Symmetry and Aesthetics in Contemporary Physics

CS-10, Spring 2016

Dr. Jatila van der Veen

CLASS 6:
FURTHER INTERROGATIONS OF REALITY

Special Relativity ushered in a new paradigm in western thought:

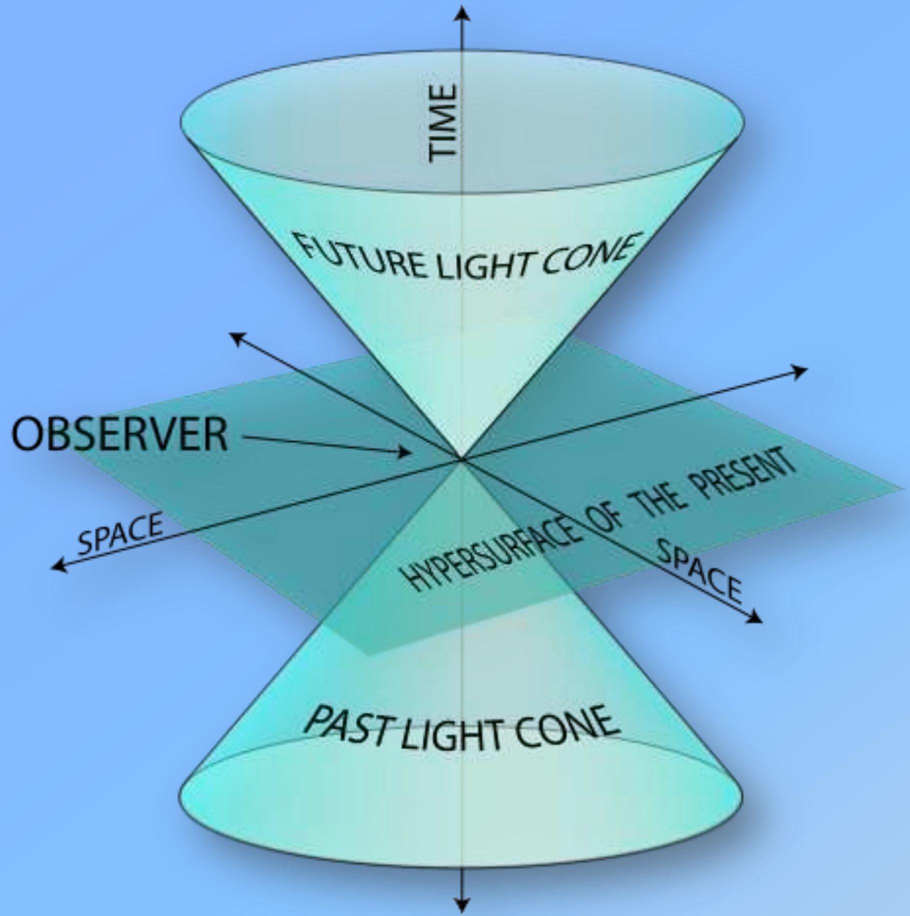
Conception over Perception

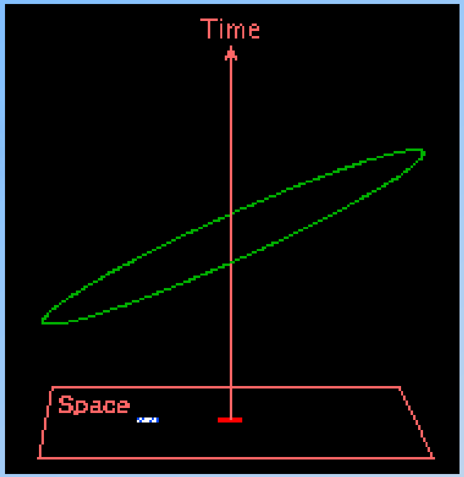
Knowing over Seeing

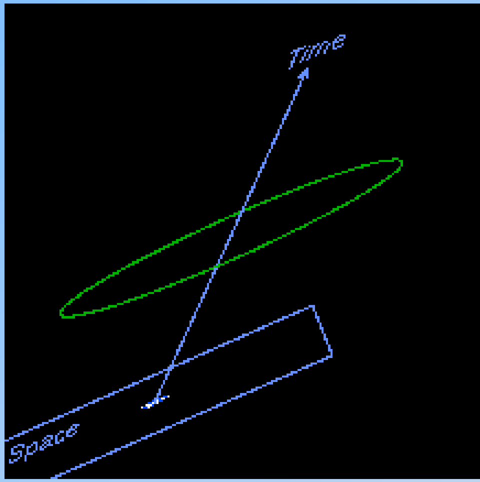
Truth = laws of Nature. We understand by reason. Math is the language of reasoning with Nature. Math gives us a way to understand what we can't experience	Reality = subjective, based on observations, which depend on the observer. "Truths" derived from perception are not universally true. Every person's reality is unique

Last week we looked at "reality" moving at high speeds:

A relativistic bike ride through Tubingen, Germany
Prof. Ute Kraus http://www.spacetimetravel.org/tuebingen/tuebingen.html







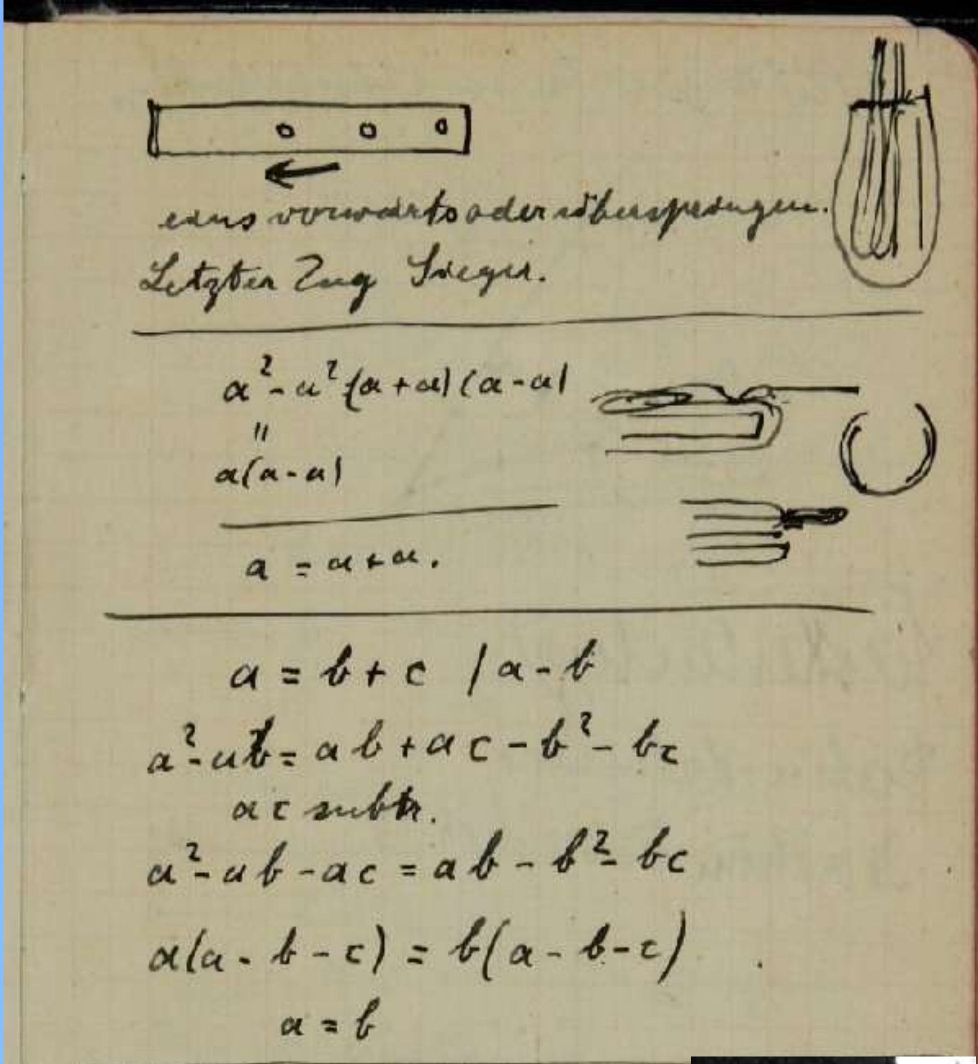
a photorealistic view of relativistic travel:

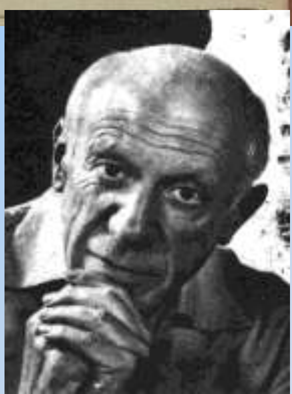
https://www.youtube.com/watch?v=JQnHTKZBTI4

lack of simultaneity in special relativity:

https://www.youtube.com/watch?v=Xrqj88zQZJg

…and artists' attempts to represent relativity on canvas





Rene Magritte

Salvador Dali

A DIFFERENT REALITY?

Nestinar, Ivailo Ayanski, Bulgaria

"On this day, June 3," explained Ivailo, "all cosmological signs of earth, fire, water, air, come together. All these forces align and create the appropriate conditions for firewalkers to go beyond the everyday, to overcome, what are considered the laws of physics. On that day, in the fire, everything turns its sign around.

Ayanski interview, 2011:

https://www.youtube.com/watch?v=fGcRgUvyxLs

Ayanski, firewalk, 2010:

https://www.youtube.com/watch?v=5rAb8GDa_x4

"I know about the power of minerals and stones; the whole of nature talks to me and resonates within me. I communicate with the world on a different vibrational level. It is a magical thing which was given to me during my childhood by my grandmother and I am immensely thankful for it."

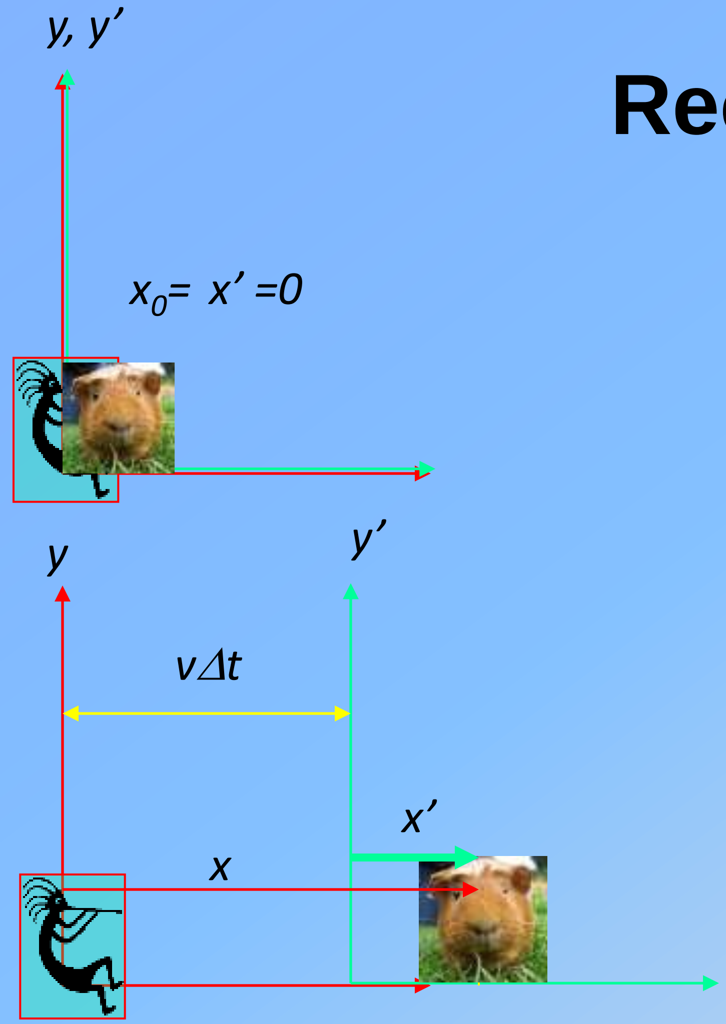
Lifted Up by the Power of the Saints: Prihvanati, Music, and Embodied Experience in the Firewalking Rituals of Two Bulgarian Nestinari
Plamena Kourtova, 2007, Masters Thesis
http://diginole.lib.fsu.edu/islandora/object/fsu:181082/datastream/PDF/view

Discussion of Zee's chapters 3 & 4, in which he recounts basically the same story as we read in Feynman and discussed in class, but with his own perspective…

Comments?
Opinions on his explanations?

What do you think of his statement in the last paragraph on p. 75?

Recall from last week:

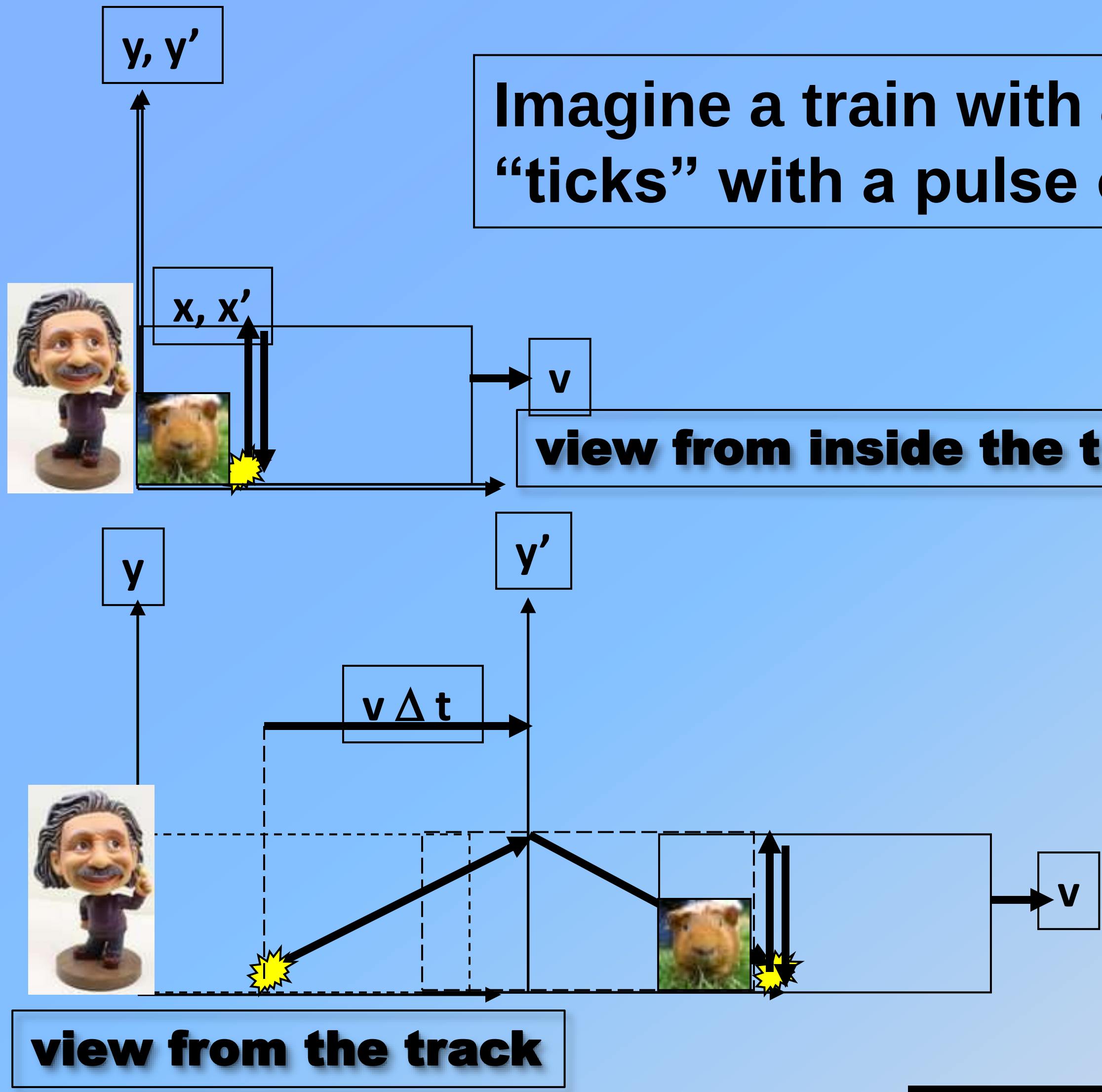


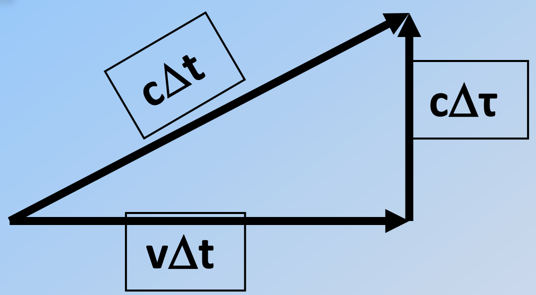
p. 54:
Relative motion as a symmetry
Galilean invariance

$$\begin{pmatrix} x' \\ y' \\ z' \\ \Delta t' \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ \Delta t \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$x' = x - v\,\Delta t$
$y' = y$
$z' = z$
$t' = t$
time is the same for everybody

Imagine a train with a light clock that “ticks” with a pulse of light once/second.





$$c^2 \Delta t^2 = v^2 \Delta t^2 + c^2 \Delta \tau^2$$

Lorentz Invariance: For motion along the x-axis:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

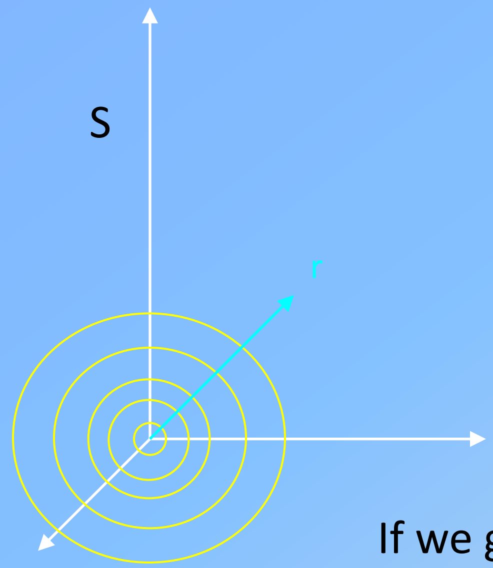
$$z' = z$$

The Lorentz Transformation looks suspiciously like a ROTATION which MIXES space and time!

Developing the idea of a new 'geometry' for spacetime:

A pulse of light spreads out in a sphere of radius r. A sphere is defined in space at any instant of time as satisfying the relation: $r^2 = x_1^2 + x_2^2 + x_3^2$.

But, since light travels at speed c, we know the sphere is expanding as its radius grows at the rate $r = ct$.



Putting these together, we have:

$$r^2 = x_1^2 + x_2^2 + x_3^2 = c^2t^2$$

or

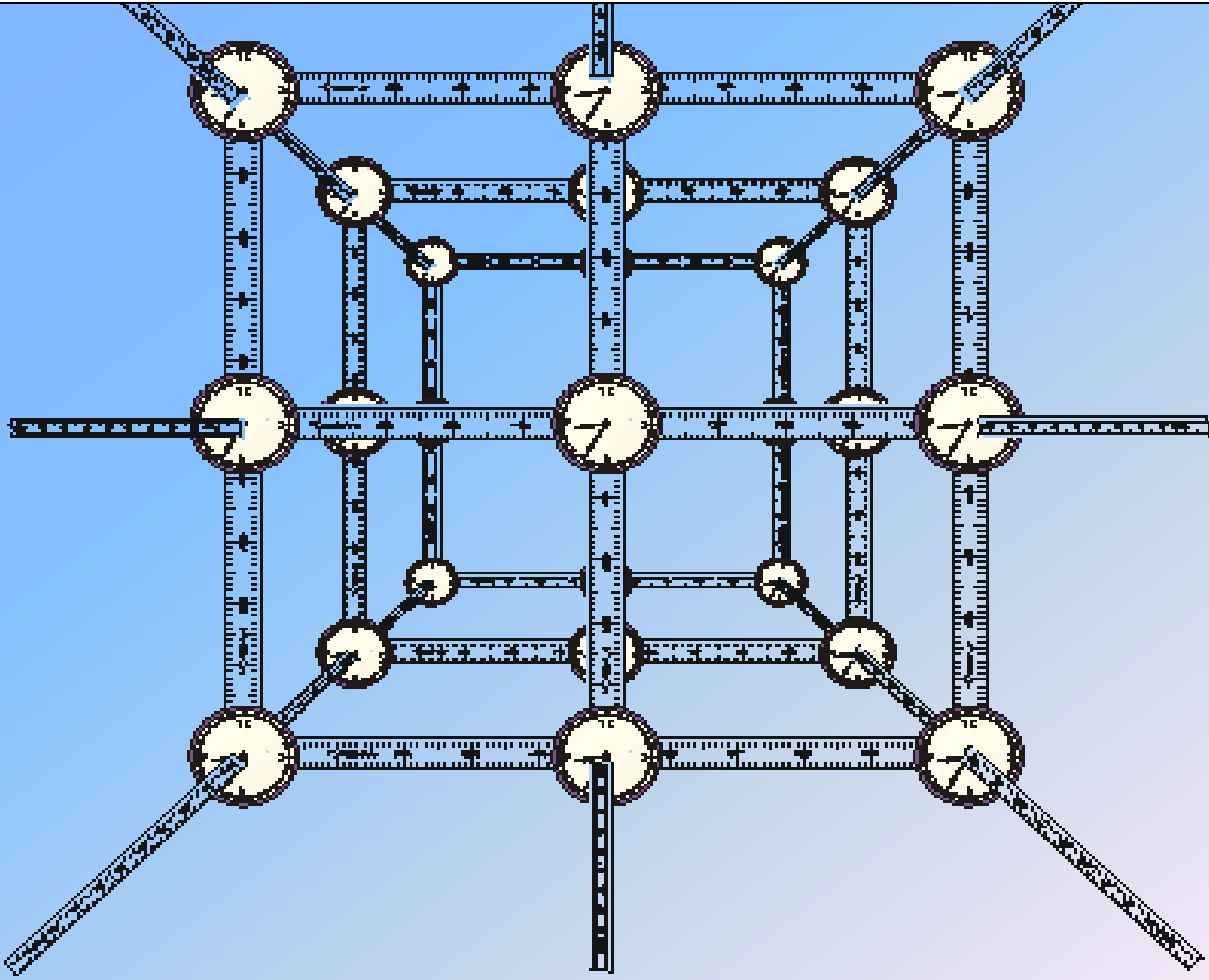
$$x_1^2 + x_2^2 + x_3^2 - c^2t^2 = 0$$

If we generalize these coordinates to x_1 , x_2 , x_3 , and x_4 we must choose $x_4 = ict$ where $i = \sqrt{(-1)}$

So, Einstein generalized space and time coordinates into a spacetime continuum in a complex geometry, and introduced the idea of 4-vectors with components x, y, z, and ict.

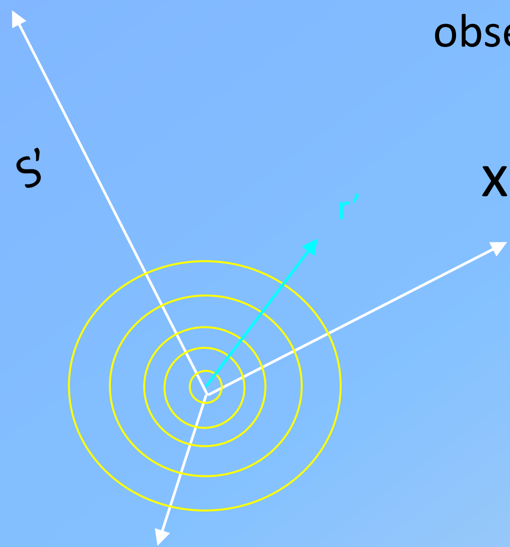
$$\boxed{\begin{aligned} x_1 &= x \\ x_2 &= y \\ x_3 &= z \\ x_4 &= ict \end{aligned}}$$

Clocks are devices that are used for measuring time-like distances.
Rulers are devices that are used for measuring space-like distances.



So, we have $x_1^2 + x_2^2 + x_3^2 + x_4^2 = x^2 + y^2 + z^2 - c^2t^2 = 0$

An observer in another frame would observe for the same light pulse:



$$x'^2 + y'^2 + z'^2 - c^2t'^2 = 0$$

We know c = constant for all observers.

So we define the invariant “spacetime interval: ”

$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\,\Delta t^2$$

From the definition $\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\,\Delta t^2$

we define $\Delta\tau^2 \equiv - \Delta s^2/c^2$ as the proper time

$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2$

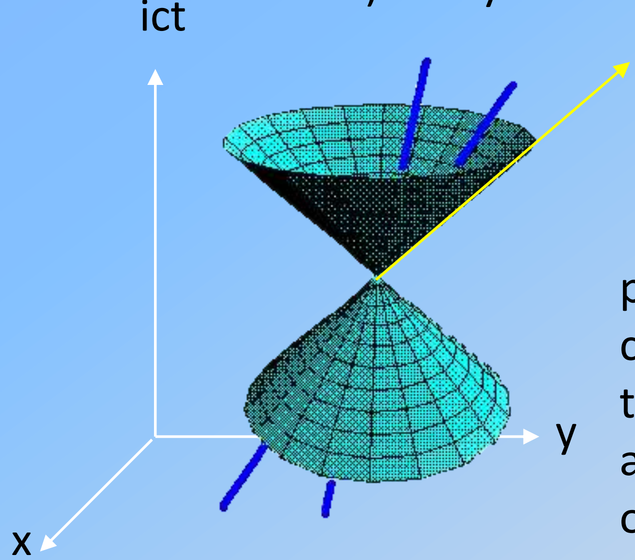
$\Delta\tau^2 \equiv - \Delta s^2/c^2$ is the proper time

For $(\Delta s^2) > 0$ points are space-like separated

For $(\Delta s^2) = 0$ this corresponds to $\Delta x^2 + \Delta y^2 + \Delta z^2 = c^2 \Delta t^2$ or traveling at the speed of light – called “null” or “light-like” separated

For $(\Delta s^2) < 0$ points are time-like separated

particles with non-zero rest mass follow time-like paths (world lines) always inside the light cone



photons with zero rest mass follow paths of $\Delta s^2 = 0$

particles which follow space-like world lines have been called tachyons. Tachyons would travel always faster than the speed of light, would have negative energy, and would violate causality...none have ever been observed!

Intuitive derivation: see pp. 70 - 75 of Fearful Symmetry…

classically,

$$E = \frac{mv^2}{2}$$

$$p = mv$$

combining these:

$$p^2 = m^2 v^2$$

$$E = \frac{p^2}{2m}$$

we get the dispersion relation between energy and momentum in classical physics

Einstein's guess:

$$\boxed{E^2 - c^2 p^2}$$ is the invariant quantity (Energy – momentum)

$$\longrightarrow \boxed{E^2 - c^2 p^2 = (m_0 c^2)^2}$$

in particle's rest frame, p=0

$$\boxed{E = \pm m_0 c^2}$$

Dirac: negative root → antimatter

We'll come back to this controversy later!

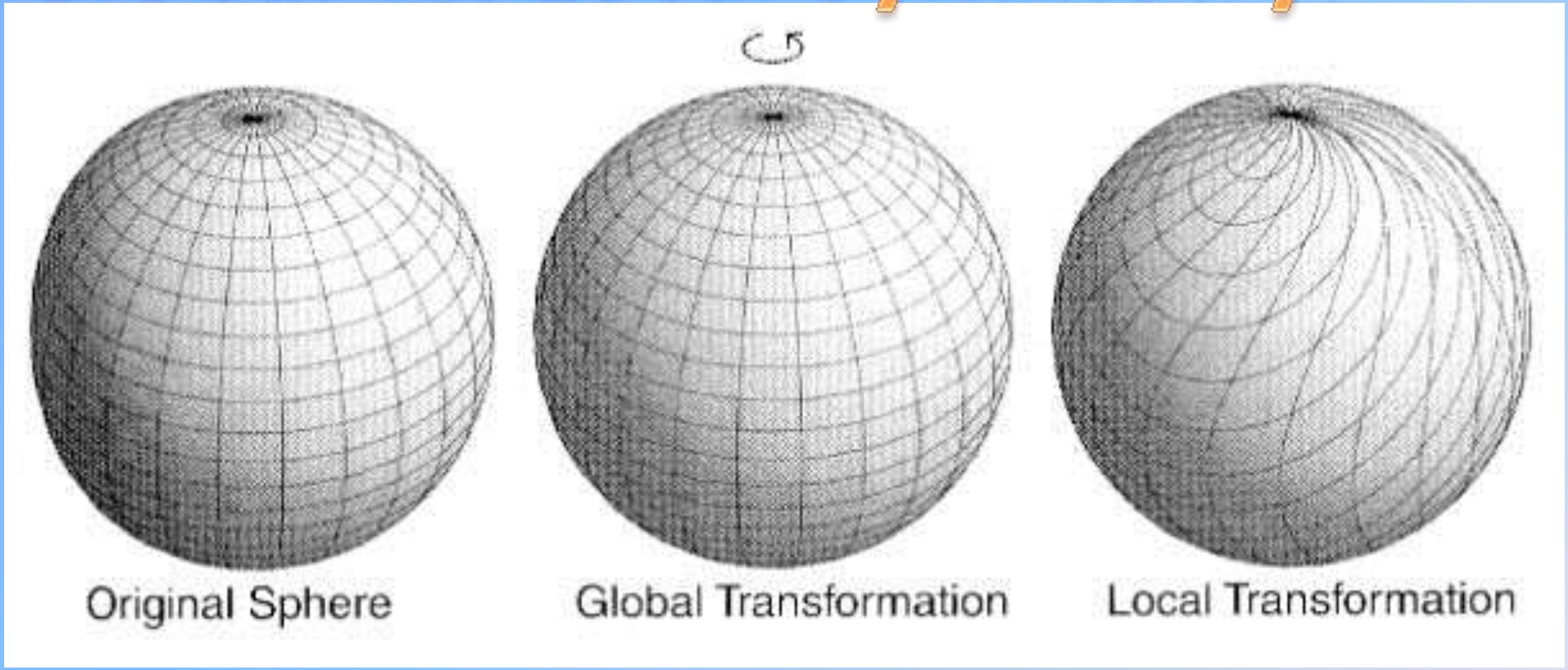
Aside: Without deriving here, Einstein's revision of Newtonian ideas of mass and momentum:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

…hence, as you approach the speed of light, mass increases without limit – i.e., only massless particles can travel at the speed of light

Global vs. Local Symmetry:



- A global symmetry does not depend on spacetime.

* A local symmetry depends on spacetime.

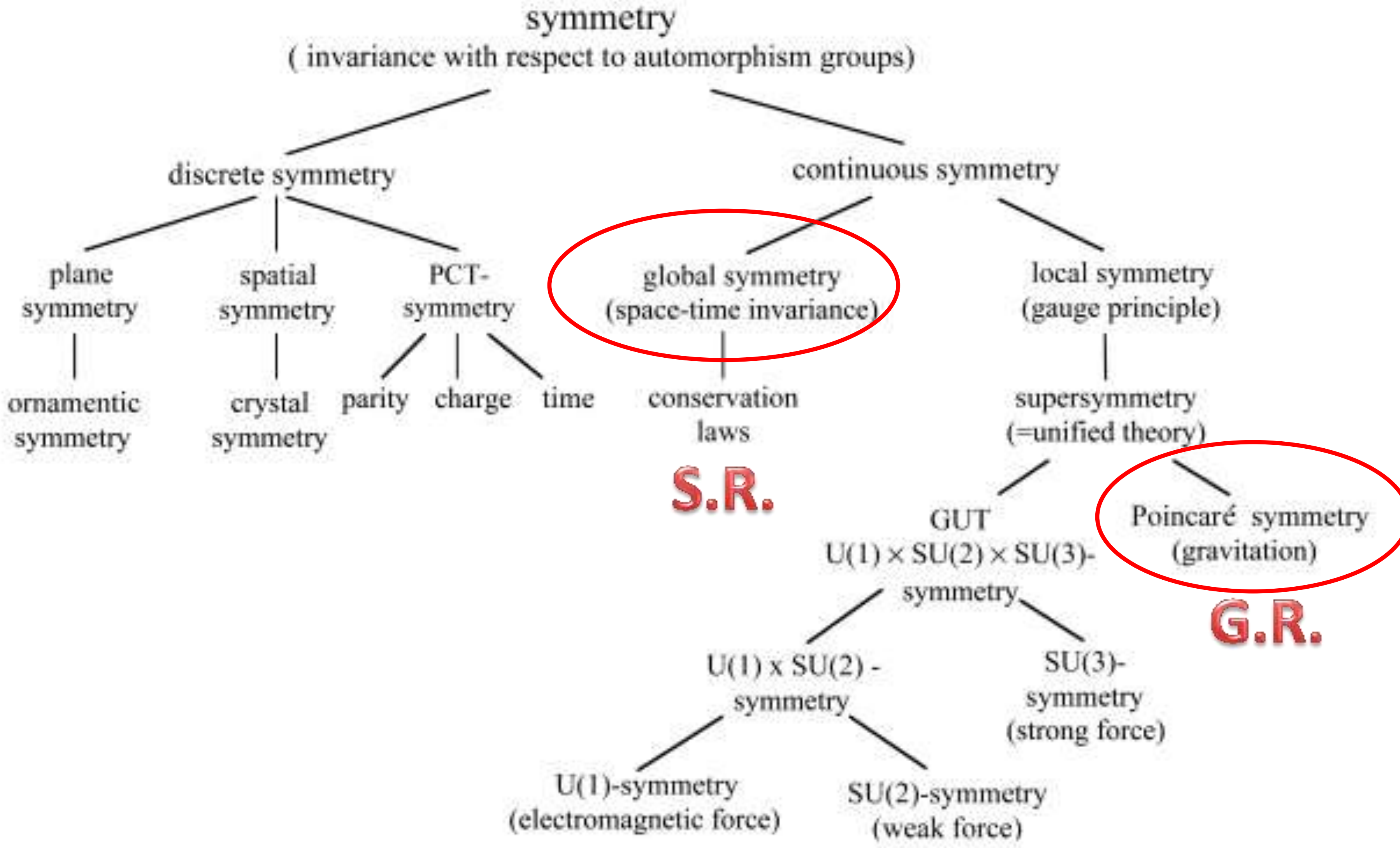
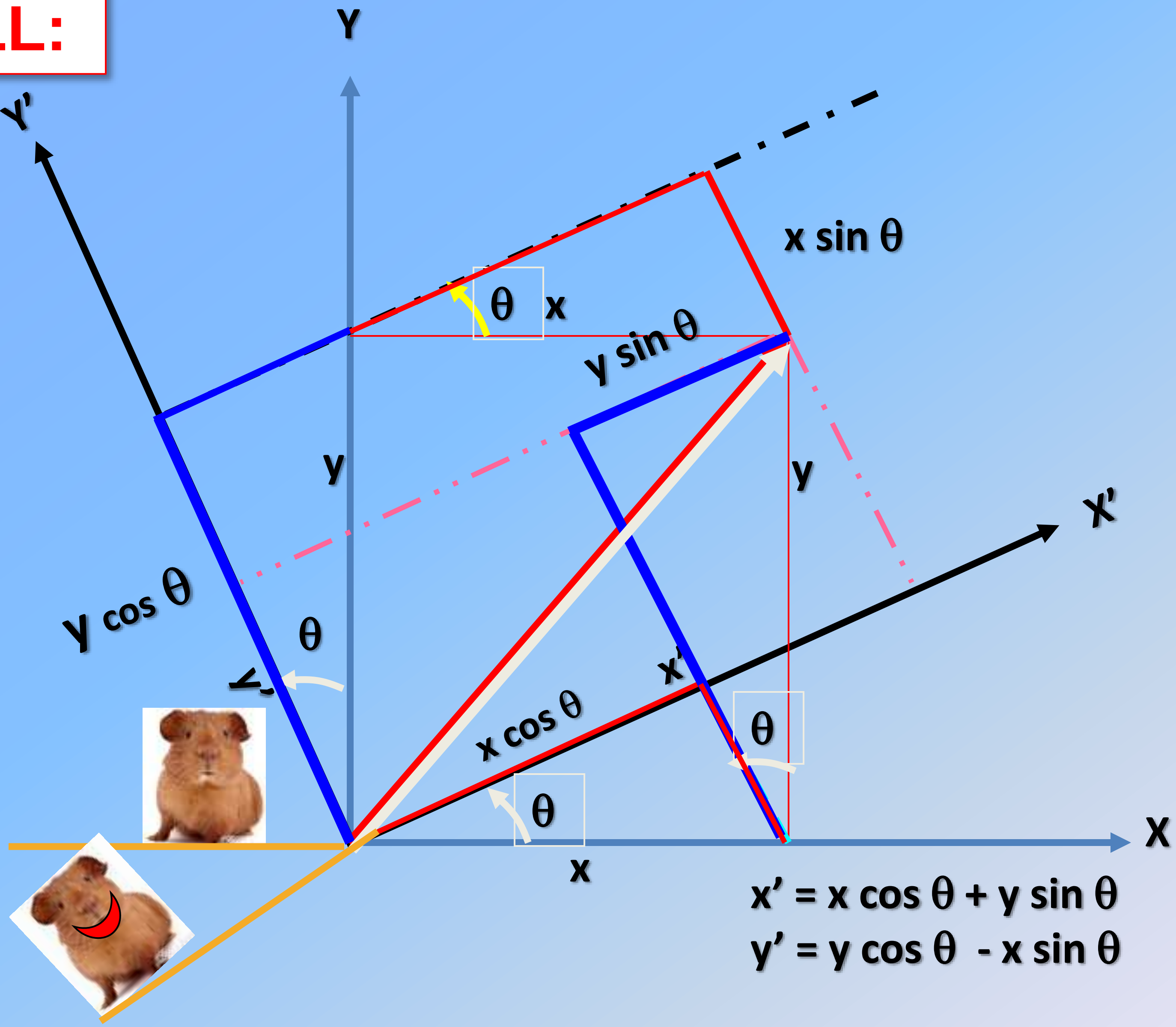


Figure 9. Classification of symmetry

RECALL:



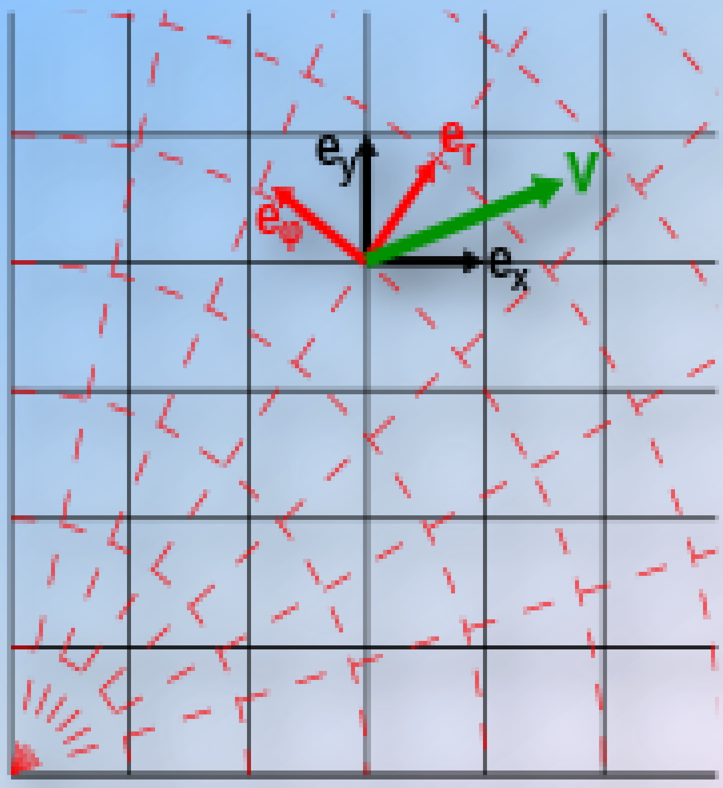
$x' = x \cos\theta + y \sin\theta$

$y' = y \cos\theta - x \sin\theta$

$$L'^2 = x'^2 + y'^2 = x^2 (\cos^2\theta + \sin^2\theta) + y^2 (\cos^2\theta + \sin^2\theta) = x^2 + y^2 = L^2$$

Call this rule "Λ" for describing the counter clockwise rotation of the reference frame:

$$\Lambda = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$



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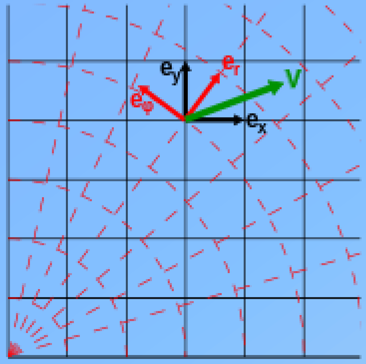


If this describes a group, it should have the 4 properties that define a group:

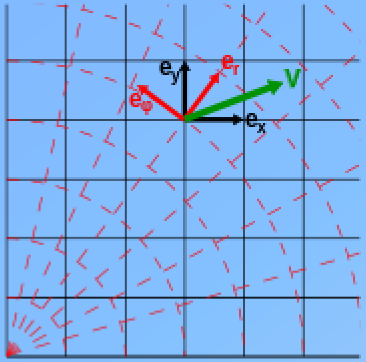
Is it closed under repeated rotations? ✓

Are repeated rotations associative? ✓

Is there an identity element ? $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ✓

Is there an inverse ? **If it is a group, there should be!**

$$\Lambda = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$



$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

general rule for finding the inverse of a matrix:

$$\Lambda^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{\det\Lambda}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\Lambda = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\det\Lambda = \cos^2\theta + \sin^2\theta = 1$$

$$\Lambda^{-1} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}^{-1} = \frac{1}{\det\Lambda}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

check: does $\Lambda\Lambda^{-1}=1$?

$$\Lambda^{-1}\Lambda = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\theta + \sin^2\theta & -\cos\theta\sin\theta + \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\boxed{\checkmark \textbf{YES!}}$

Summary of SO(2):

Transpose

$$\Lambda = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\Lambda^T = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$\Lambda^T \Lambda = \begin{pmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \cos^2\theta + \sin^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

condition of orthogonality:
the inverse is the transpose

plus the special property:

$$\det\Lambda = \cos^2\theta + \sin^2\theta = 1$$

This is a representation of the infinite group of rotations in the real plane – Special Orthogonal Group of order 2.
Conditions for all SO(n) groups: Orthogonal, and determinant = 1

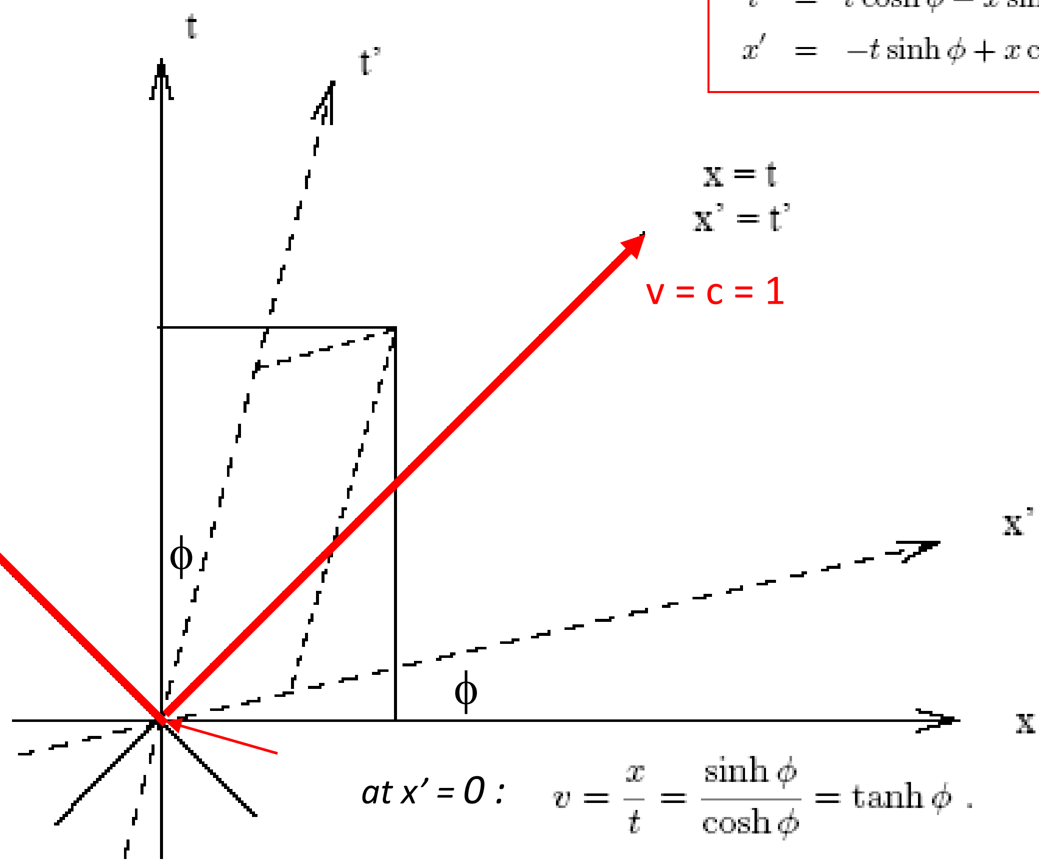
Galilean Symmetry: All inertial reference frames are identical. If you don't look out the window, you can't tell if you're moving or not. Global symmetry in 'flat' space.

Galilean Transformation written out as equations (left) and in short hand (matrix) notation (right). ***Global symmetry IN Euclidean space***

$$
\begin{aligned}
x' &= x - v\Delta t \\
y' &= y \\
z' &= z \\
\Delta t' &= \Delta t
\end{aligned}
$$

$$
\begin{pmatrix} x' \\ y' \\ z' \\ \Delta t' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & -v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ \Delta t \end{pmatrix}
$$

In the flat, 4D Lorentz-invariant reference frame (Minkowski spacetime) in which we envision ourselves 'at rest' in a moving (but still inertial) frame, at some velocity which is a large fraction of the speed of light, we have 'four vectors' (-t, x, y, z).



$$t' = t\cosh\phi - x\sinh\phi$$
$$x' = -t\sinh\phi + x\cosh\phi\ .$$

$$v = \frac{x}{t} = \frac{\sinh\phi}{\cosh\phi} = \tanh\phi\ .$$

Diagram of a Lorentz boost taken from Sean Carroll's on-line notes on General Relativity, available at http://arxiv.org/PS_cache/gr-qc/pdf/9712/9712019v1.pdf.

Lorentz Invariance:

For motion along the x-axis:

DEFINE:

$$\gamma = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$\beta = \frac{v}{c}$$

$$or, v = \beta c$$

$$x' = \gamma(x - vt)$$

$$t' = \gamma\left(t - \frac{\beta x}{c}\right)$$

Lorentz Transformation: the rule that translates between inertial reference frames

$$\begin{bmatrix} x' \\ y' \\ z' \\ t' \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta c \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{\gamma\beta}{c} & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix}$$

Global symmetry in Minkowski spacetime (no mass, no gravity)

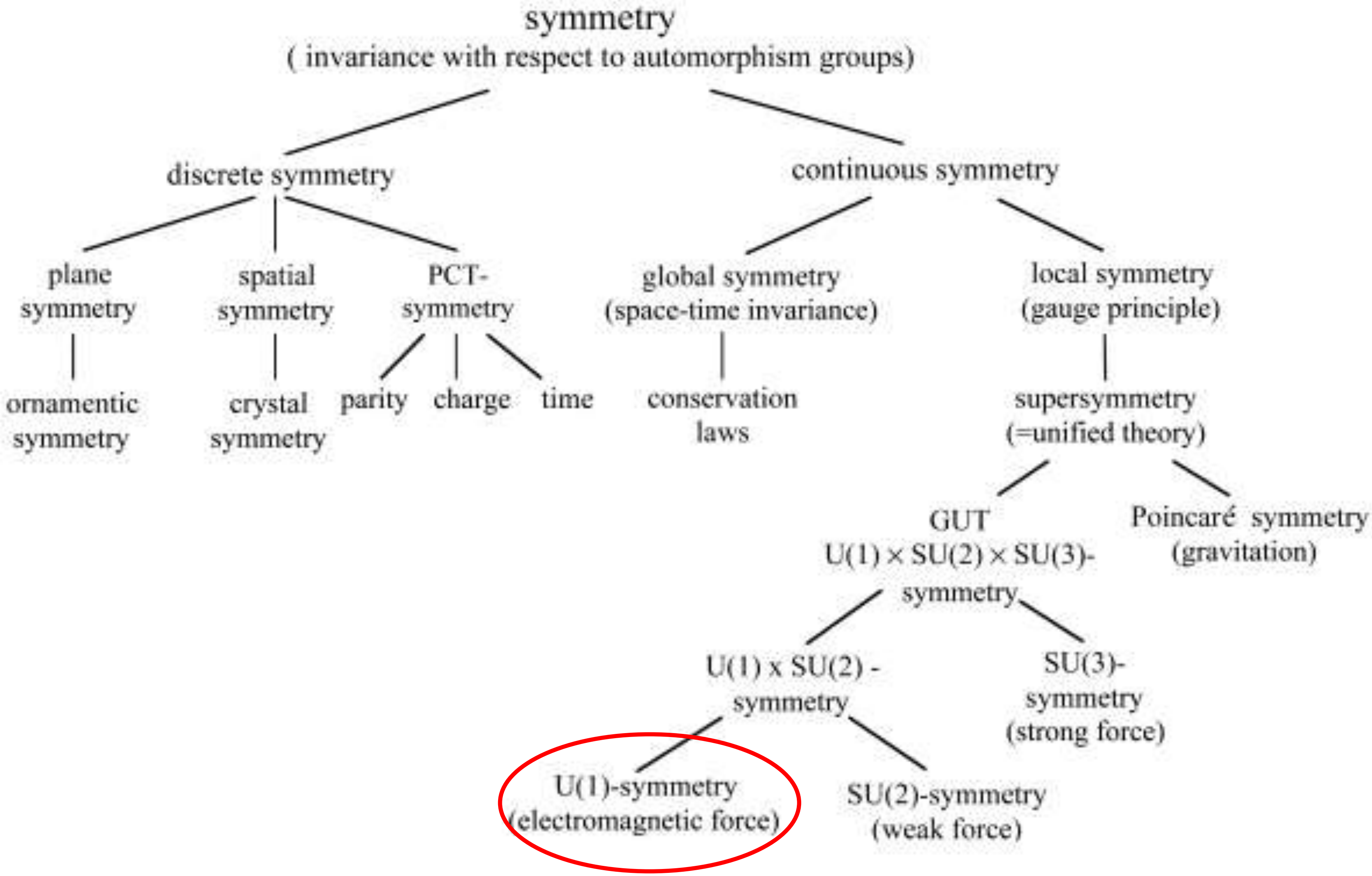
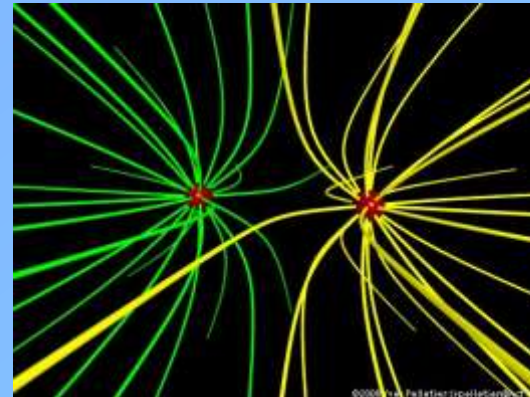
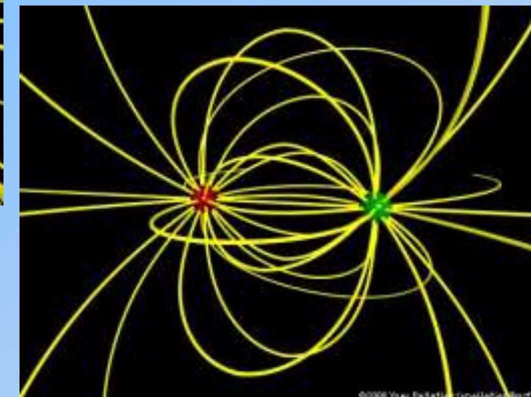


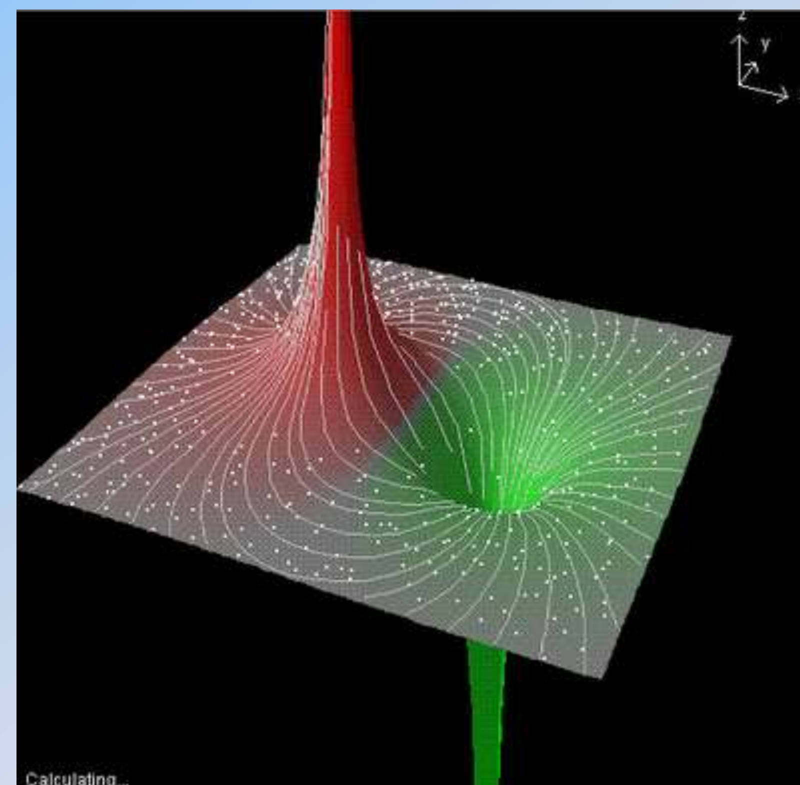
Figure 9. Classification of symmetry





Michael Faraday in his lab
Painting by Harriet Jane Moore

Faraday's Field Lines:
The first idea that a charge creates a field which influences the shape of the space around it, and effects other charges.



Definition of unitary groups U(n):

$U^* = U^{-1}$ … i.e., the complex conjugate transpose is equal to the inverse

Special unitary groups SU(n):

satisfy the additional condition that |det | = 1.

A gauge theory is a theory where the action is invariant under a continuous group symmetry that depends on spacetime.

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The transformation that depends on spacetime is called a gauge transformation.

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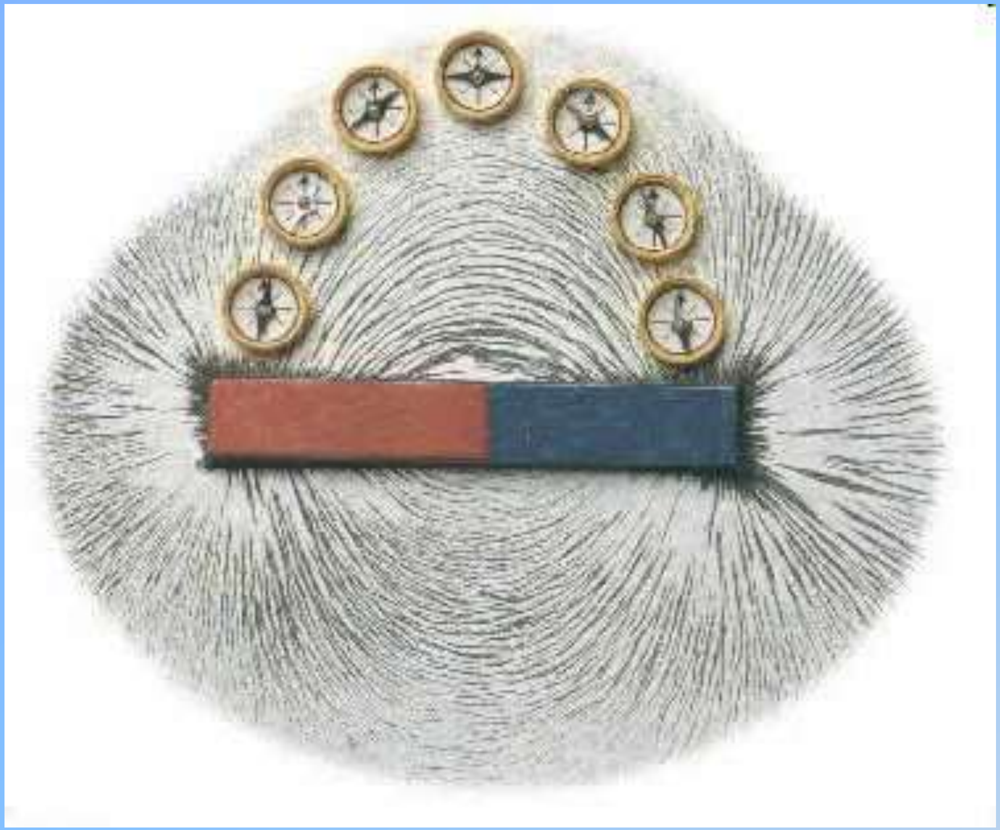
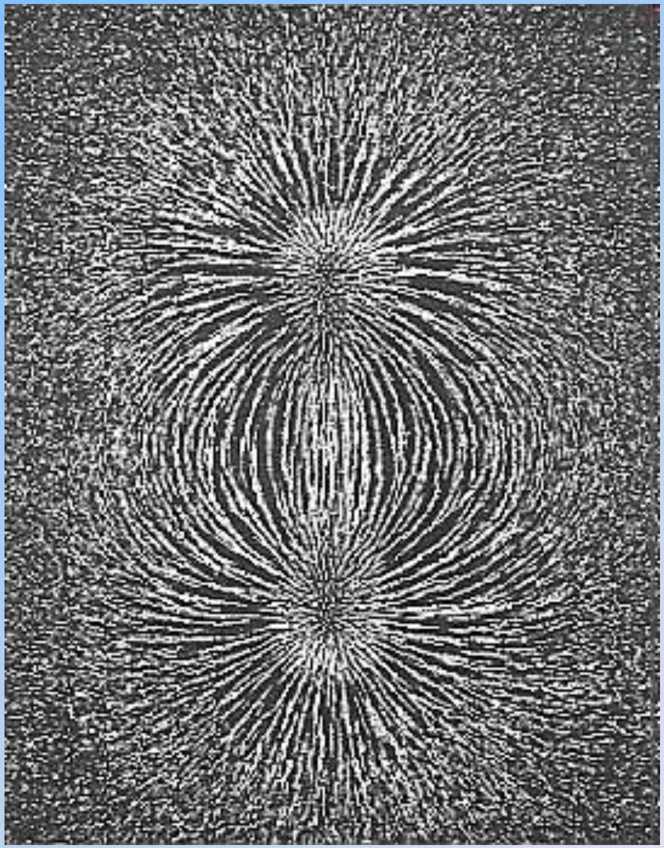
Gauge symmetries introduce gauge fields to the theory which mediate a force.

Iron filings follow magnetic field lines of a dipole magnet





An electron *at rest* in a magnetic field does not "feel" a magnetic force. Only an electron moving in a magnetic field is deflected by a force: F = qv x B.

U**(1) symmetry** ***= rotation by phase angle, describes the*** **Electromagnetic interaction.**

E&M fields are described by **A**, a vector potential, and V, a scalar potential.

Invariance under gauge transformation means that making a small change in **A** must be accompanied by a corresponding small change in V.

For example:

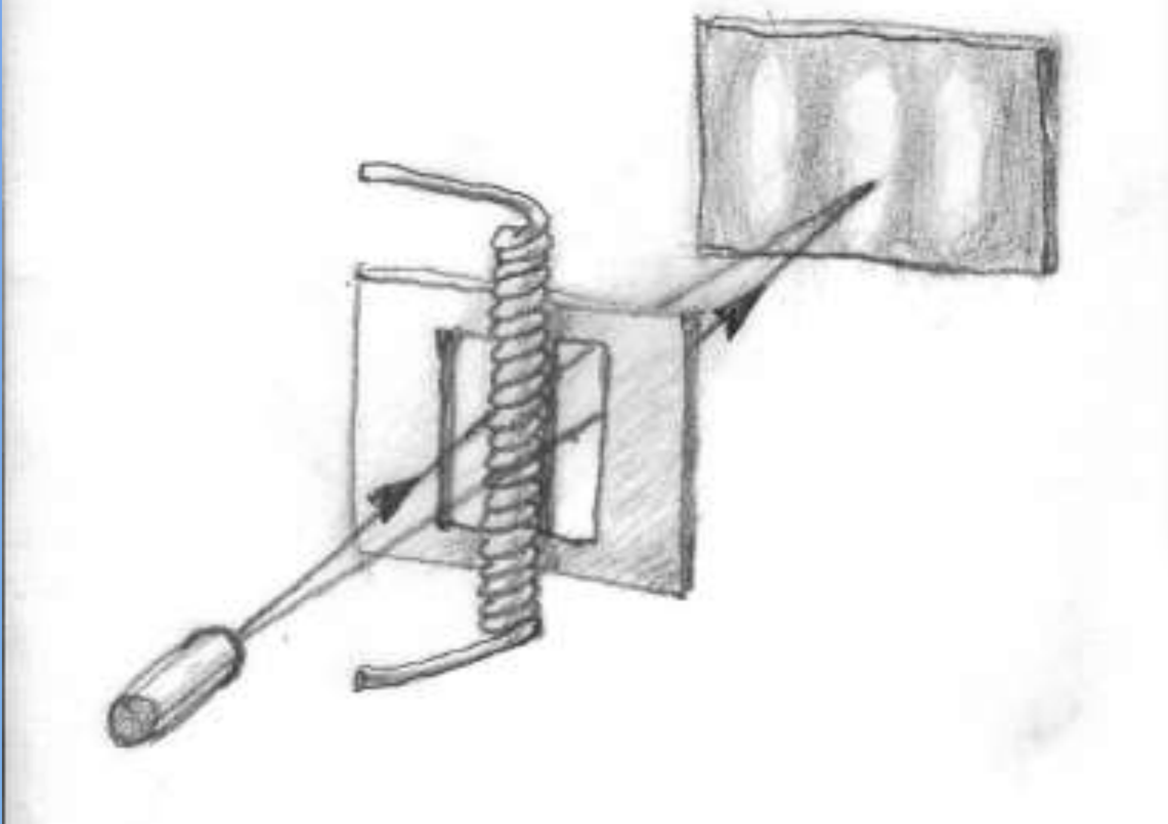
E&M fields are described by **A**, a vector potential, and V, the scalar potential.

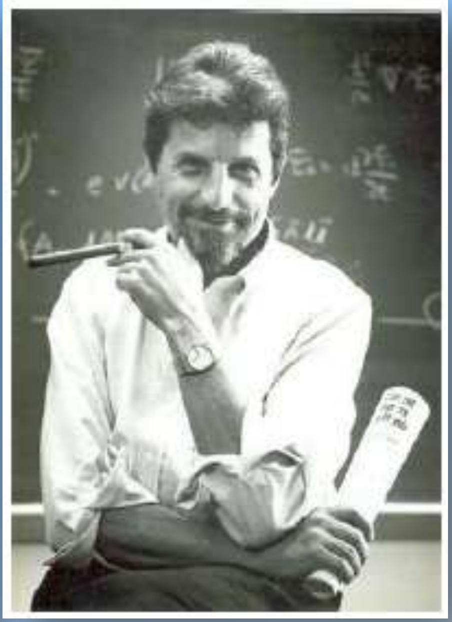
Invariance under gauge transformation means that making a small change in **A** must be accompanied by a corresponding small change in V:

$$\overline{A}' = \overline{A} + \nabla \lambda$$

$$V' = V - \frac{\partial \lambda}{\partial t}$$

Aharonov-Bohm Experiment





Aharonov

Bohm

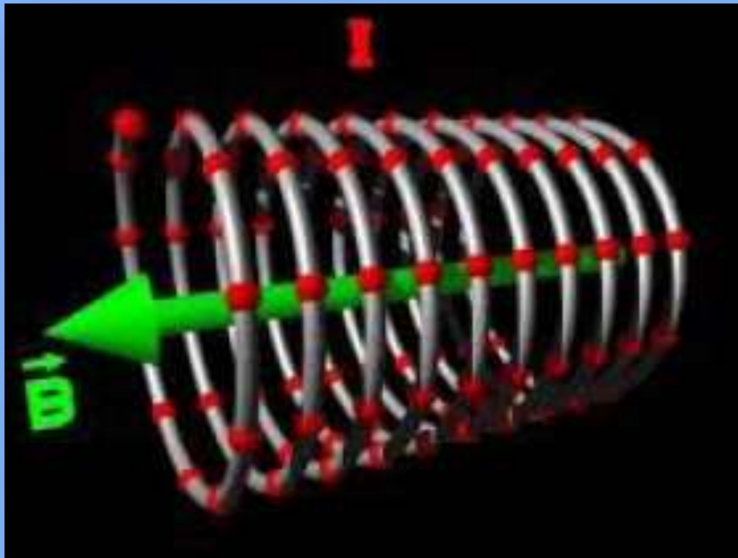
Shot electrons through a slit with a solenoid in front of it. With no current, the electrons arrived at the screen in phase. With the current on, the electrons arrived out of phase.

Electrons behave both like particles and like waves.

$$\Psi(\vec{x},t) = |\Psi(\vec{x},t)|e^{i\theta(\vec{x},t)}$$

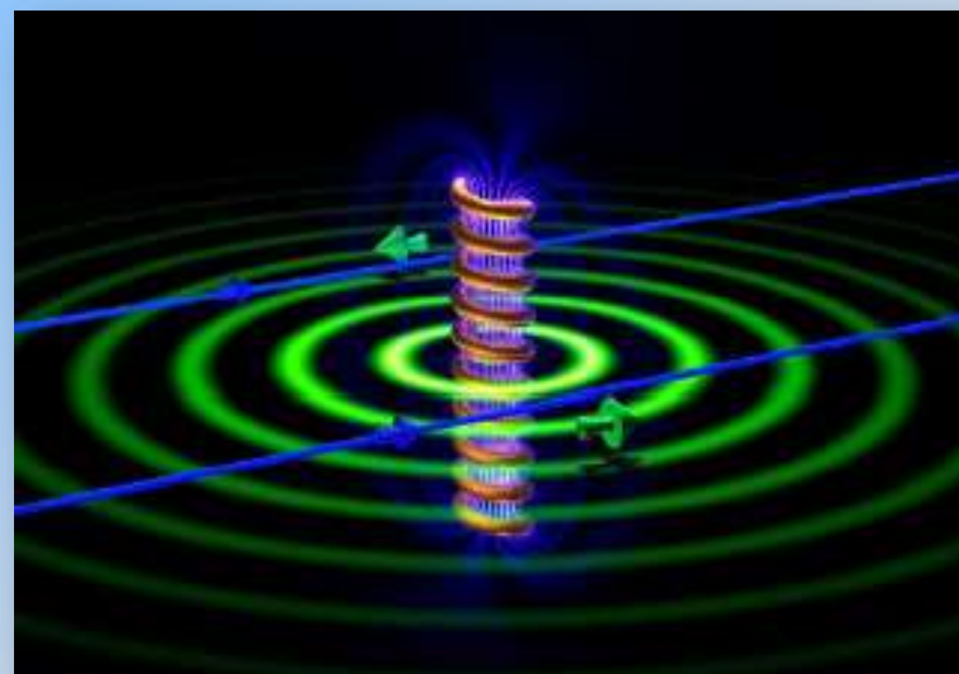
Louis de Broglie

WAVE-PARTICLE DUALITY



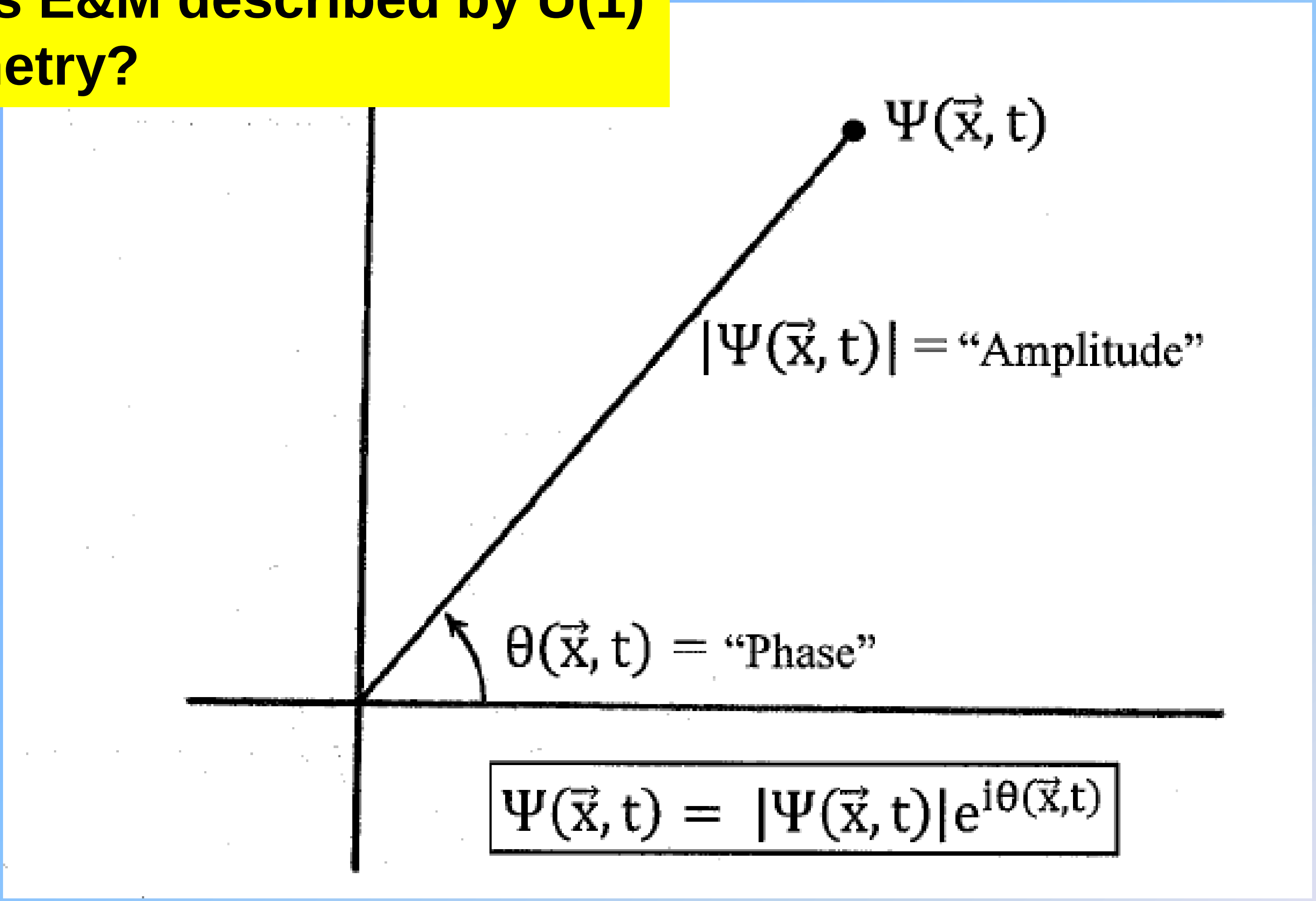
Magnetic field outside a solenoid is zero, but there is a magnetic *potential* (A) which affects the phase of the electrons, so they arrive out of phase at the screen.

A does not affect the motion of the electrons, but affects their phase.



A is the first-discovered example of a gauge field.

Why is E&M described by U(1) symmetry?

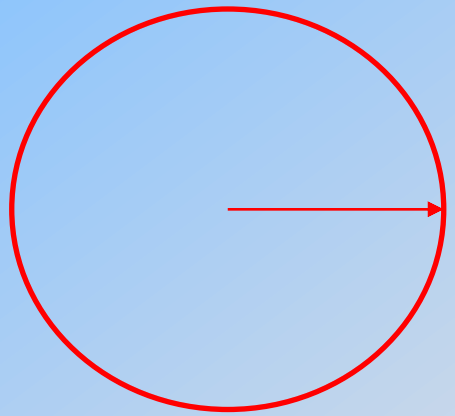


Electrons described as a vector in the complex plane

The phase factors are just complex numbers with amplitude 1.

$$e^{i\theta(x,t)}$$

So we can picture them as points on a circle of radius 1 in the complex plane.

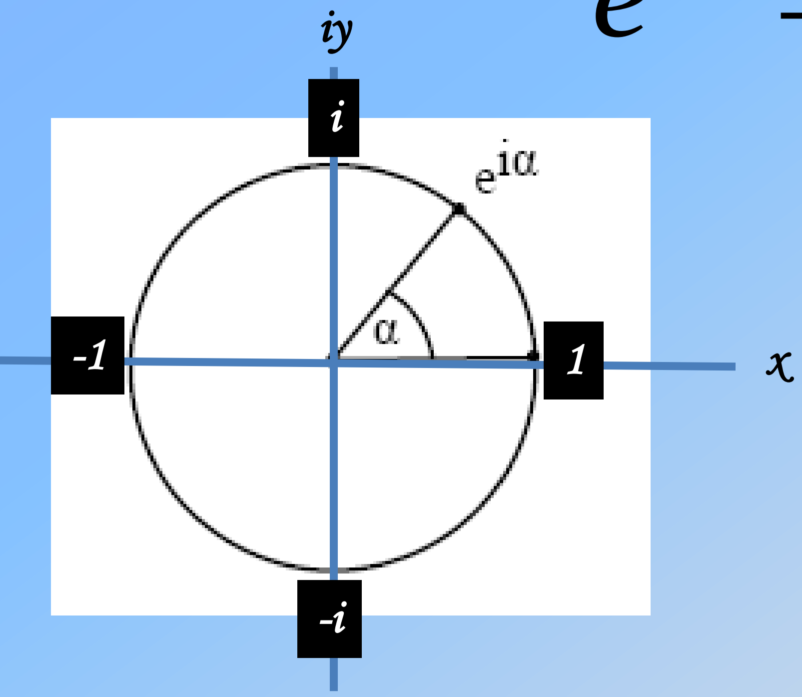


This collection of complex numbers is a representation of the group U(1). It is the complex equivalent of SO(2), rotations in the real plane.

U(1) is the group of all possible phase multiplications $e^{i\alpha}$

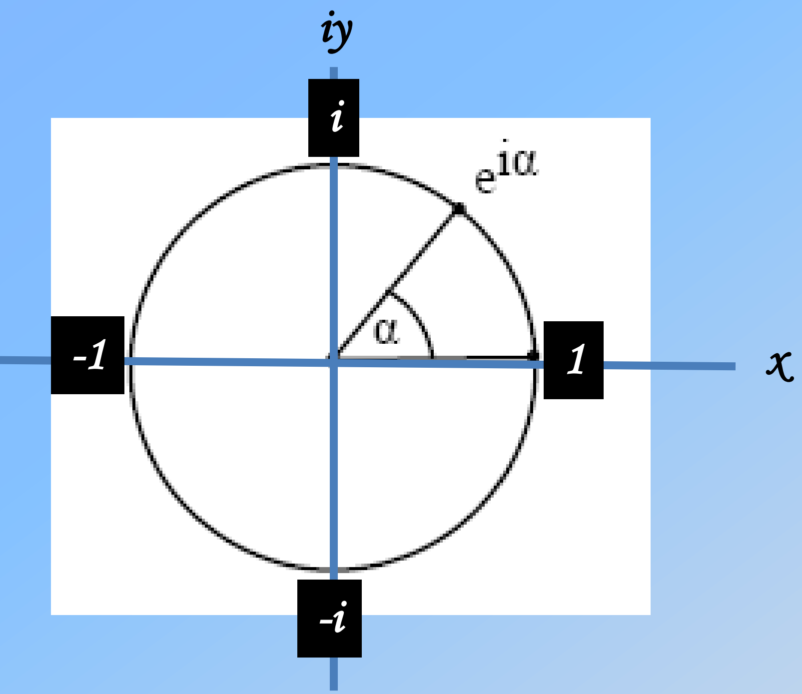
$$\Psi(x) \to e^{i\alpha}\Psi(x) \quad ; \quad \overline{\Psi}(x) \to e^{-i\alpha}\overline{\Psi}(x)$$

$$e^{i\alpha} = \cos\alpha + i\sin\alpha$$



The U(1) group has the property that the internal rotation operations are commutative: $e^{i\,\theta\,\alpha 1}\,e^{i\,\theta\,\alpha 2} = e^{i\,\theta\,\alpha 2}\,e^{i\,\theta\,\alpha 1}$.

Hence it is called Abelian, after the mathematician Niels Henik Abel.



Local *U*(1) symmetry *describes the* Electromagnetic interaction.

***U*(1) local symmetry → a gauge field mediates the interactions between the charge fields.**

Invariance under phase transformations requires a compensating change in the E-M field.

Similar gauge invariance exists for the strong and weak interactions, the "internal rotation" depends on more than one parameter in these cases. Group of objects can be formed from these generalized "rotational displacements".

However, these elements are no longer commutative. Such groups are called non-Abelian.

More about these later…

Time for a break!

A look at some contemporary artists attempting to express concepts in contemporary theoretical physics…

A review of the history of western music – a progression from symmetry to symmetry breaking???

https://www.youtube.com/watch?v=lExW80sXsHs

A collaboration between cosmologist George Smoot and musician Mickey Hart: Rhythms of the Universe

https://www.youtube.com/watch?v=qu1JCQj5HLY

Dancing in the Quantum World: interactive installation

https://www.youtube.com/watch?v=y4MGYFeZ0AE

Quantum trailer - a piece performed at CERN:

https://www.youtube.com/watch?v=spiT35AQWXw

SYMMETRY - CERN dance-opera filmed inside the collider (official trailer):

https://vimeo.com/120676848

numerical simulation of gravity wave detection by LIGO earlier this year:

http://www.aei.mpg.de/1824987/?page=2

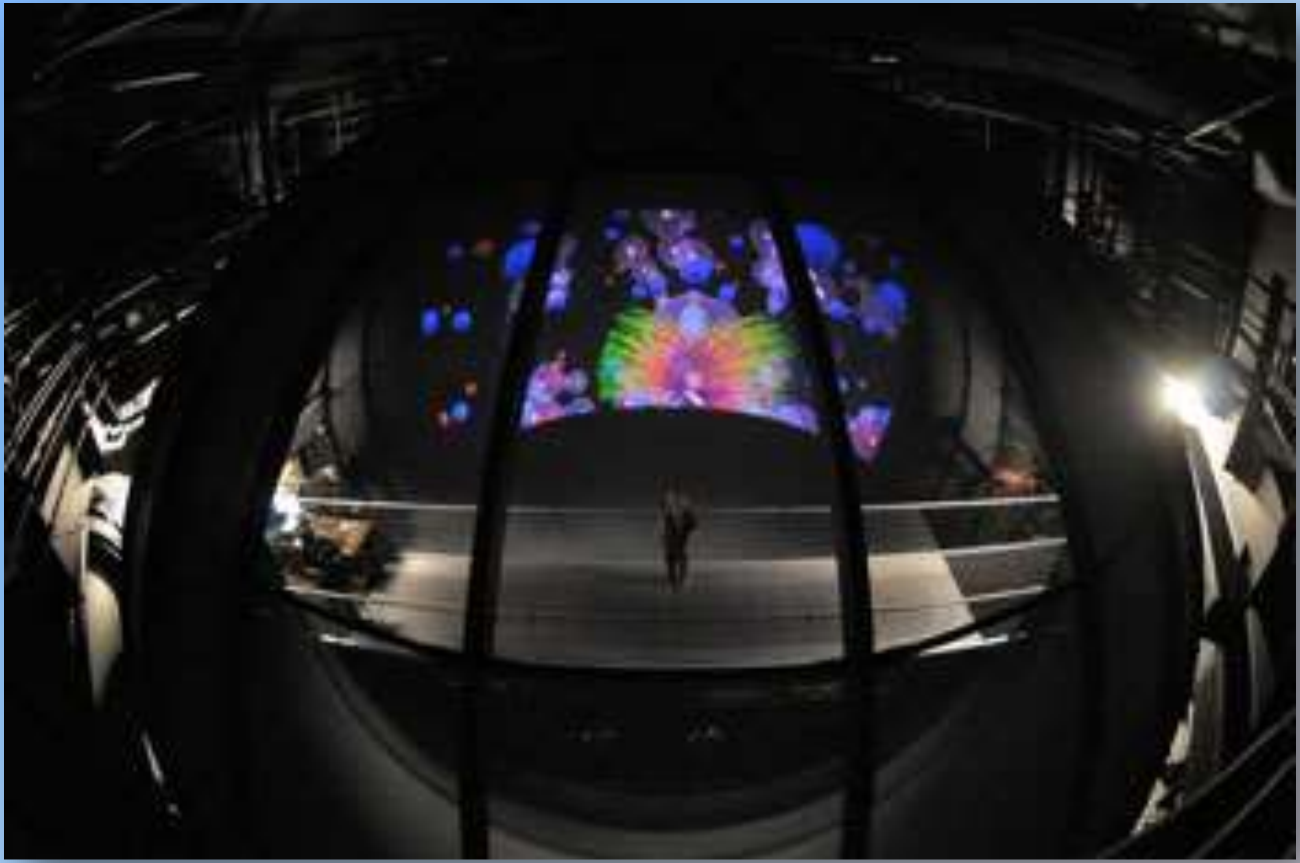
mixed media: portrayal of symmetry and broken symmetry in dance with multimedia:

https://www.youtube.com/watch?v=xdklMiMfRAA

(Note: this one was done with kids at a summer camp at Notre Dame University by …er…yours truly.)

The AlloSphere at UCSB: the ultimate instrument for creating interactive visualizations of science and art:

http://www.allosphere.ucsb.edu/about.php



For next time, chapter 5: A Happy Thought

Consequences of Faraday-Maxwell-Lorentz Invariance: Gravity is also visualized as a FIELD. Einstein showed that gravity is a deformation of spacetime.