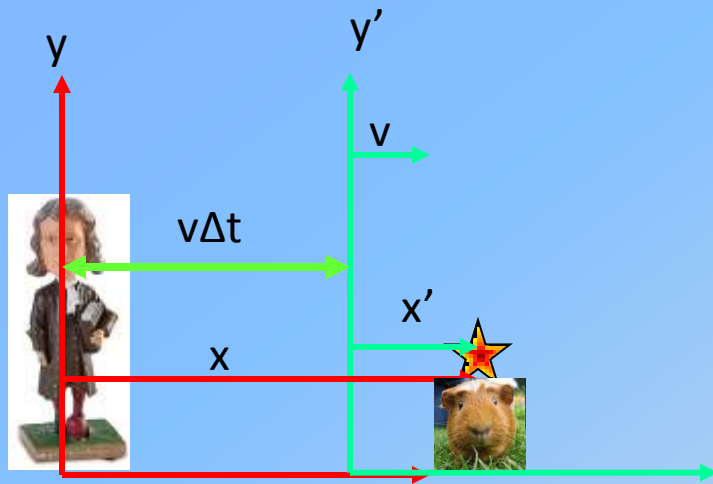
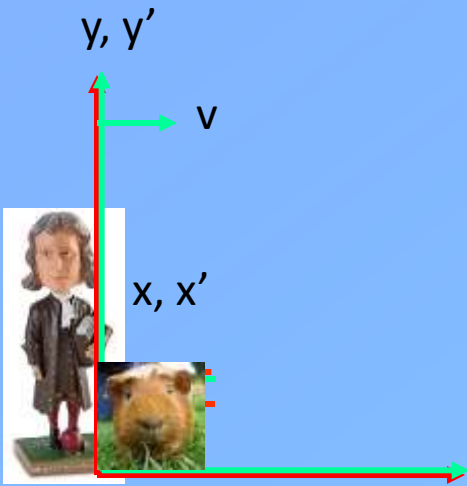


**lengths and angles
are preserved under
rotations and translations**

**SO(4)
rotation
in
4-D
just
for fun!**



Newtonian mechanics is based on the assumption that space is SO(3) and time is an independent variable.



Galilean Invariance:

$$x' = x - v\Delta t$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

Galilean Symmetry: All inertial reference frames are identical. If you don't look out the window, you can't tell if you're moving or not.

Galilean Transformation written out as equations (left) and in short hand (matrix) notation (right).

$$x' = x - v\Delta t$$

$$y' = y$$

$$z' = z$$

$$\Delta t' = \Delta t$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ \Delta t' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & -v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ \Delta t \end{pmatrix}$$



Michael Faraday
1791-1867



Heinrich Lenz
1804-1865



J. C. Maxwell
1831 – 1879

Towards the end of the 19th century, James Clerk Maxwell, building on previous work of Faraday, Gauss, Lenz, and others, discovered a new symmetry of Nature:

**A changing electric field produces a magnetic field.
A changing magnetic field produces an electric field.**

$$\vec{F}_e = q\vec{v} \times \vec{B}$$

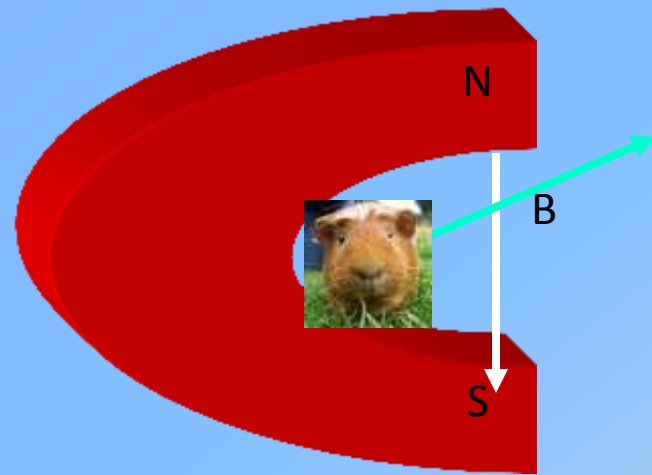
Imagine being at rest on an electric charge moving at constant v in a magnetic field.

In this case the magnet appears to be moving relative to you.

A static charge feels only an electric field.

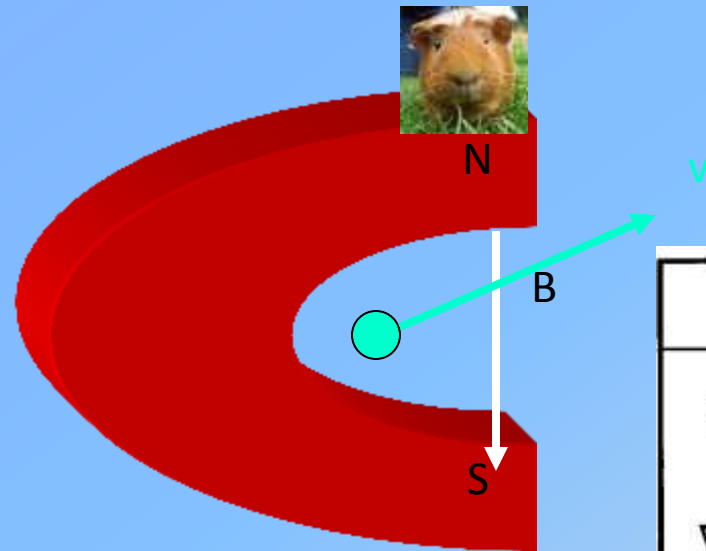
So you conclude that the moving magnet

must be producing an electric field because there is a force that is accelerating you in a circular path.



Now suppose you are at rest on the magnet. You feel a magnetic force on you when the charge whizzes by. A magnet cannot feel an electric force, so you conclude that the moving charge must produce a magnetic field.

Hence, Maxwell's equations:

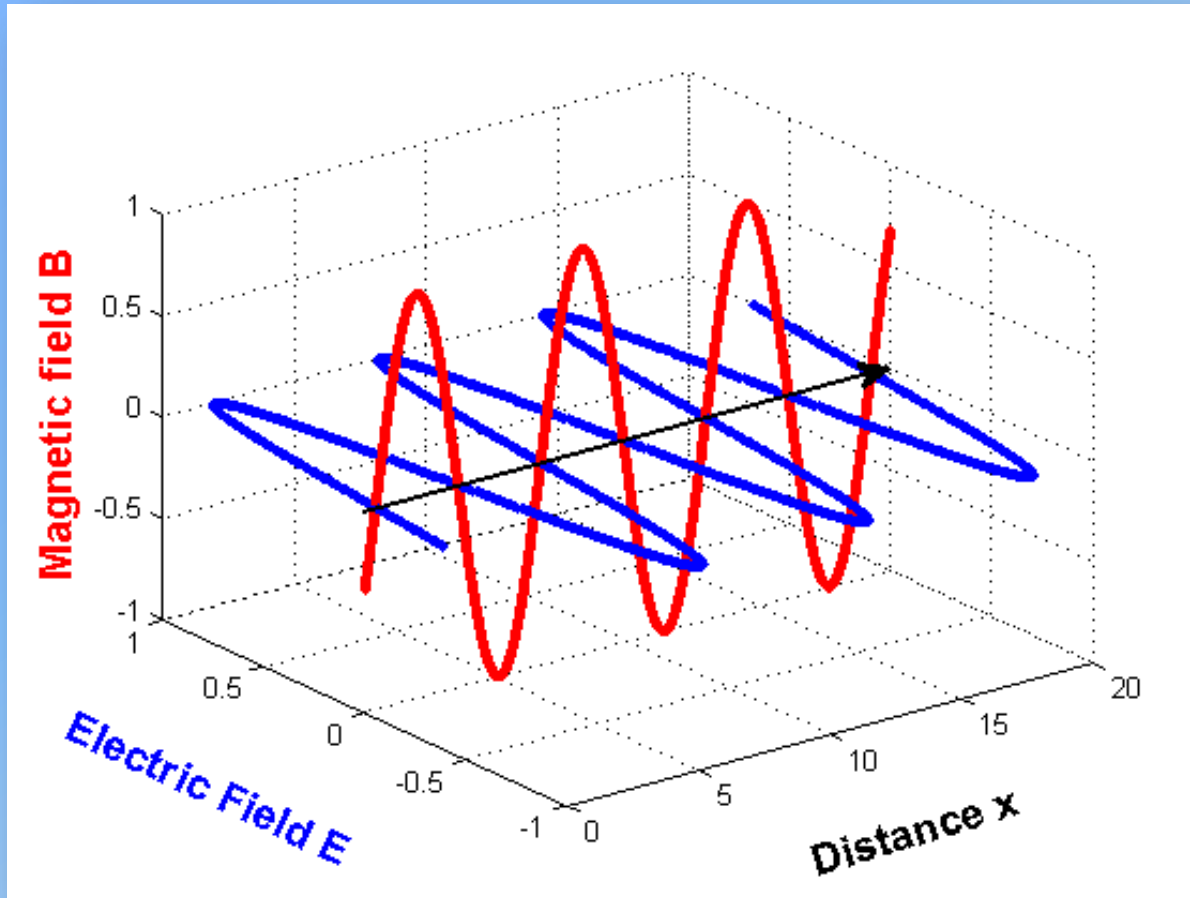


Point Form	Integral Form
$\nabla \times \mathbf{H} = \mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t}$	$\oint \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{S}$ (Ampère's law)
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint \mathbf{E} \cdot d\mathbf{l} = \int_S \left(-\frac{\partial \mathbf{B}}{\partial t} \right) \cdot d\mathbf{S}$ (Faraday's law; S fixed)
$\nabla \cdot \mathbf{D} = \rho$	$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho dv$ (Gauss' law)
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$ (nonexistence of monopole)

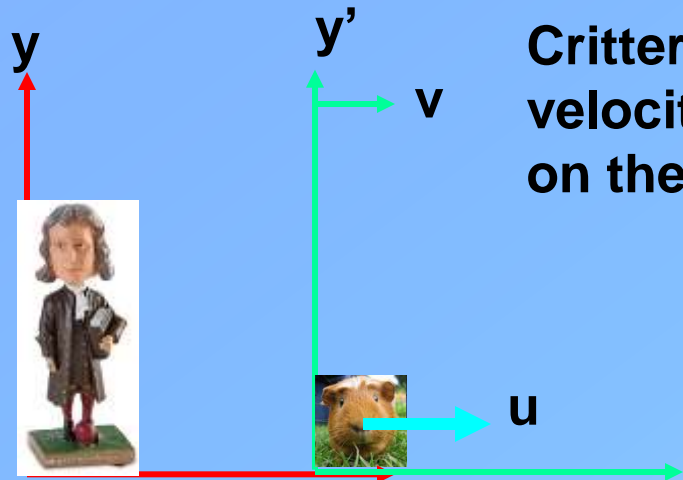
And Maxwell is credited with figuring out that light is an electromagnetic wave that travels at a constant speed in a vacuum, depending only on the properties of the vacuum:

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

ϵ_0 = electric permittivity
 μ_0 = magnetic permeability

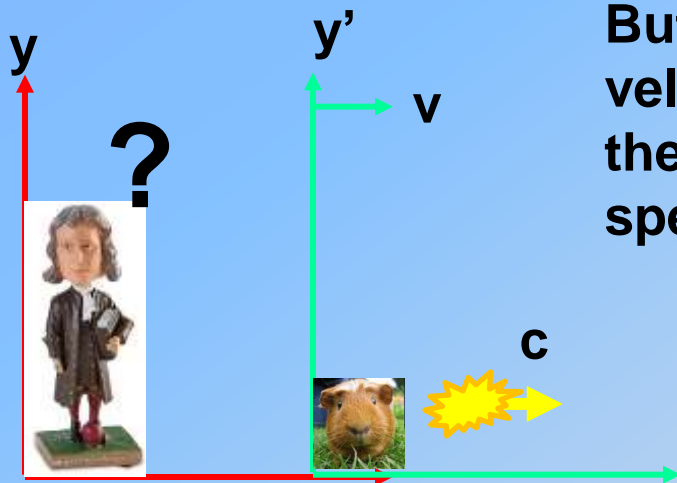


Applying Galilean reasoning to this led to a contradiction:



Critter running at velocity v in a train moving with velocity u has velocity V relative to an observer on the platform:

$$V = v + u$$



But shining a light in the train moving with velocity u does not result in an observer on the platform measuring a velocity V for the speed of light!

$$V \neq v + c$$

But folks thought that if light is a wave, there must be some medium for it to propagate through. So they decided that there must be an “ether” permeating space, through which light can travel.



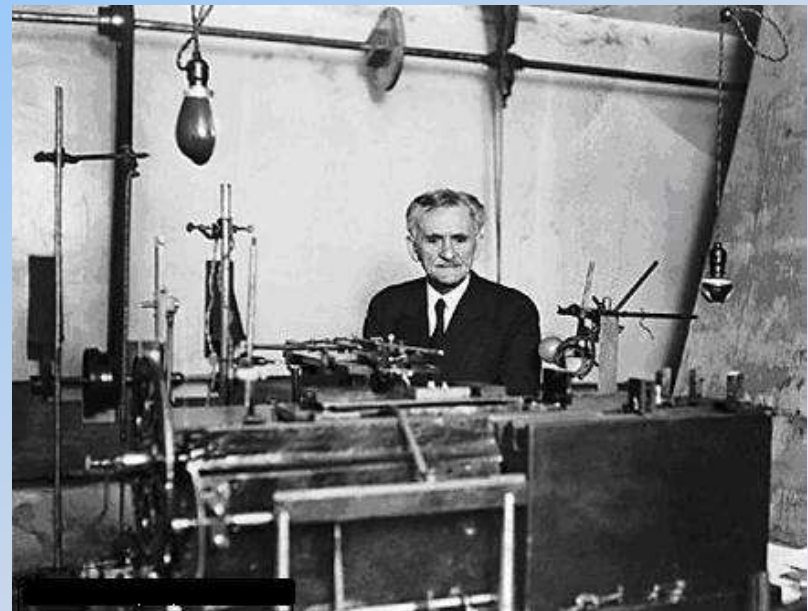
A. A. Michelson
1852-1931

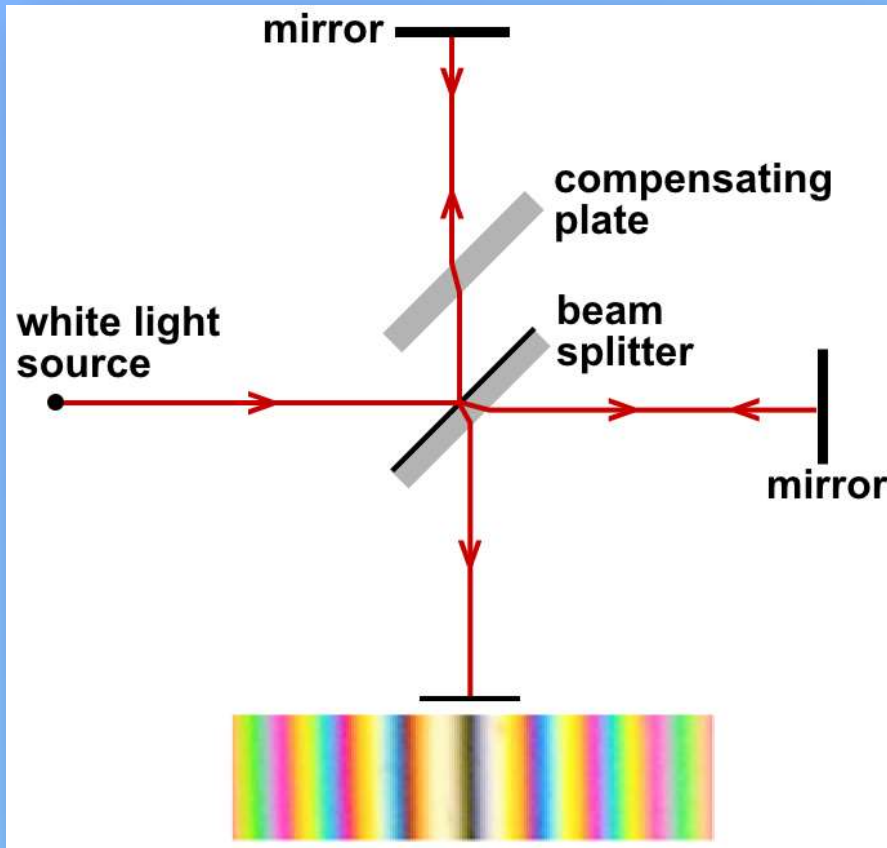


Edward Morley
1828 - 1923

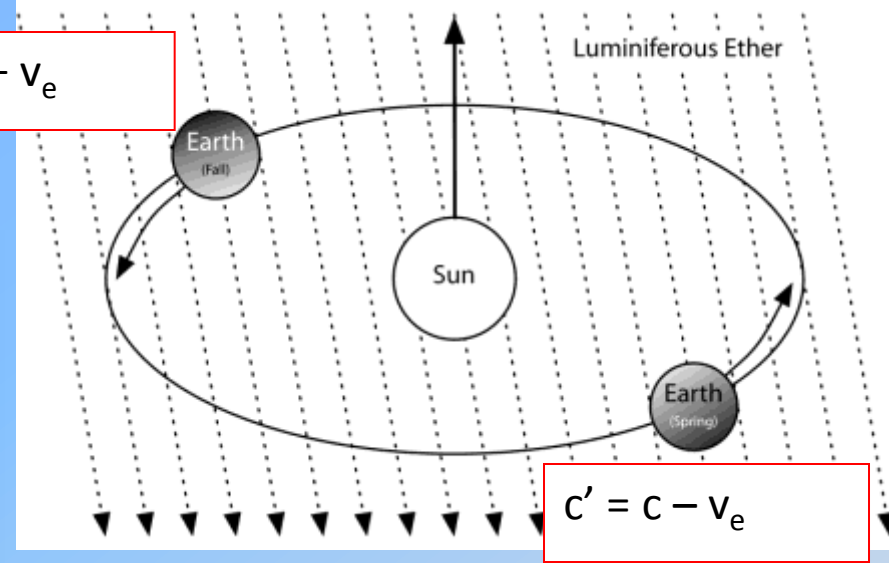
Famous Michelson – Morley experiment of 1887 tried to demonstrate the presence of “ether” that permeates space, and was thought to alter the speed of light depending on the direction, which should be seen as the Earth changed direction of travel...

THE MOST SUCCESSFUL FAILED EXPERIMENT IN HISTORY





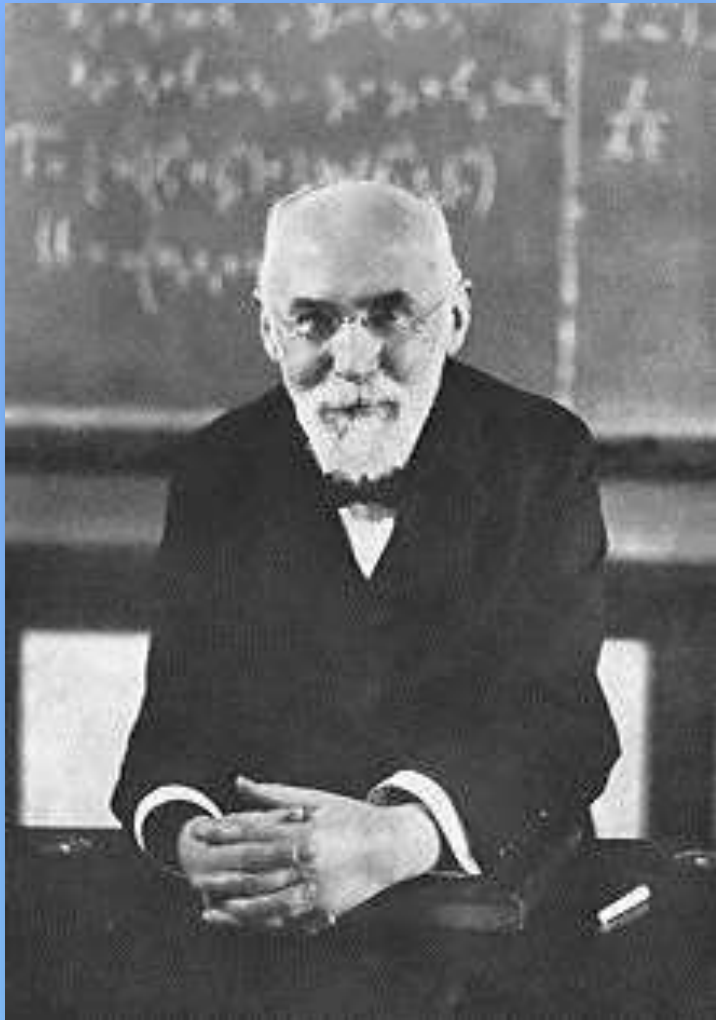
$$c' = c + v_e$$



Expectation: speed of light should be different at different times of the year, depending on relative velocity of Earth through the ether, which was presumed to have a velocity of its own, similar to a flowing river.



Michelson & Moreley's interferometer

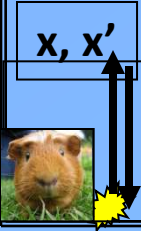


**Hendrik Antoon Lorentz
(1853 – 1928)**

Hendrik Lorentz proposed that moving bodies experience a time dilation relative to a ‘local time’ and a length contraction relative to a local observer.

He concluded that it would be impossible for either observer to tell which one was moving, and which one was not.

Imagine a train with a light clock that “ticks” with a pulse of light once/second.



view from inside the train

y, y'

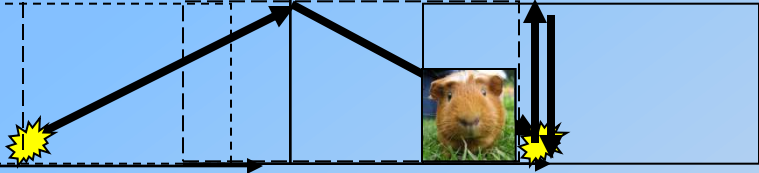
x, x'

v

y

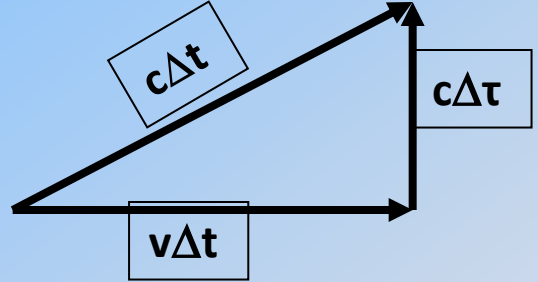
y'

$v \Delta t$



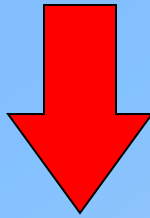
view from the track



v





$$c^2 \Delta t^2 = v^2 \Delta t^2 + c^2 \Delta \tau^2$$

$$c^2 \Delta t^2 = v^2 \Delta t^2 + c^2 \Delta \tau^2$$



$$\Delta \tau = \Delta t \sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$


$$\Delta t = \frac{\Delta \tau}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$


Conclusion: Time is NOT the same for each observer, when they try to compare measurements in each other's frame.

$$\Delta \tau = \Delta t \sqrt{1 - \frac{v^2}{c^2}}$$



$$\Delta t = \frac{\Delta \tau}{\sqrt{1 - \frac{v^2}{c^2}}}$$



What happens to time as v approaches c ?

As $v \rightarrow c$, $\Delta \tau \rightarrow 0$

Time ceases to exist for a light beam.

As $v \rightarrow c$, $\Delta t \rightarrow \infty$

The length of a second on a light beam approaches infinity as seen by an observer who is NOT on the light beam.

What happens as v approaches zero?

As $v \rightarrow 0$, $\Delta \tau \rightarrow \Delta t$ which is just the Galilean transformation

Lorentz Invariance: For motion along the x-axis:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

**The Lorentz Transformation looks suspiciously like a ROTATION
which MIXES space and time!**

$$\begin{aligned}x' &= x \cos \theta + y \sin \theta \\y' &= -x \sin \theta + y \cos \theta\end{aligned}$$

for any angle θ

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Rotation in space of x and y to x' and y'

Lorentz Invariance: For motion along the x-axis:

DEFINE:

$$\gamma = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$
$$\beta = \frac{v}{c}$$

or, $v = \beta c$

$$x' = \gamma(x - vt)$$
$$t' = \gamma\left(t - \frac{\beta x}{c}\right)$$

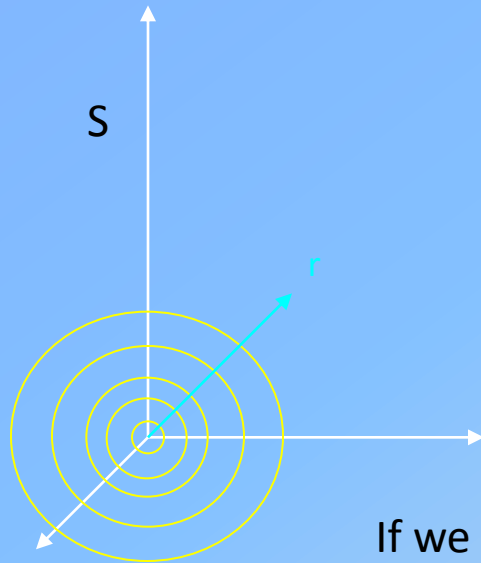
We can write it in the same notation we used for rotations in space:

$$\begin{bmatrix} x' \\ y' \\ z' \\ t' \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta c \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{\gamma\beta}{c} & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix}$$

Developing the idea of a new 'geometry' for spacetime:

A pulse of light spreads out in a sphere of radius r . A sphere is defined in space at any instant of time as satisfying the relation: $r^2 = x_1^2 + x_2^2 + x_3^2$.

But, since light travels at speed c , we know the sphere is expanding as its radius grows at the rate $r = ct$.



Putting these together, we have:

$$r^2 = x_1^2 + x_2^2 + x_3^2 = c^2t^2$$

or

$$x_1^2 + x_2^2 + x_3^2 - c^2t^2 = 0$$

If we generalize these coordinates to x_1, x_2, x_3 , and x_4 we must choose $x_4 = ict$ where $i = \sqrt{-1}$

So, Einstein generalized space and time coordinates into a spacetime continuum in a complex "geometry:"

$$x_1 = x$$

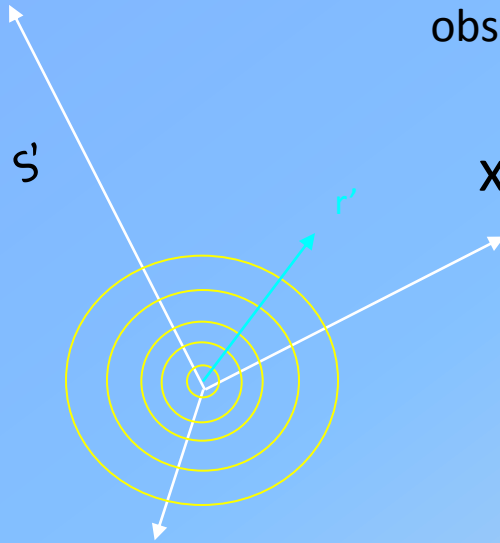
$$x_2 = y$$

$$x_3 = z$$

$$x_4 = ict$$

So, we have $x_1^2 + x_2^2 + x_3^2 + x_4^2 = x^2 + y^2 + z^2 - c^2t^2 = 0$

An observer in another frame would observe for the same light pulse:



$$x'^2 + y'^2 + z'^2 - c^2t'^2 = 0$$

We know $c = \text{constant}$ for all observers.

So we define the invariant “spacetime interval:”

$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2$$

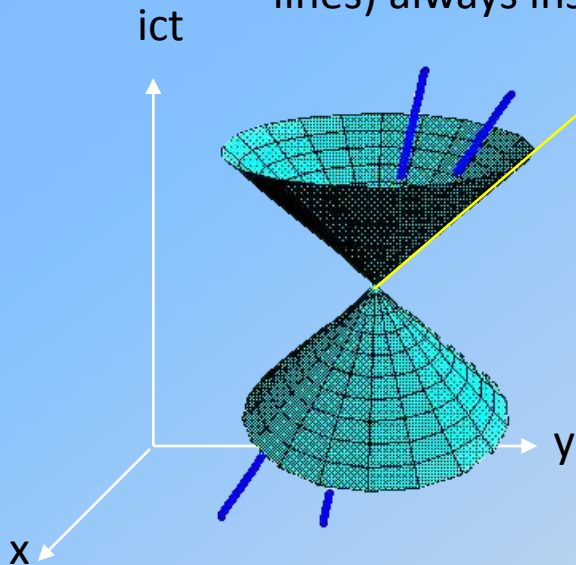
$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2$$

For $(\Delta s^2) > 0$ points are space-like separated

For $(\Delta s^2) = 0$ this corresponds to $\Delta x^2 + \Delta y^2 + \Delta z^2 = c^2 \Delta t^2$ or traveling at the speed of light – called “null” or “light-like” separated

For $(\Delta s^2) < 0$ points are time-like separated

particles with non-zero rest mass follow time-like paths (world lines) always inside the light cone



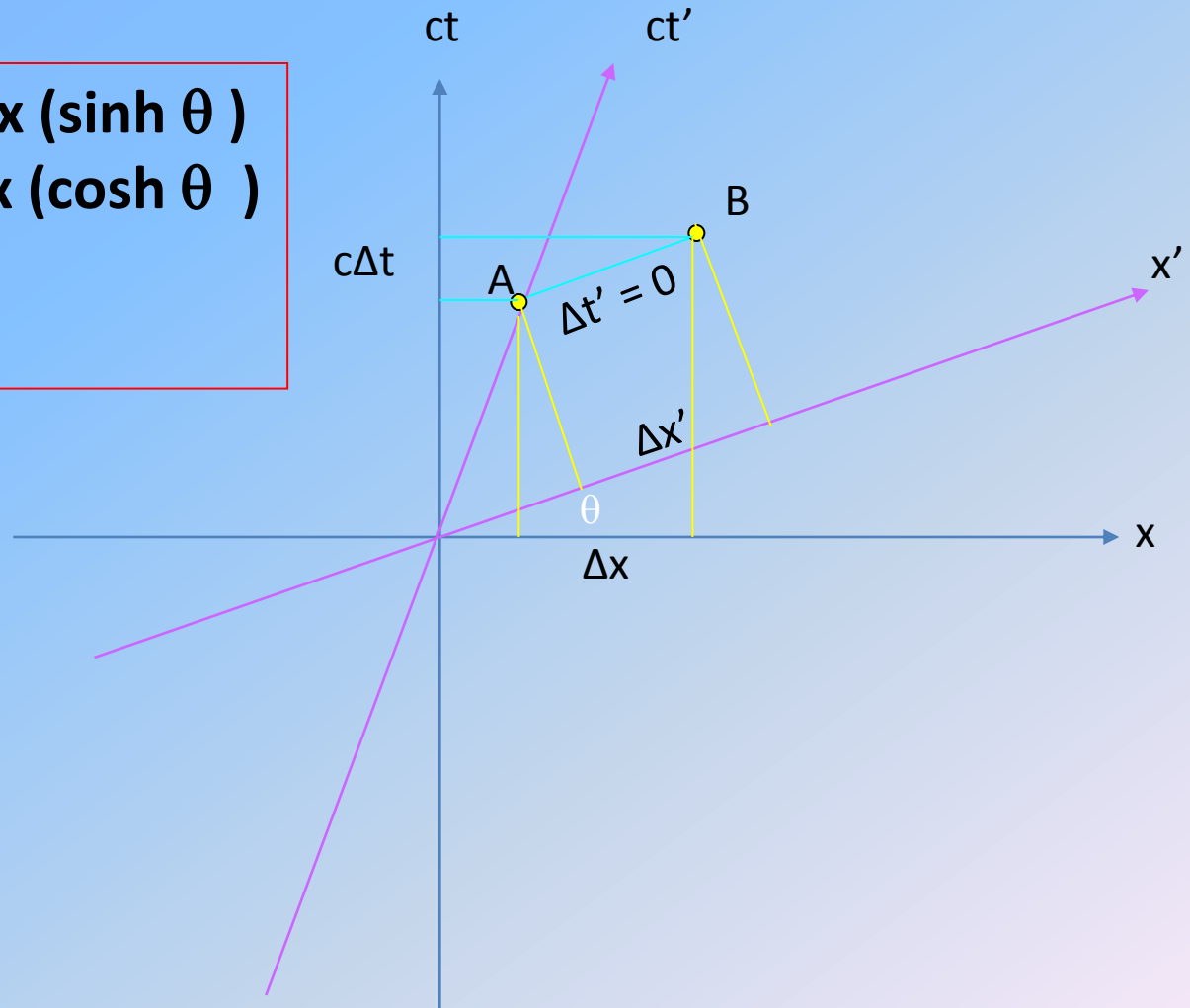
photons with zero rest mass follow paths of $\Delta s^2 = 0$

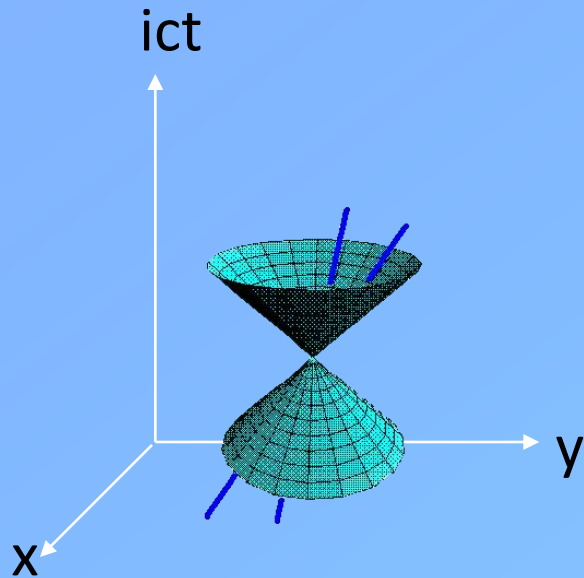
particles which follow space-like world lines have been called tachyons. Tachyons would travel always faster than the speed of light, would have negative energy, and would violate causality...none have ever been observed!

Without deriving here, for motion in the x-direction (like we looked at before), the analog is a rotation of the ct and x axes, and instead of “regular” sine and cosine, we must use the hyperbolic sinh and cosh.

$$\begin{aligned} ct' &= ct (\cosh \theta) - x (\sinh \theta) \\ x' &= -ct (\sinh \theta) + x (\cosh \theta) \\ y' &= y \\ z' &= z \end{aligned}$$

We call this a
Lorentz boost.





The light cone is the locus of points that would be traced out by a pulse of light emitted at P or converging on it. The surface of the pulse would be an expanding or contracting sphere in three spatial dimensions. In this diagram, showing only two spatial dimensions and time, it appears as the circular cross section of a cone.

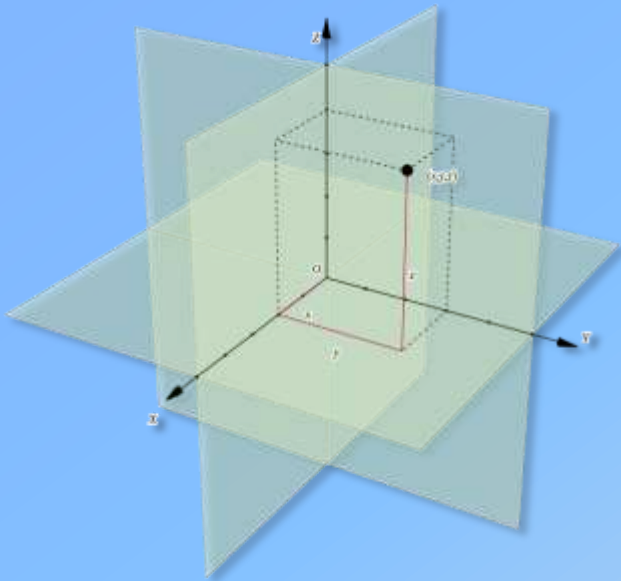
Clocks are devices that are used for measuring time-like distances.
Rulers are devices that are used for measuring space-like distances.

From the definition $\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2$

we define $\Delta \tau^2 \equiv -\Delta s^2/c^2$ as the proper time

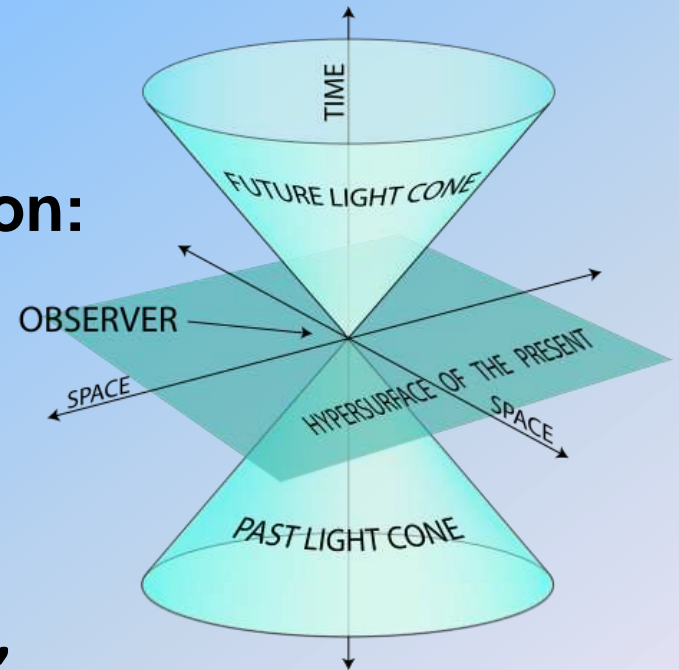
Old perception:

- * Euclidean space
- * Rotations, translations in 3-space
- * Euclidean Group



New perception:

- * Minkowski space
- * Rotations, translations, and “Lorentz boosts” in 4-space (3 space, 1 time dimension)
- * Poincare Group



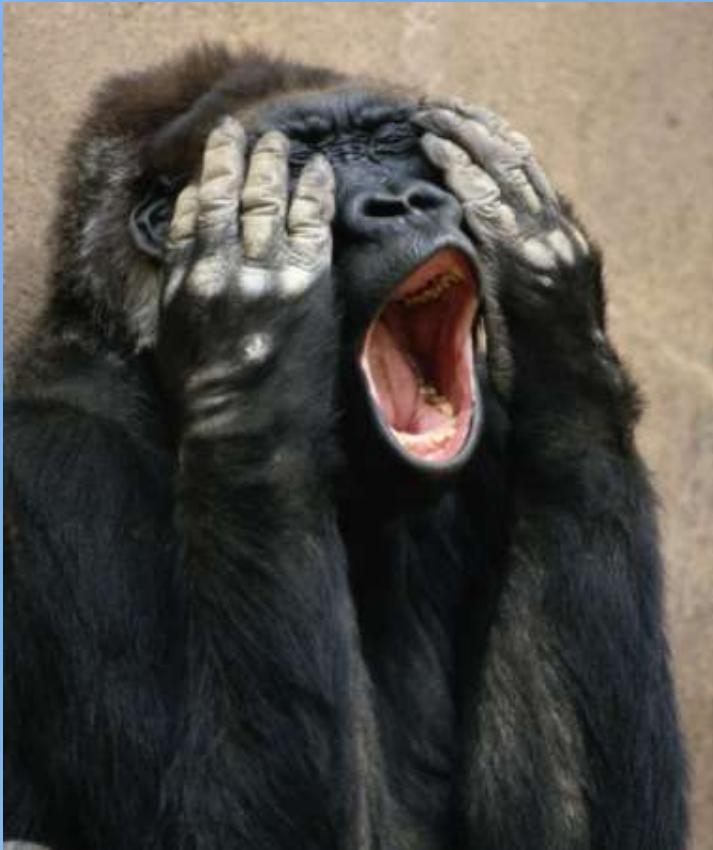
The laws of physics are not violated.

Our perception of space and time must be restructured to understand SPACETIME!

Thus we have a NEW SYMMETRY. Space is not really SO(3) with independent time. Spacetime is SO(4).

Group	Representations	Degrees of freedom
SO(2)	Circle, motion in a plane	$[2(2+1)/2] = 3$ d.f. 1 rotation angle, 2 directions of translation
SO(3)	Rotations on a sphere	$[3(3+1)/2] = 6$ d.f. 3 rotation angles, 3 directions of translation
SO(4) “Poincare Group”	Spacetime	$[4(4+1)/2] = 10$ d.f. 3 rotation angles, 3 directions of translation, 3 ‘boosts’ 1 direction of time

Time for a break!

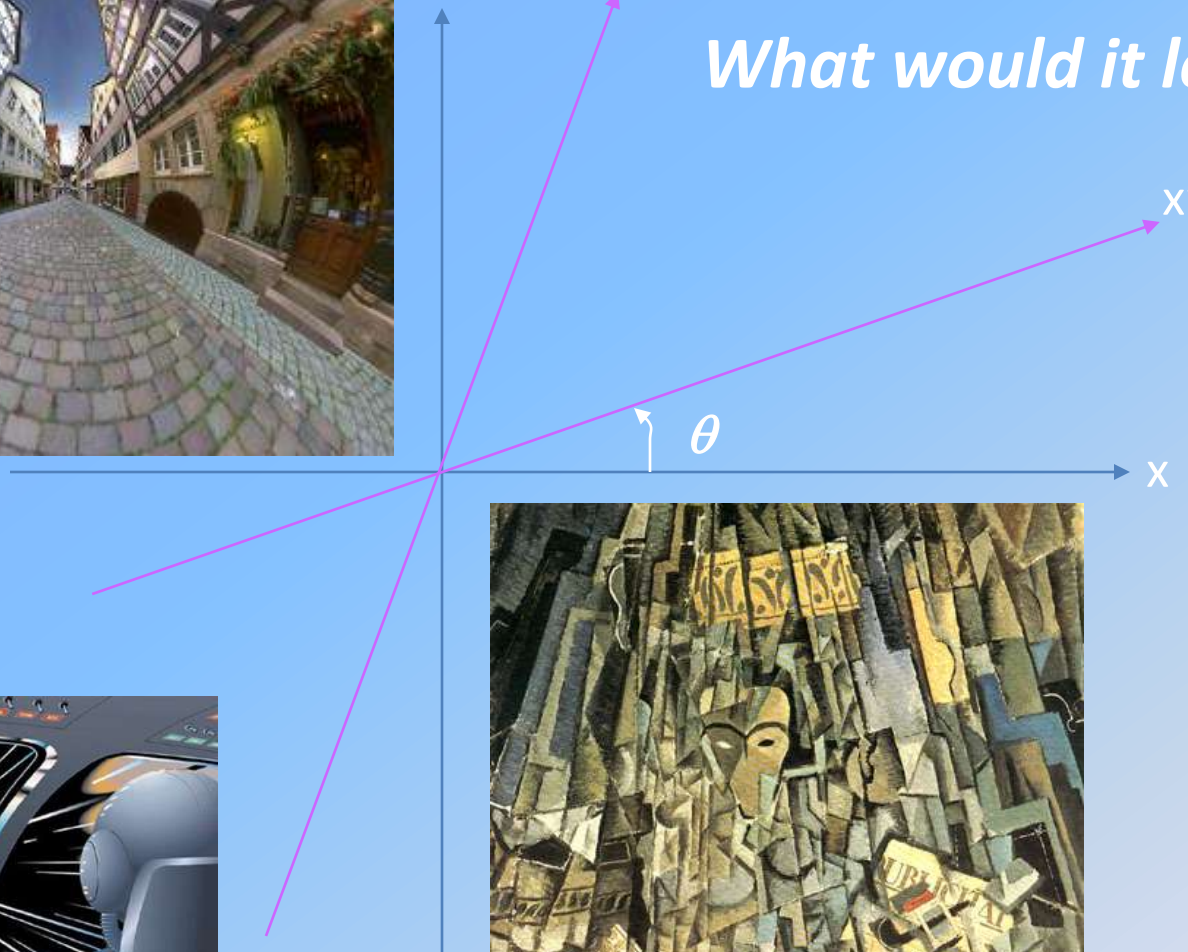




ct

ct'

What would it look like ?



Some consequences:

We define the “proper” frame as the frame in which the observer is at rest.

Define: τ = proper time measured by observer in his/her rest frame

Define: L_0 = proper length measured by observer in his/her rest frame

Define: t' = time as measured in the “other” frame

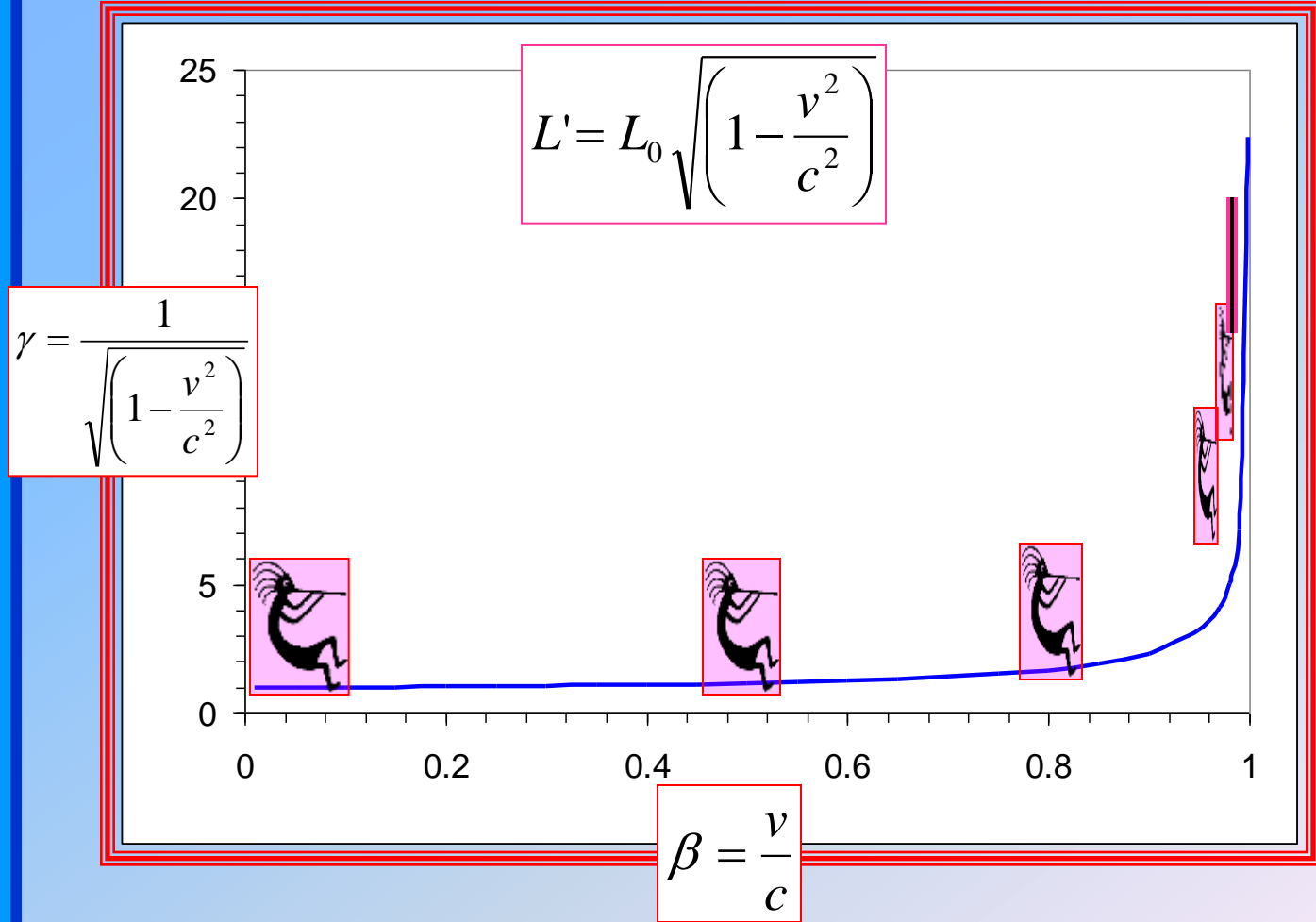
Define: L' = length as measured in the “other” frame

$$L' = L_0 / \gamma = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$t' = \gamma \tau = \frac{\tau}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Foreshortening of length in the direction of travel as observer approaches the speed of light

v/c	gamma
0.01	1.00005
0.05	1.001252
0.1	1.005038
0.15	1.011443
0.2	1.020621
0.25	1.032796
0.3	1.048285
0.35	1.067521
0.4	1.091089
0.45	1.119785
0.5	1.154701
0.55	1.197369
0.6	1.25
0.65	1.315903
0.7	1.40028
0.75	1.511858
0.8	1.666667
0.85	1.898316
0.9	2.294157
0.95	3.202563
0.96	3.571429
0.97	4.11345
0.98	5.025189
0.99	7.088812
0.999	22.36627
0.9999	70.71245
0.99999	223.6074



EXAMPLE: MUONS IN THE UPPER ATMOSPHERE

$$L' = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$t' = \frac{\tau}{\sqrt{1 - \frac{v^2}{c^2}}}$$

In Earth rest frame:

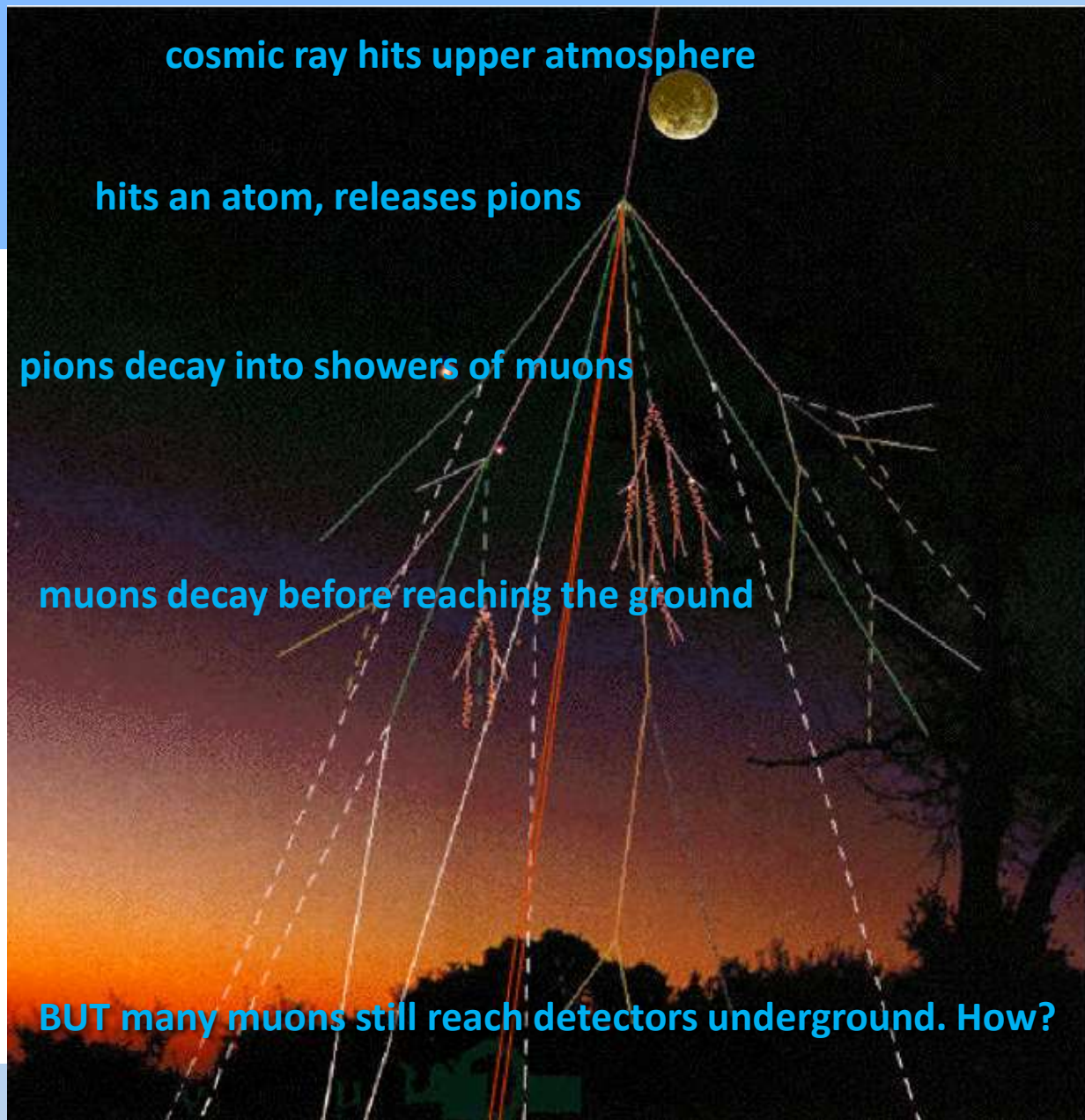
$L_0 = 10 \text{ km} =$ height of atmosphere

In rest frame of muons:

$\tau =$ half life $= 2.2 \times 10^{-6} \text{ sec.}$

What is L' of atmosphere, as seen by muons which travel at $.98c$?

What is the half life of muons as observed in the Earth frame?



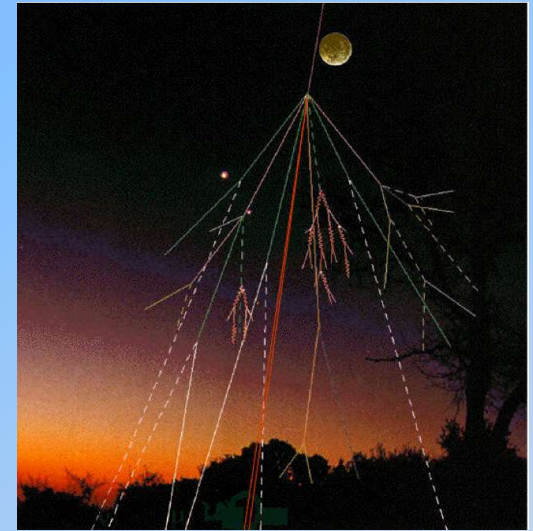
What is L' of atmosphere, as seen by muons which travel at $.98c$?

$$\sqrt{1 - \frac{(.98c)^2}{c^2}} = \sqrt{1 - .98^2}$$
$$\cong \sqrt{.04} = .2$$

So $L' = .2L_0 = (.2) \times 10\text{km} = 2 \text{ km}$

$$L' = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$t' = \frac{\tau}{\sqrt{1 - \frac{v^2}{c^2}}}$$



This result tells us that from the reference frame of the muons, moving at $.98c$ relative to the ground, the length of the atmosphere appears to be only 2 km instead of 10 km!

What is the half life of muons as observed in the Earth frame?

Half life as seen by an observer on Earth is longer than the half life as measured in the muons' rest frame:

$$t' = \frac{2.2 \times 10^{-6} \text{ sec}}{.2} = 1.1 \times 10^{-5} \text{ sec}$$

$$L' = \frac{L_0}{\gamma} = .199(10km) = 1.99km \approx 2km$$

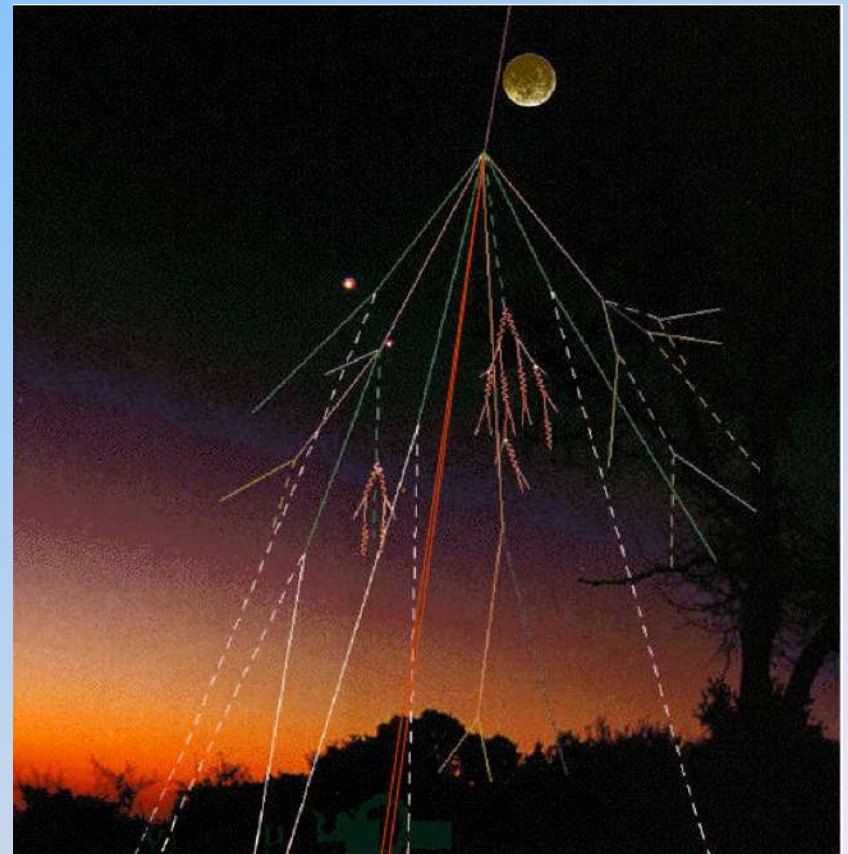
length of atmosphere as seen by muons in their frame

$$t' = \gamma\tau = 5(2.2 \times 10^{-6} \text{ sec}) = 1.1 \times 10^{-5} \text{ sec}$$

half-life of muons as measured in Earth frame

How many muon half-lives pass before the muon shower hits the ground?

Remember: The Earth observer sees the muons traveling at .98c “down” and the muons see the ground traveling “up” at .98c!



- 3a. How much time passes in each observer's frame?
 3b. How many half lives go by in each observer's frame?

Earth observer sees the muons moving down at a constant speed of $.98c$, and the muons see the ground moving up towards them at a constant speed of $.98c$. Using the relationship that time elapsed = distance/speed:

Earth observer measures:

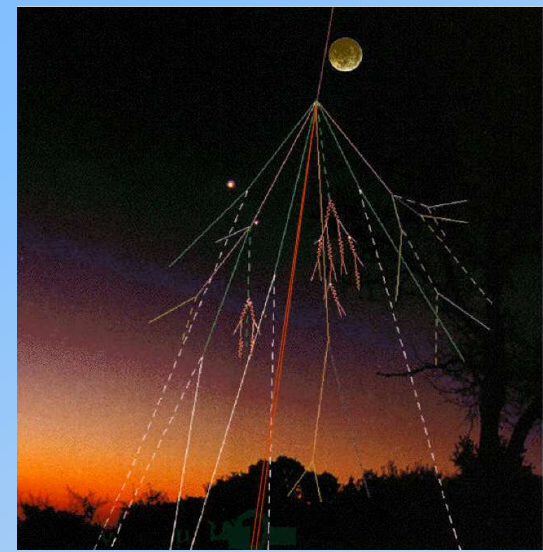
$$t = \frac{L_0}{v} = \frac{10^4 \text{ m}}{(.98 \times 3 \times 10^8 \text{ m/sec})} = 3.4 \times 10^{-5} \text{ sec} \text{ for the muons to traverse the atmosphere}$$

$$\frac{3.4 \times 10^{-5} \text{ sec}}{1.1 \times 10^{-5} \text{ sec}} = 3.09 \text{ half lives}$$

muons measure:

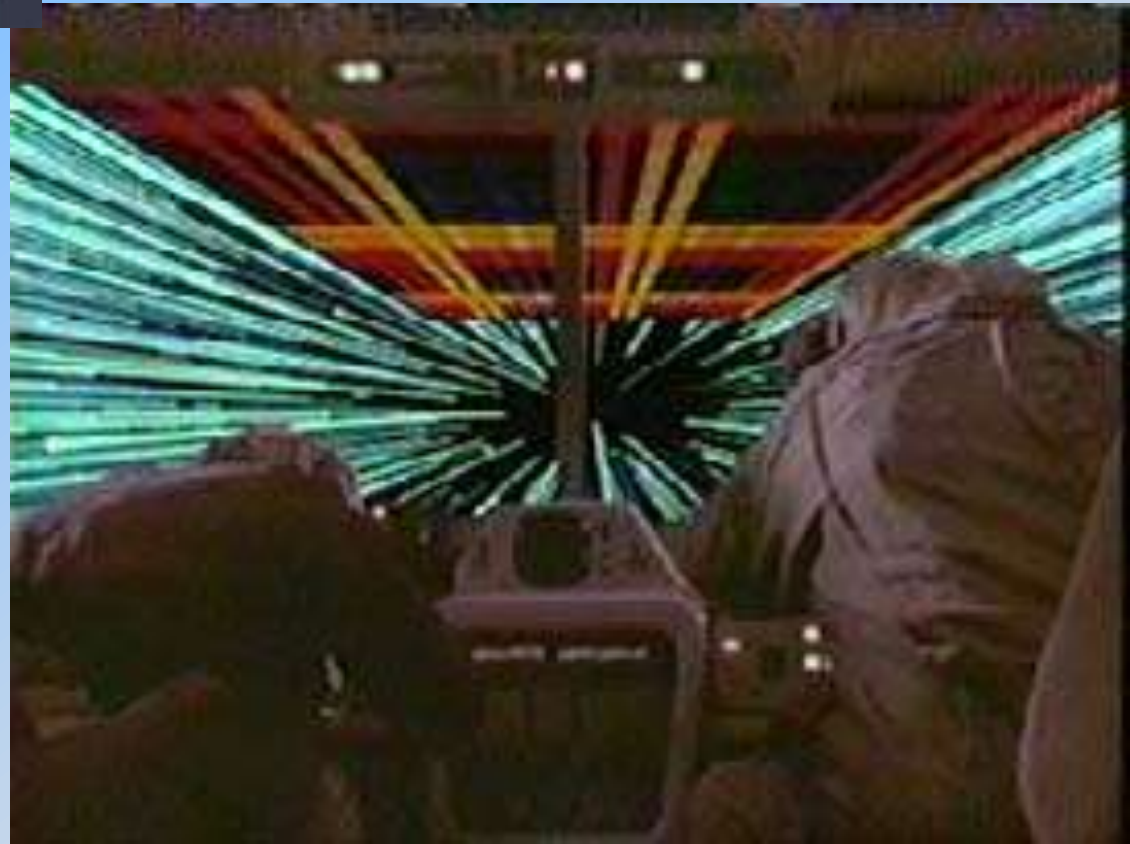
$$t_0 = \frac{L'}{v} = \frac{1.99 \times 10^4 \text{ m}}{(.98 \times 3 \times 10^8 \text{ m/sec})} = 6.77 \times 10^{-6} \text{ sec} \text{ for the muons to traverse the atmosphere}$$

$$\frac{6.77 \times 10^{-6} \text{ sec}}{2.2 \times 10^{-6} \text{ sec}} = 3.09 \text{ half lives}$$



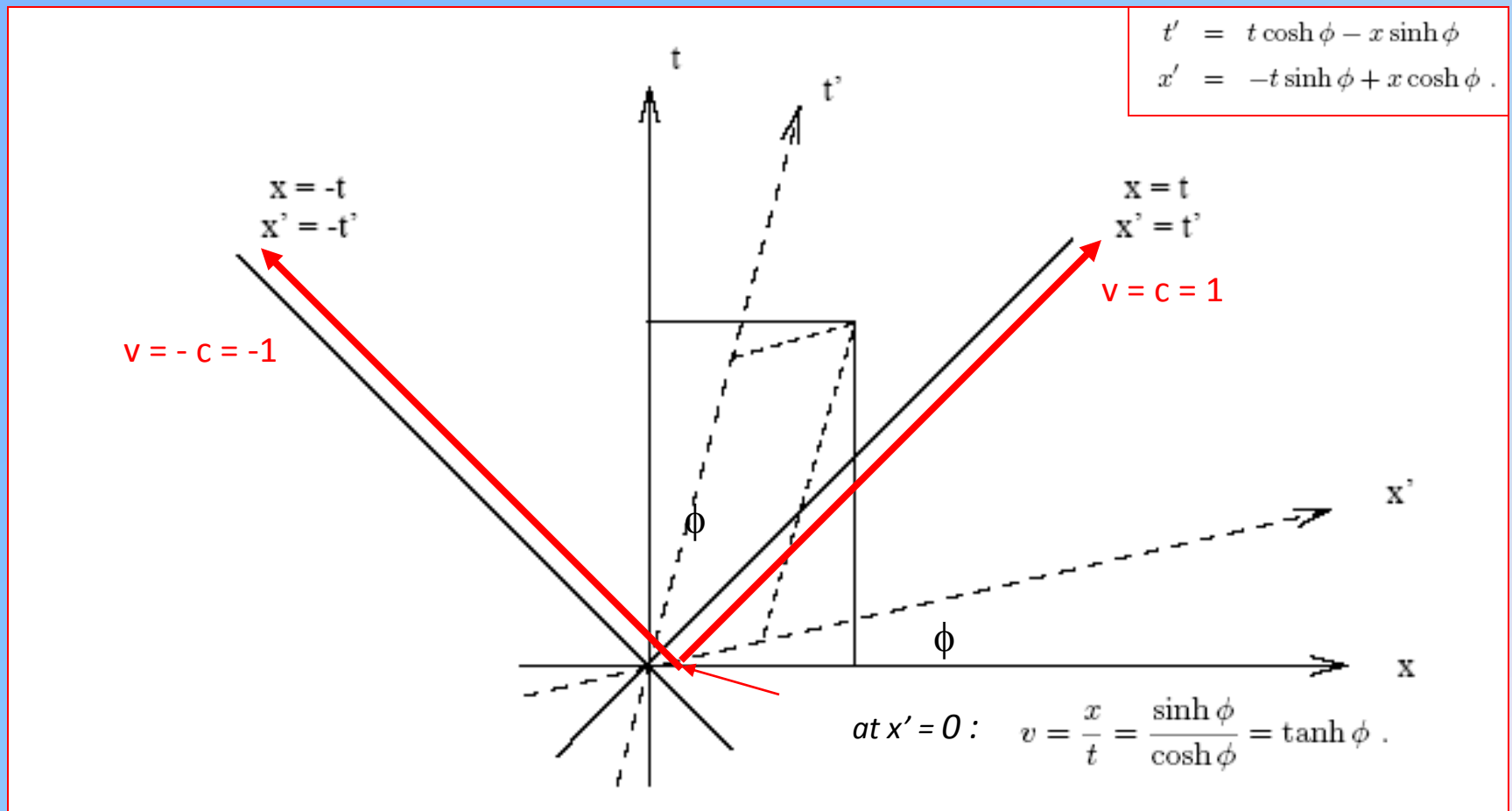


Artists' renditions of the view through the front window of a space vehicle traveling near light speed.

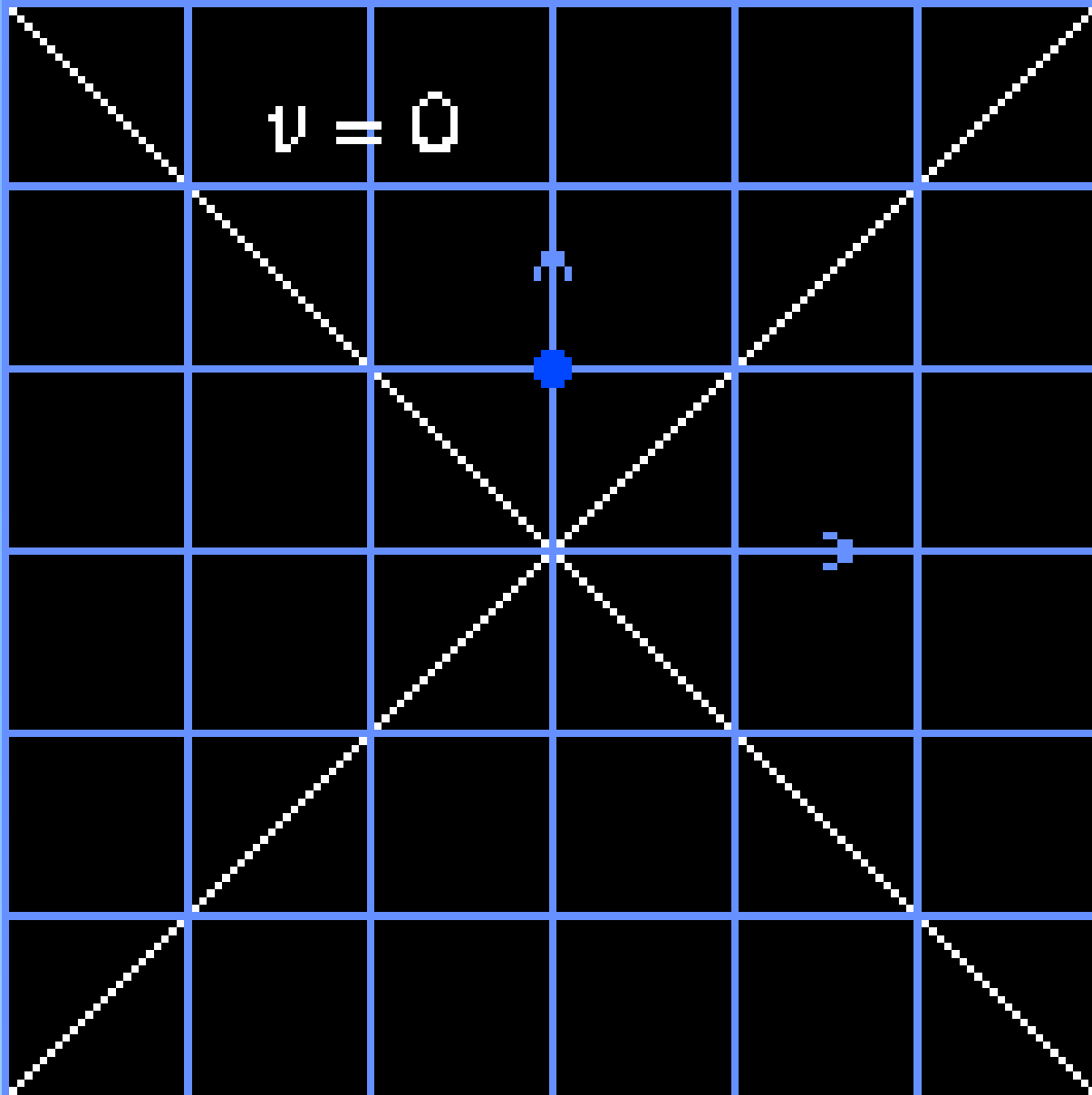


“They’ve gone to plaid!”

Diagram of a Lorentz boost taken from Sean Carroll's on-line notes on General Relativity, available at http://arxiv.org/PS_cache/gr-qc/pdf/9712/9712019v1.pdf.



As $v \rightarrow c$ the axes collapse!

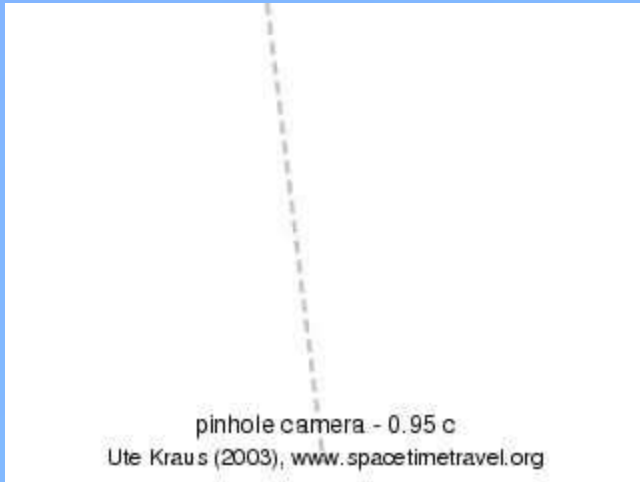


http://casa.colorado.edu/~ajsh/sr/congridbig_gif.html



A relativistic bike ride through Tübingen, Germany

van de Ree, 2010
Prof. Ute Kraus <http://www.spacetime.travel.org/tuebingen/tuebingen.html>





camera standing still

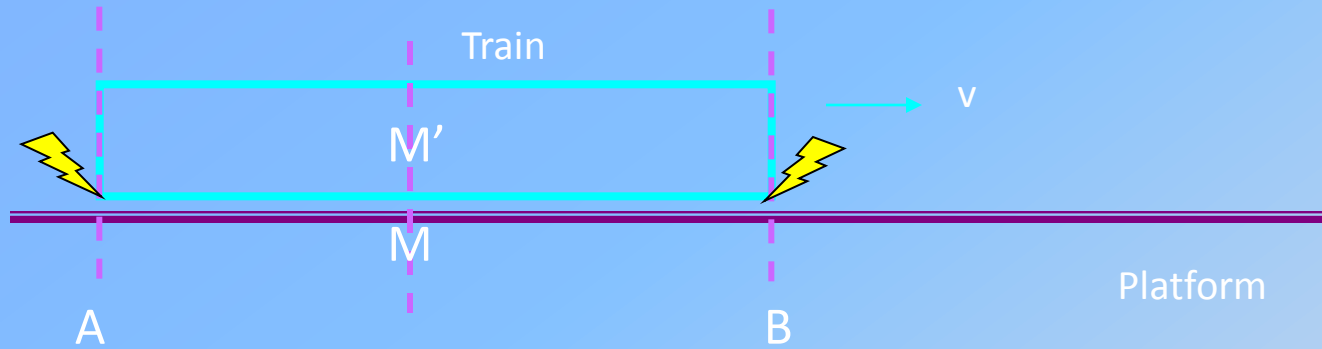


camera moving at .8c

camera moving at .95c (left) and .99c (right)

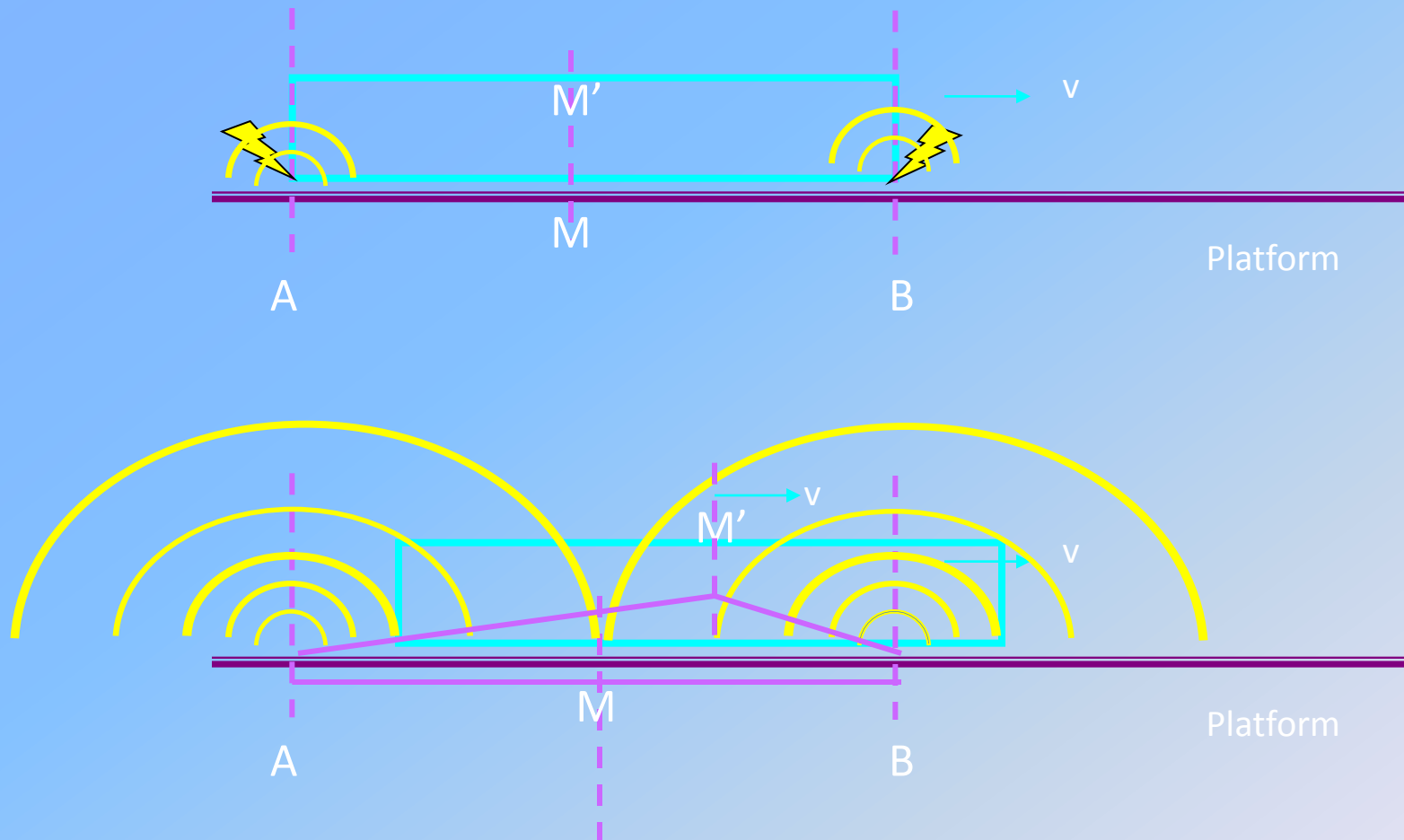


Events are not simultaneous in Special Relativity:



Imagine a long train moving at speed v relative to a platform. At the moment that the front and back of the train coincide with points A and B on the platform, and the center of the train M' coincides with the midpoint of A and B (call it M) on the platform, lightning strikes the front and back of the train, as seen by the station master.

The light from each lightning bolt travels at c in all directions from each strike.

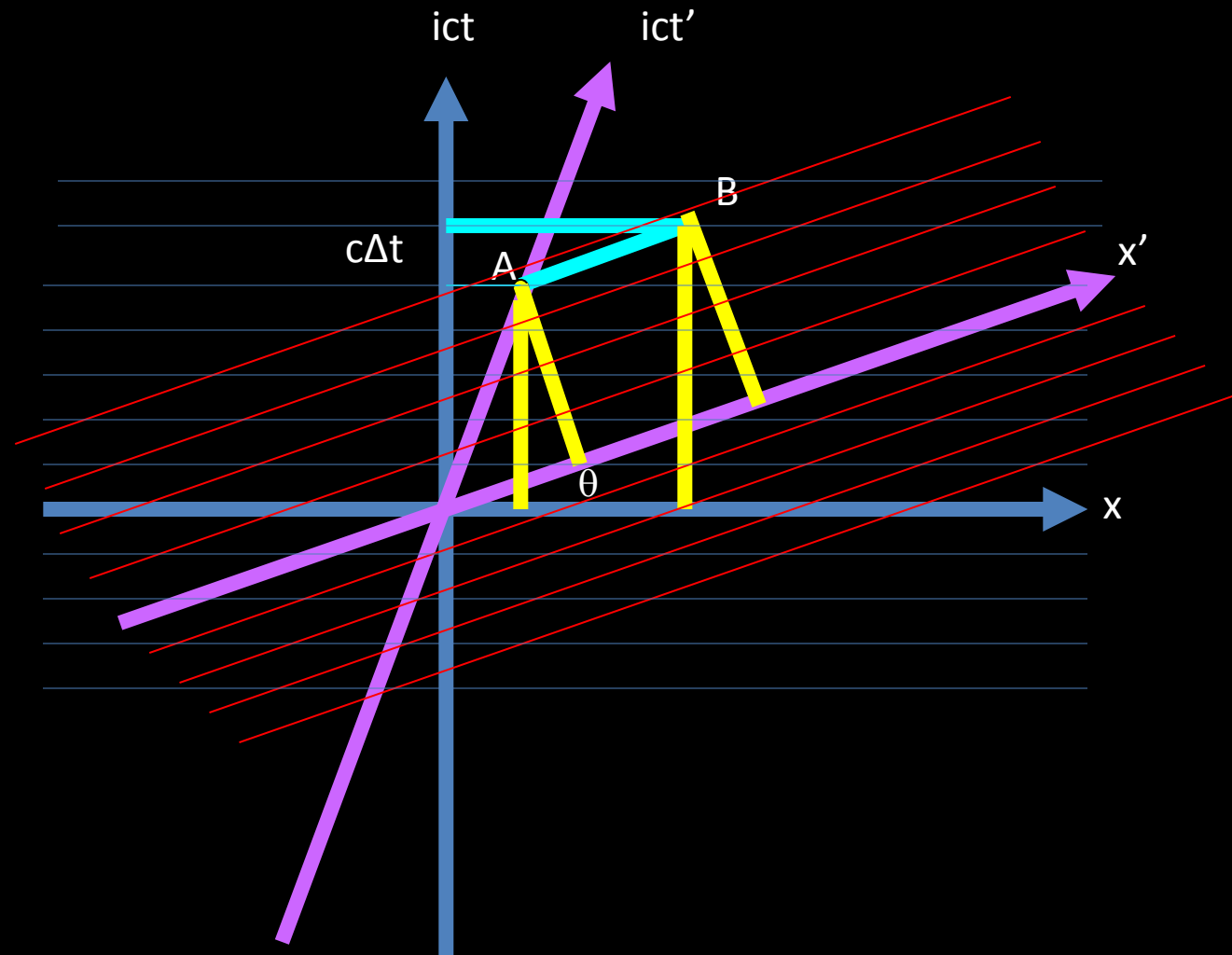


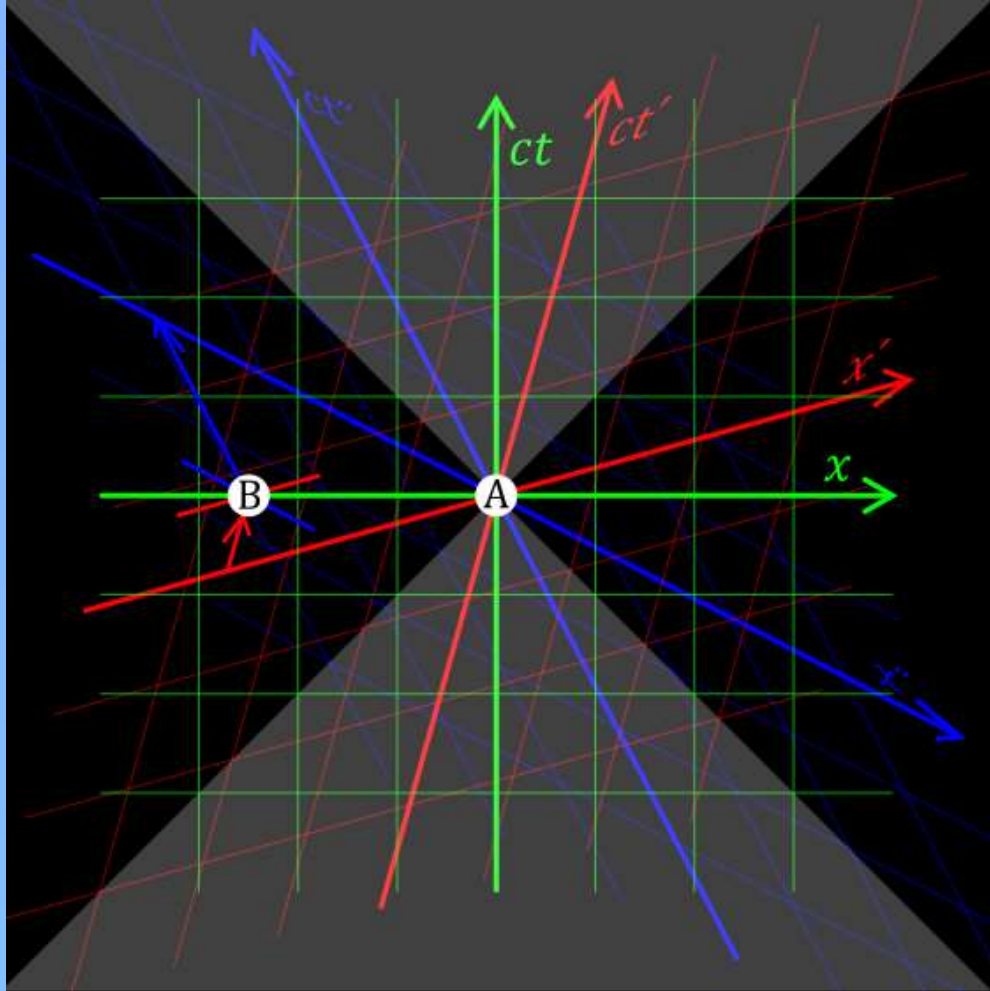
The observer at M sees the light from the lightning reach him simultaneously, but the observer at M' sees the light from the strike at the front of the train before she sees the light from the strike at the back of the train.

Graphical depiction of the Relativity of Simultaneity:

Events A and B are simultaneous in the primed frame, but not in the unprimed frame. $\Delta t' = 0$ but Δt is not. A and B are spacetime separated by $c\Delta t$ in the x, ct frame

Lines of constant t are parallel to the x axis.
Lines of constant t' are parallel to the x' axis.





Green frame: A and B are simultaneous

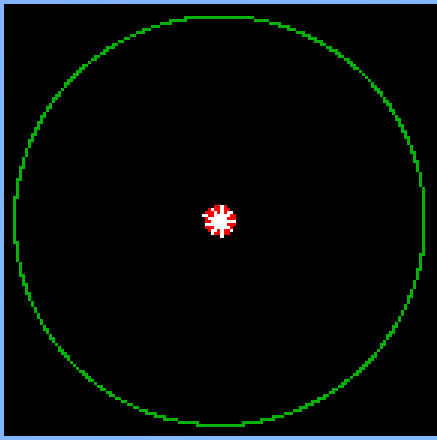
Red frame: A occurs before B

Blue frame: B occurs before A

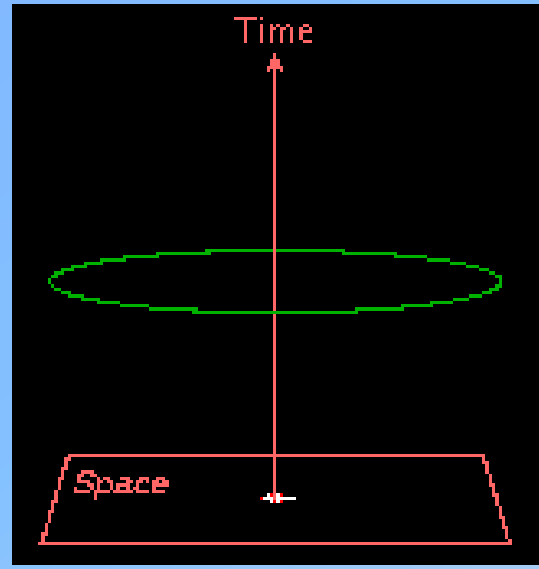
Events that are simultaneous (constant t) in one frame will not be seen as simultaneous in another which is moving relative to the first.

The speed of light will always be measured the same in any reference frame.

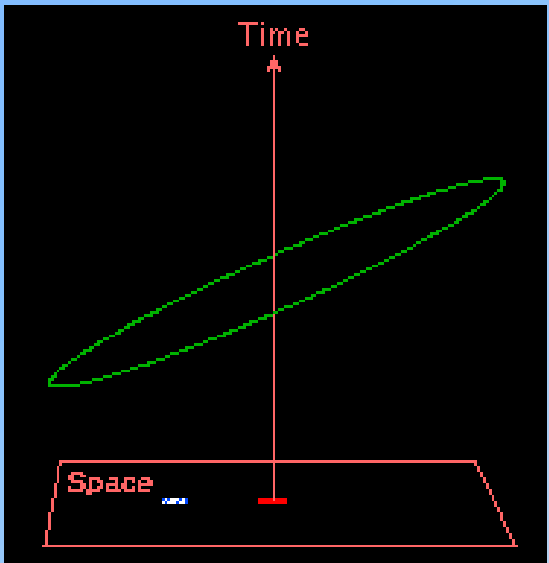
Light pulse viewed by observer in its rest frame – light is emitted at the center, bounces off spherical mirror, and returns to the center



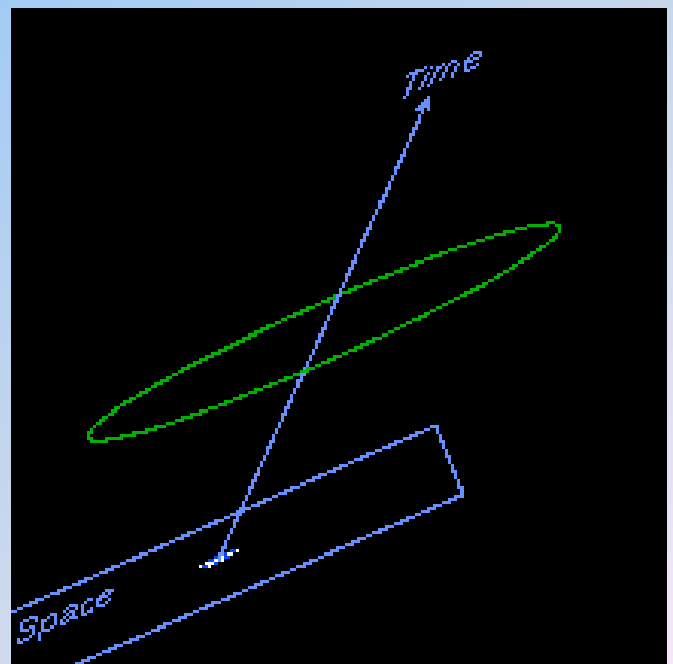
Light cone in observer's rest frame



Light cone for a moving observer seen by non-moving observer



<http://casa.colorado.edu/~ajsh/sr/simultaneous.html>



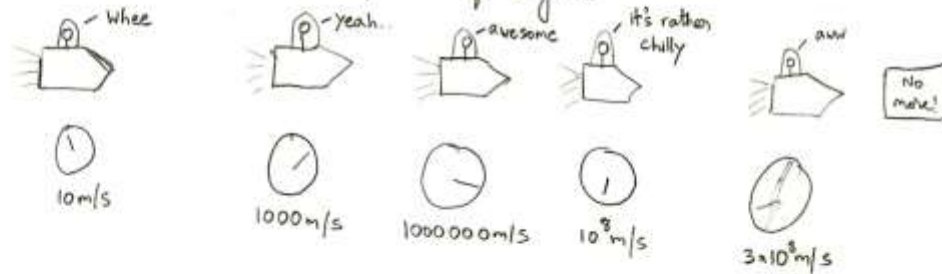
**Anand
Das,
2013**



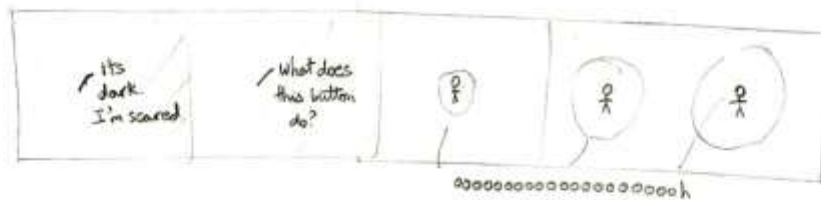
Whats so special about Special Relativity?

-Anand Das

The postulate: All of special relativity can be derived from the simple fact that the speed of light is constant, motivated by Maxwell's theory of electromagnetism. As a result of this, it can be shown that a particle can never exceed the speed of light.



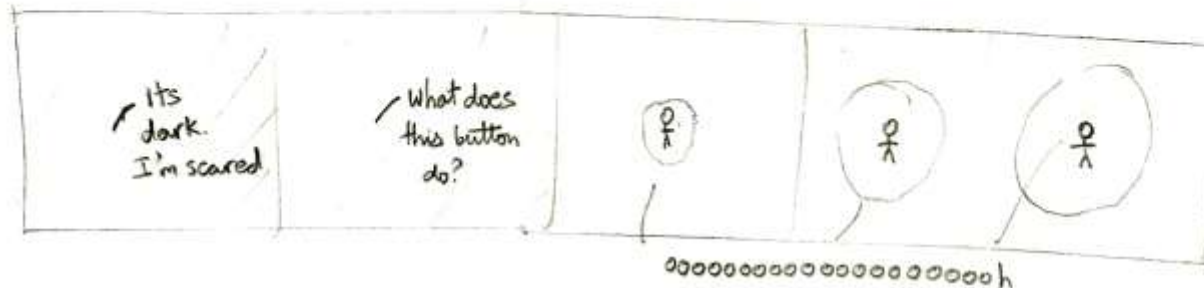
This leads to some possible paradoxes. In particular consider a person sitting in the dark who emits a flash of light.



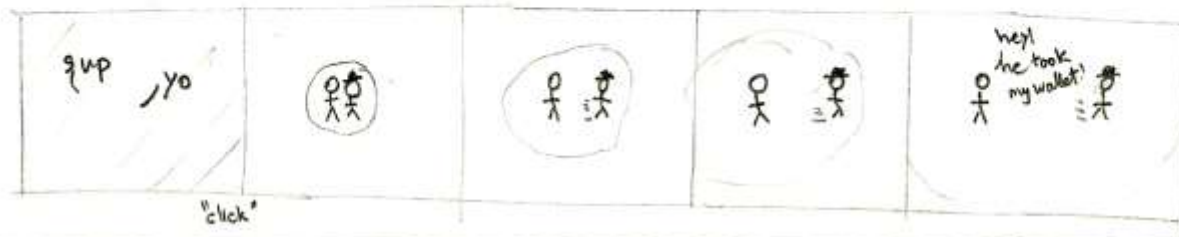
The person would see a sphere of light expanding away from him at 3×10^8 m/s. Not that big a deal. Now introduce a second person, running away from the first



consider a person sitting in the dark who emits a flash of light.



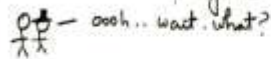
The person would see a sphere of light expanding away from him at 3×10^8 m/s. Not that big a deal. Now introduce a second person, running away from the first



Person A (Let's call him Bob) still sees light expanding away from him in a sphere whose center at which he is standing. But what about person B (Let's call him... Robert) According to special relativity, Robert also thinks light is moving away at the same speed in all directions. So Robert is in the center of the sphere. How is this possible?

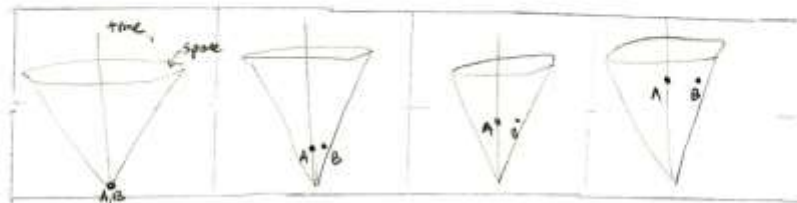


The answer is that movement can rotate your perception of space and time.

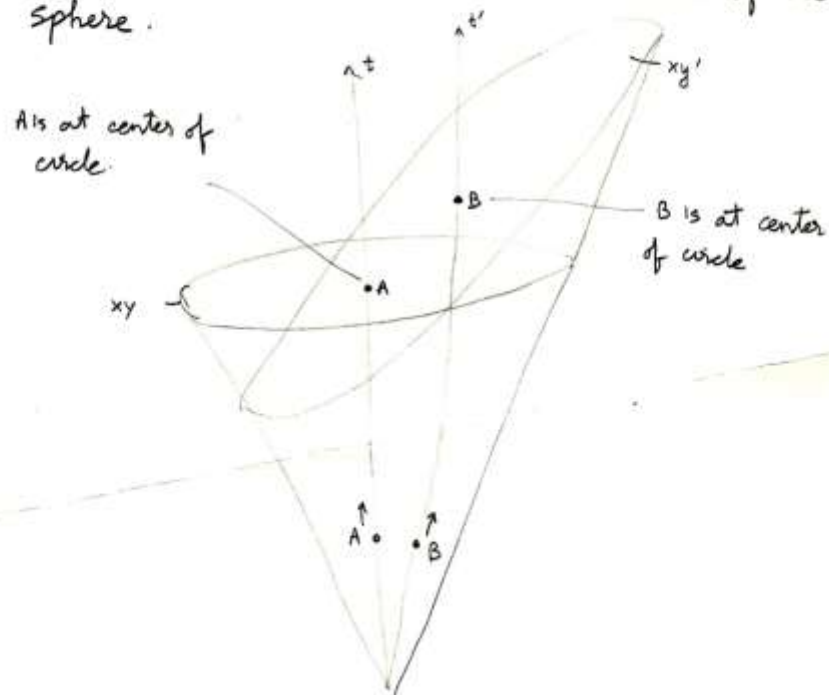


An easy way to visualize this is in the form of a light cone. If we restrict space to two dimensions and add time as the third, we can see how rotating spacetime can satisfy the requirement that both Bob and Robert be at the center of the sphere (now a circle in 2d)

From Bob's perspective, he is stationary while Robert moves away. Thus, he stays on the time axis (the only coordinate changing for him is time) while Robert moves away.



But there's a frame in which Robert is the stationary one and moves up the axis. Combining these two frames, rotating them as required, it's easy to see how both can be at the center of the sphere.



Thus, because of Special Relativity, movement rotates space and time in such a way so as to prevent things from exceeding the speed of light. Now, who wants ice cream?

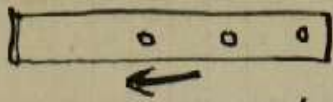
Special Relativity ushered in a new paradigm in western thought:

Conception over Perception

Knowing over Seeing

Truth = laws of Nature. We understand by reason. Math is the language of reasoning with Nature. Math gives us a way to understand what we can't experience

Reality = subjective, based on observations, which depend on the observer. "Truths" derived from perception are not universally true. Every person's reality is unique

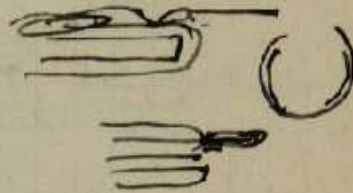


aus vorwärts oder rückwärts.
 Letzter Zug Siegen.



$$\frac{a^2 - a^2 (a+a)(a-a)}{a(a-a)}$$

$$a = a + a.$$



$$a = b + c \quad | \quad a - b$$

$$a^2 - ab = ab + ac - b^2 - bc$$

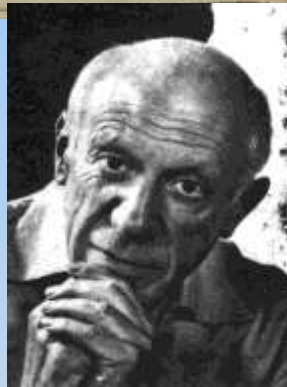
ac subtr.

$$a^2 - ab - ac = ab - b^2 - bc$$

$$a(a - b - c) = b(a - b - c)$$

$$a = b$$

In the early 20th century, relativity became a popular theme in art, music, and literature





African mask, Fang people



Nok sculpture,
Louvre exhibit



Detail from Les Demoiselles

Cubist movement was heavily influenced by primitive art as an abstract geometric formulation of perceived reality.



Rene Magritte



Salvador Dali

Picasso explored the problem of representing simultaneous viewpoints on one canvas.





Les Femmes d'Alger
Pablo Picasso, 1907

Guernica, painted by Picasso in 1939



Attempt to portray simultaneous viewpoints from 3 or 4 dimensions onto 2.



Guitar and Flowers
Juan Gris, 1912



Harbor in Normandy
Georges Braque, 1906



**Escher –
playing with rotations
through 4D?**



Representing intervals of time at one time, over a certain spatial interval

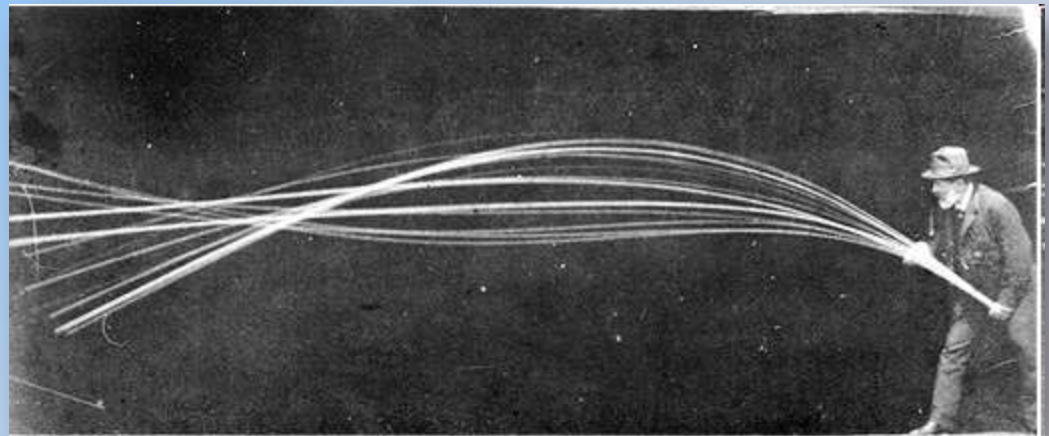
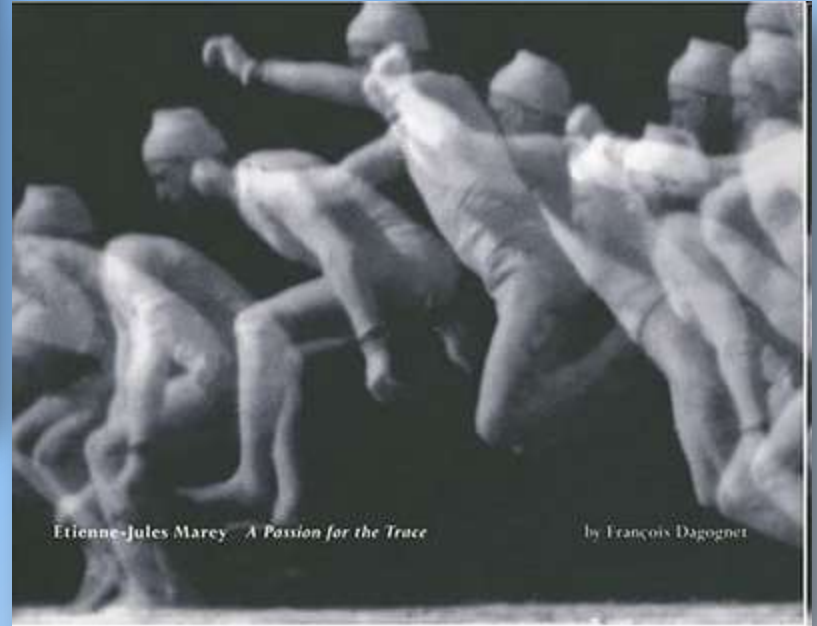


Umberto Boccioni (1882-1916)
Dynamism of a Soccer Player (oil on canvas, 1913)



Pelican in Flight

Multiple exposure photographs of Etienne-Jules Marey – technology for representing temporal sequences simultaneously





Marcel Duchamp
descending a flight of
stairs.



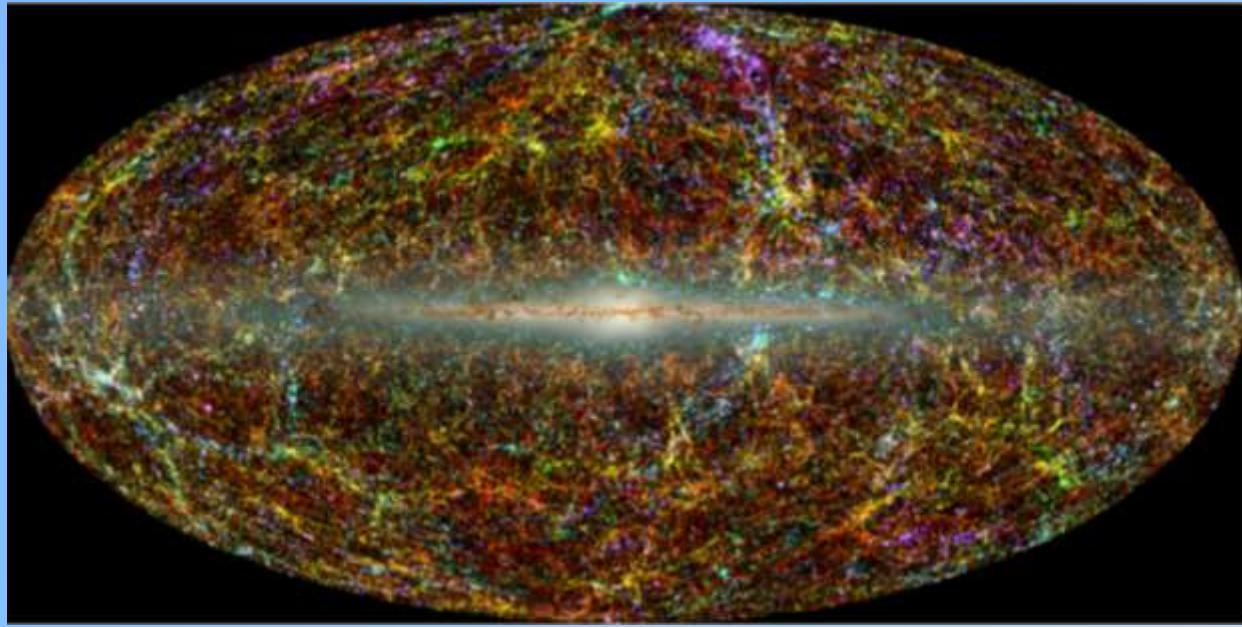
Nude descending a flight
of stairs, by Marcel Duchamp,
1916



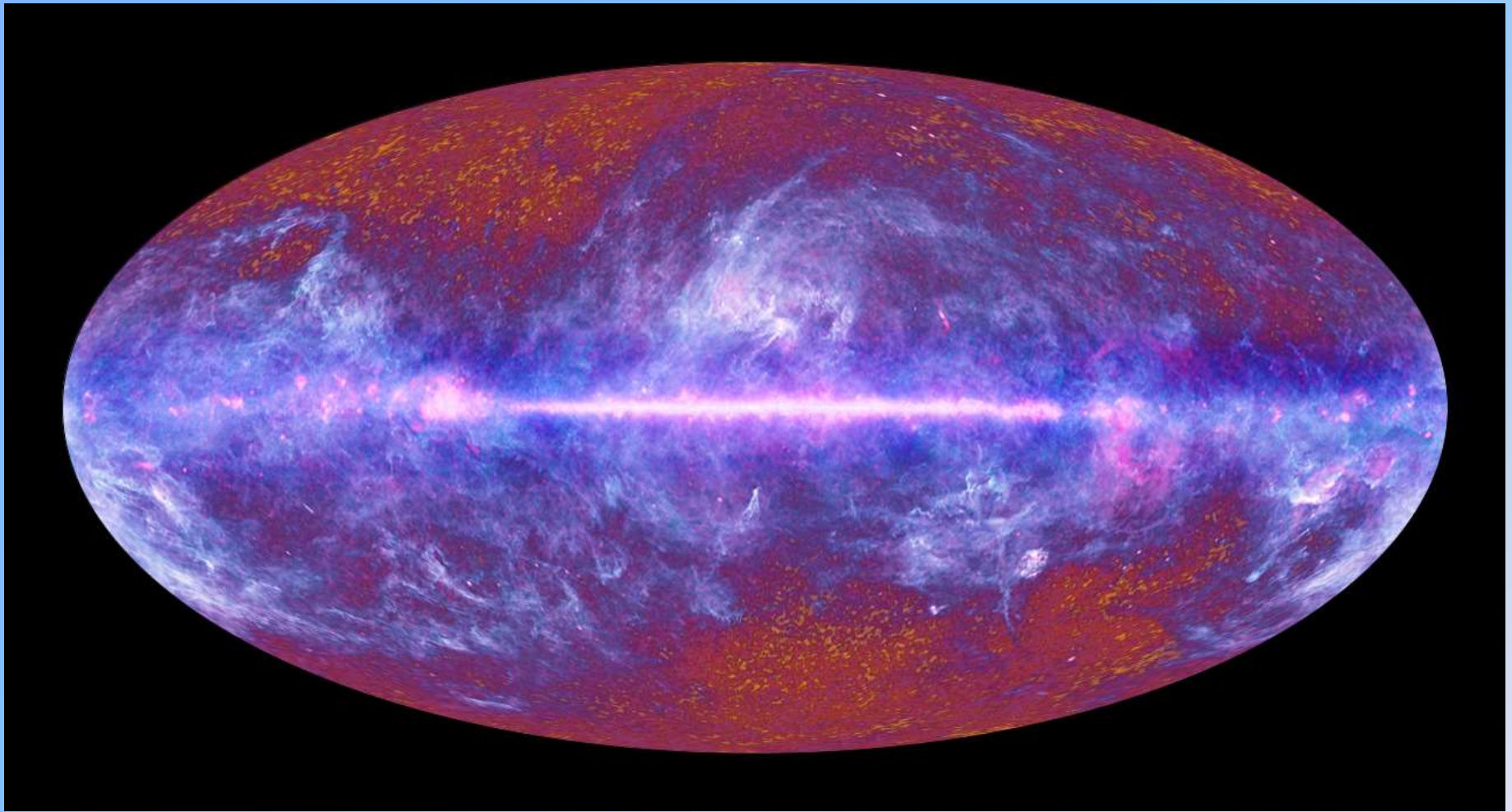
Andreas Gianopoulos, 3rd year math major, CCS-120, 2011



X-rays: looking through multiple layers in one view



modern galaxy surveys: looking back through a slice of spacetime



**Planck All-Sky map – composite of 9 frequencies from 30 to 857 GHz
Looking back through 13.7 billion years of time on one image**