Symmetry and Aesthetics in Contemporary Physics

CS-10, Spring 2016

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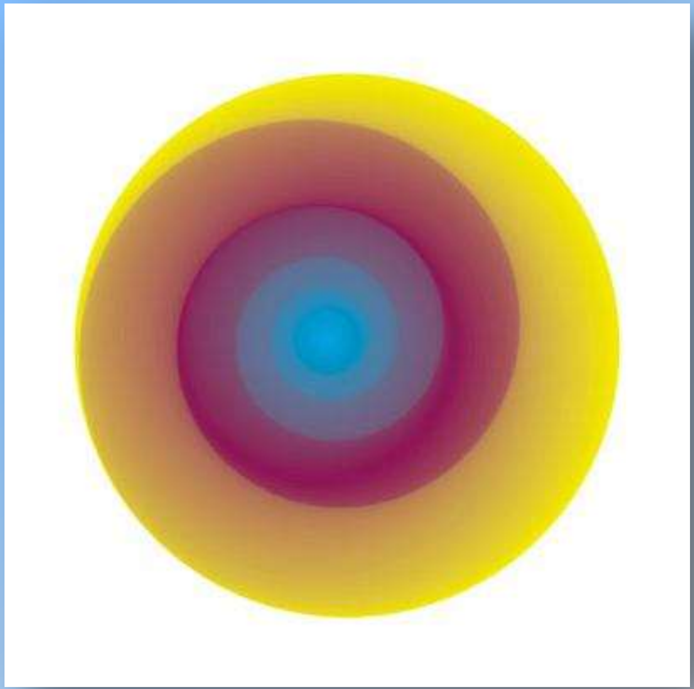
CLASS 8:

SYMMETRY DICTATES DESIGN

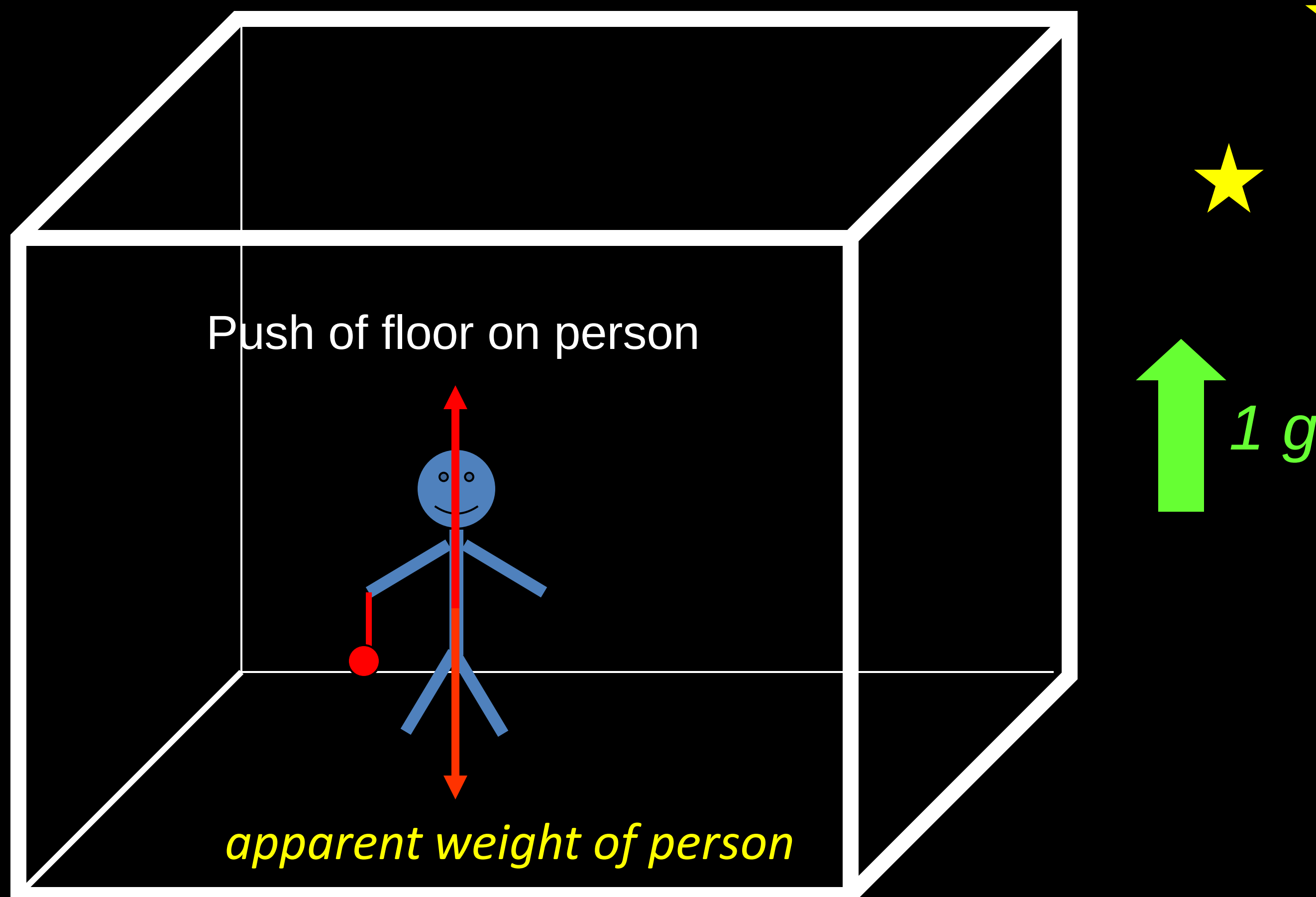
Today:

Discussion of Zee, chapters 5 & 6 and a bit more

“Field Trip” to the KITP for a tour of the Physics Art, led by the KITP Artist in Residence, Jean-Pierre Hebert



Einstein's Happy Thought



In a small enough region of spacetime, such that the gravitational field strength does not vary, a person accelerating at 1g "in outer space" cannot distinguish this from standing still on the surface of the Earth, at sea level.

What is meant by general covariance as a symmetry?

The laws of physics must preserve their structural form under general coordinate transformation.

i.e., for something to be a law of physics, coordinates must transform in the same way on both sides when you apply a Lorentz transformation.

An accelerating observer and a non-accelerating observer can interpret the different physical realities that each perceives as being due to a gravitational field.

illustration on p. 110:
The Action of the Universe on a cocktail napkin:

$$S = \int dx \sqrt{g}\left[\frac{1}{G}R + \frac{1}{g^2}F^2 + \bar{\psi}\not{D}\psi + \left(D\varphi\right)^2 + V\left(\varphi\right) + \bar{\psi}\varphi\psi\right]$$

mmmwah!

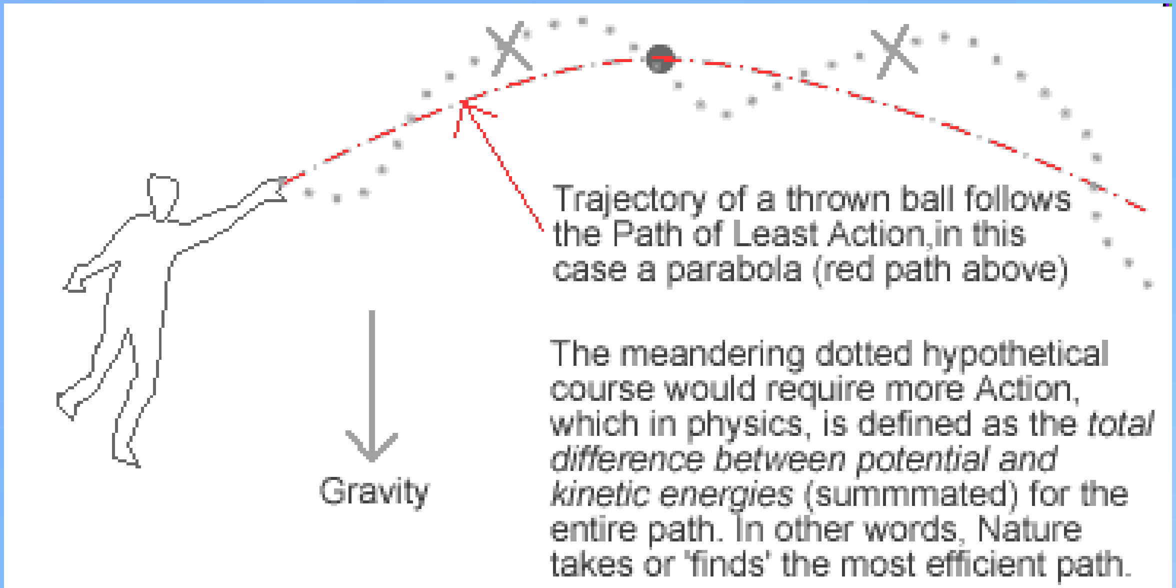
p. 111: To say that physics possesses a certain symmetry, is to say that the Action is invariant under the transformation associated with that Symmetry.

$$S = \int L(x, \dot{x}, t)\,dt = \int_{t_1}^{t_2} (T - V)\,dt$$

S is the Action.
L is called the Lagrangian.
L = T – V
T = kinetic energy
V = potential energy

- **The Principle of Stationary Action:**

  The path of a particle is the one that yields a stationary value of the action.



$$S = \int_{t_1}^{t_2} L dt = const$$

Humpty Dumpty will always follow a geodesic in spacetime! That is, he will always follow a path such that the difference between his kinetic and potential energies is stable to small perturbations.

i.e., his $\frac{mv^2}{2} - mgh$ is constant over his path.

$$\frac{d}{dt}\left[\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i}\right] = 0$$

An attempt to visualize the principle of stationary action with soap films.

Try this: Using soap solution and wire, show that for any shape of wire you make, the soap film will always settle onto one stable surface, and if you disturb this surface ‘a little’ – say, by blowing on it very gently (without popping it!) – that it will return to its stable position.

Einstein's Realizations:

1) Energy and momentum must be Lorentz invariant

$$p'^{\mu} \equiv \Lambda^{\mu}_{\nu} p^{\nu} = \begin{bmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{bmatrix} \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix}.$$

primed frame

rule

unprimed frame

in units where c = 1:

$$x^{\mu} = \begin{bmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{bmatrix} = \begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix}.$$

(If $c \neq 1$ then $x^0 = ct$.) This vector is an element of a 4-dimensional vector space called **Minkowski space**. Then we have

$$ds^2 = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2$$

defining the **Lorentz** (or **Minkowski**) **metric**

$$g_{\mu\nu} = \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$$

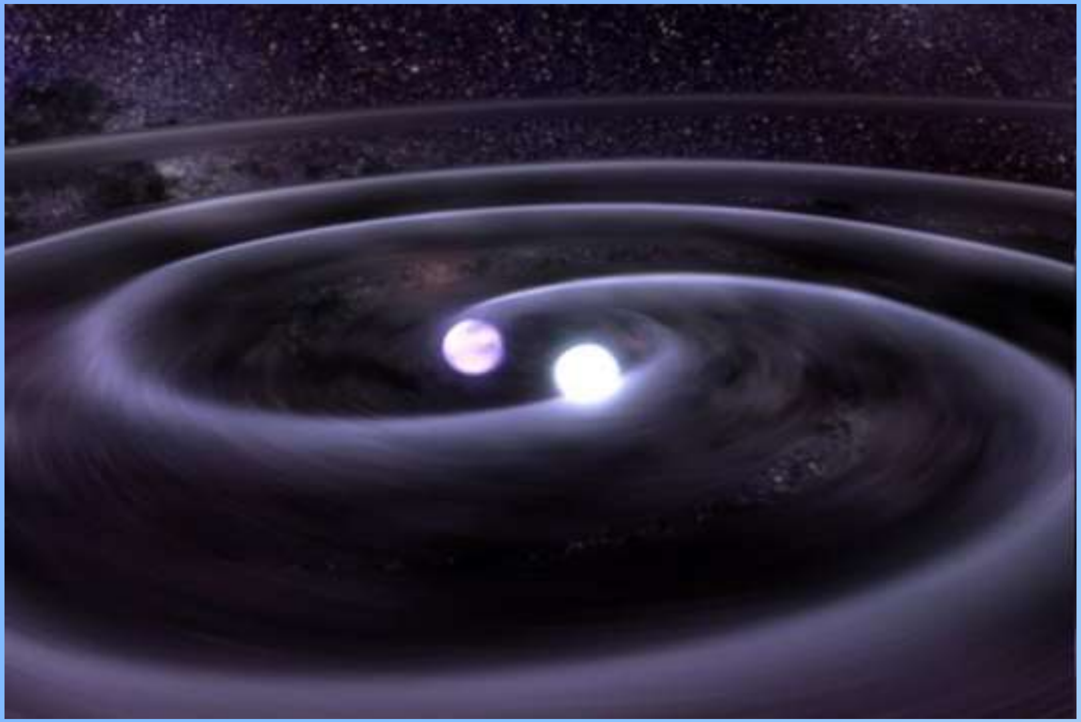
many authors choose the opposite: (-1, 1,1,1)

$$\Lambda^{\mu}{}_{\nu} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Lorentz transformation is a rotation in Minkowski space

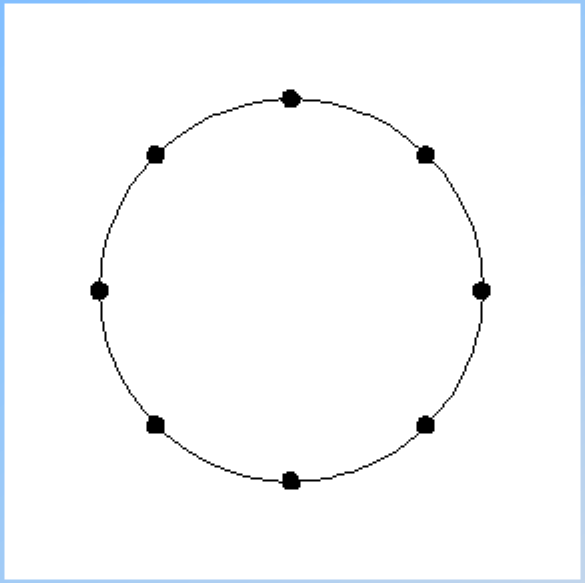
$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}.$$

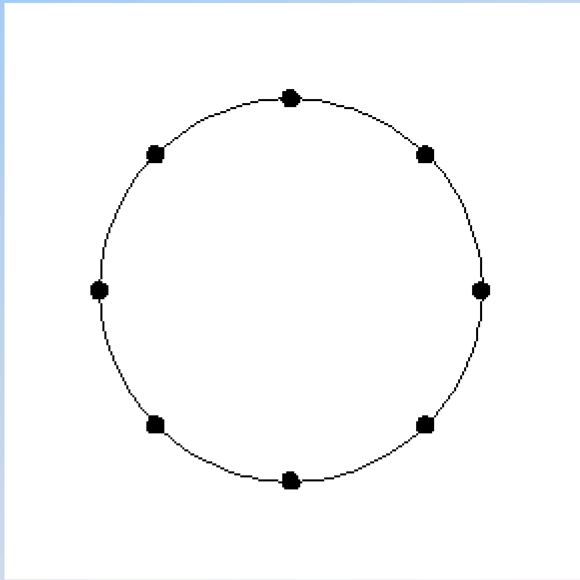
s *is* the path of stationary action



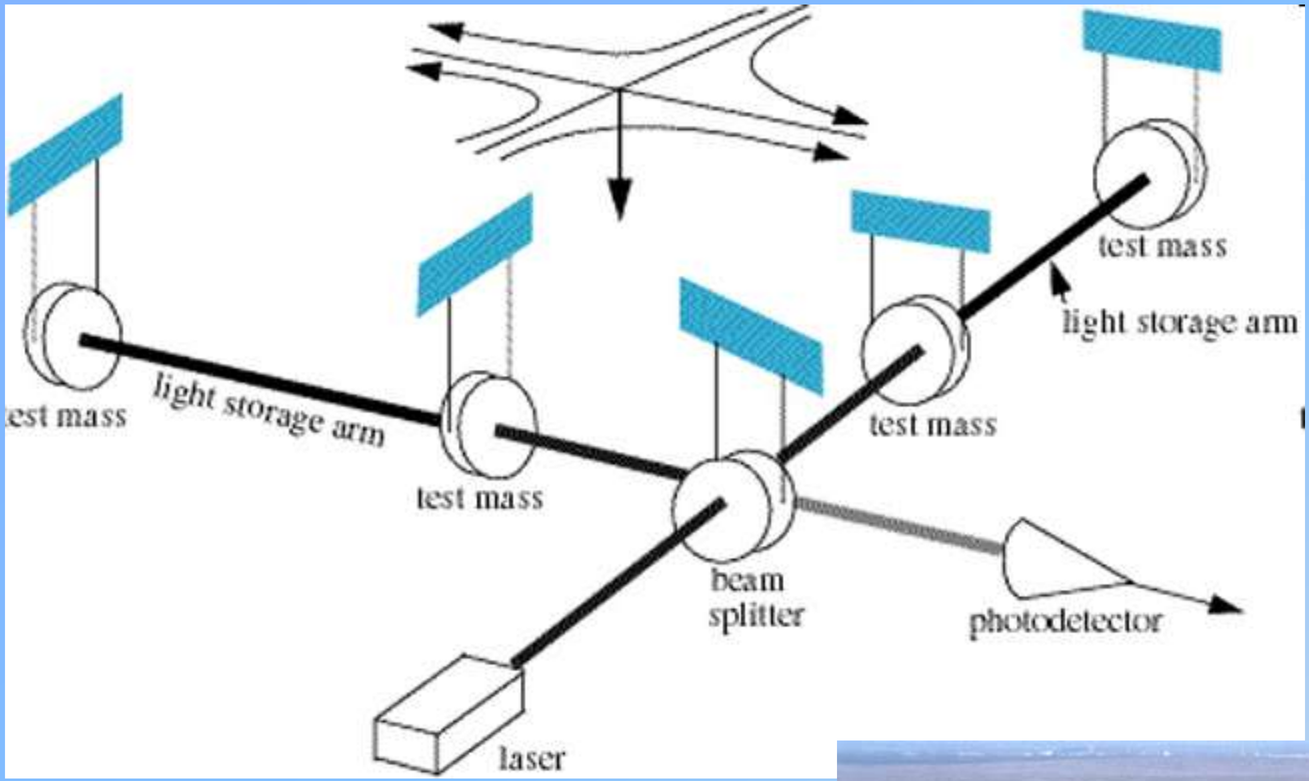
Gravity waves produced by two rotating massive objects

Meanwhile: in 2016 the detection of gravity waves was announced!





disturbance of test particles due to passage of gravity waves



and the design of instruments that can measure small deformations of spacetime:

the LIGO gravity wave detector

Einstein's Realizations:

2) The Laws of Electromagnetism must be Lorentz invariant

$$\nabla \cdot \mathbf{E} = 4\pi\rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = 4\pi\mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}$$

$$x'^{\mu} = \Lambda^{\mu}{}_{\nu}\, x^{\nu}$$

primed frame

rule: the LT!

unprimed frame

Progression of symmetry from obvious to subtle:

1. **Rotation of coordinate axes in space ~ invariance of the length of a line**
2. **Relative motion of inertial observers at slow speeds ~ Galilean invariance**
3. **Speed of light is constant for all observers - Lorentz invariance**
4. **Equivalence of Mass & Energy – General Relativity - General covariance, curvature of spacetime**
5. **Gauge theories – explain "internal" symmetries of particles**
6. **What's next? Super symmetry? String theory? Something else?**

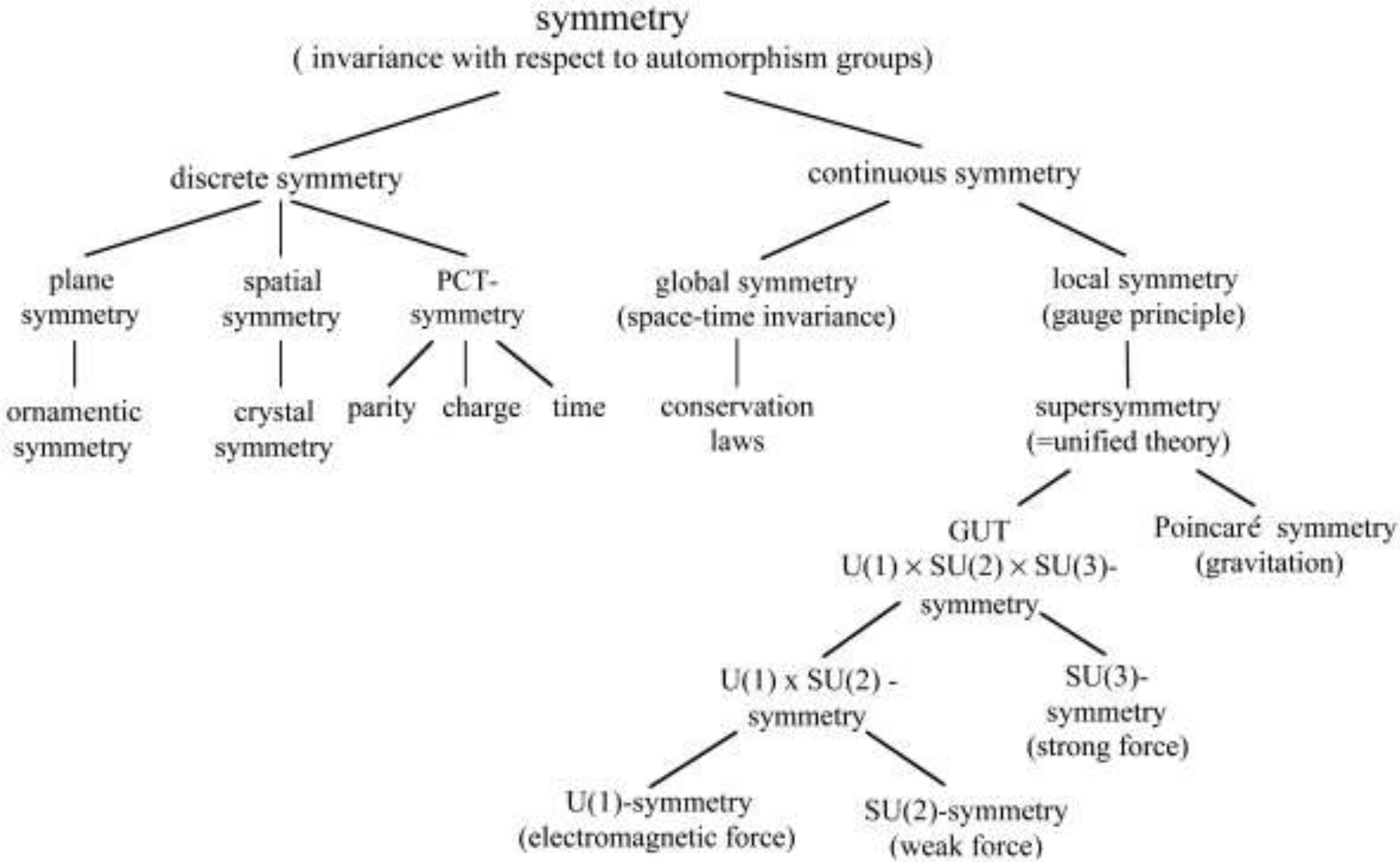


Figure 9. Classification of symmetry

A gauge theory is a type of field theory in which the Lagrangian is invariant under a continuous group of local transformations.

Yang-Mills Theory: a gauge theory in which a field is defined everywhere in space, mediated by the exchange of virtual particles

First it was noticed that groups of particles were related to each other in a way that matched the representation theory of SU(3).

SU(3): transforms 3 objects into each other via rotation and has n^2-1 = 8 degrees of freedom

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These are the three lightest: up, down, and strange.

Postulate: There is an abstract three-dimensional vector space in which the 3 quarks which make up spin ½ baryons can be described:

$$up \to \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad down \to \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad strange \to \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

The laws of physics are *approximately invariant under applying a unitary transformation to this space, sometimes called a *flavor rotation*:**

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to A \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

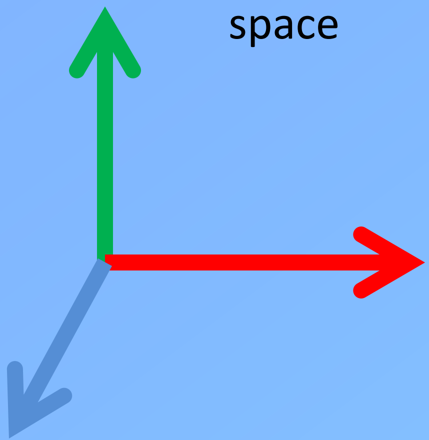
$$A = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where A is in SU(3)

* ***because the quark masses are not exactly identical.***

Quarks as fundamental representation of the 3-dimensional color group SU(3)

quark color space



$$r = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$g = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

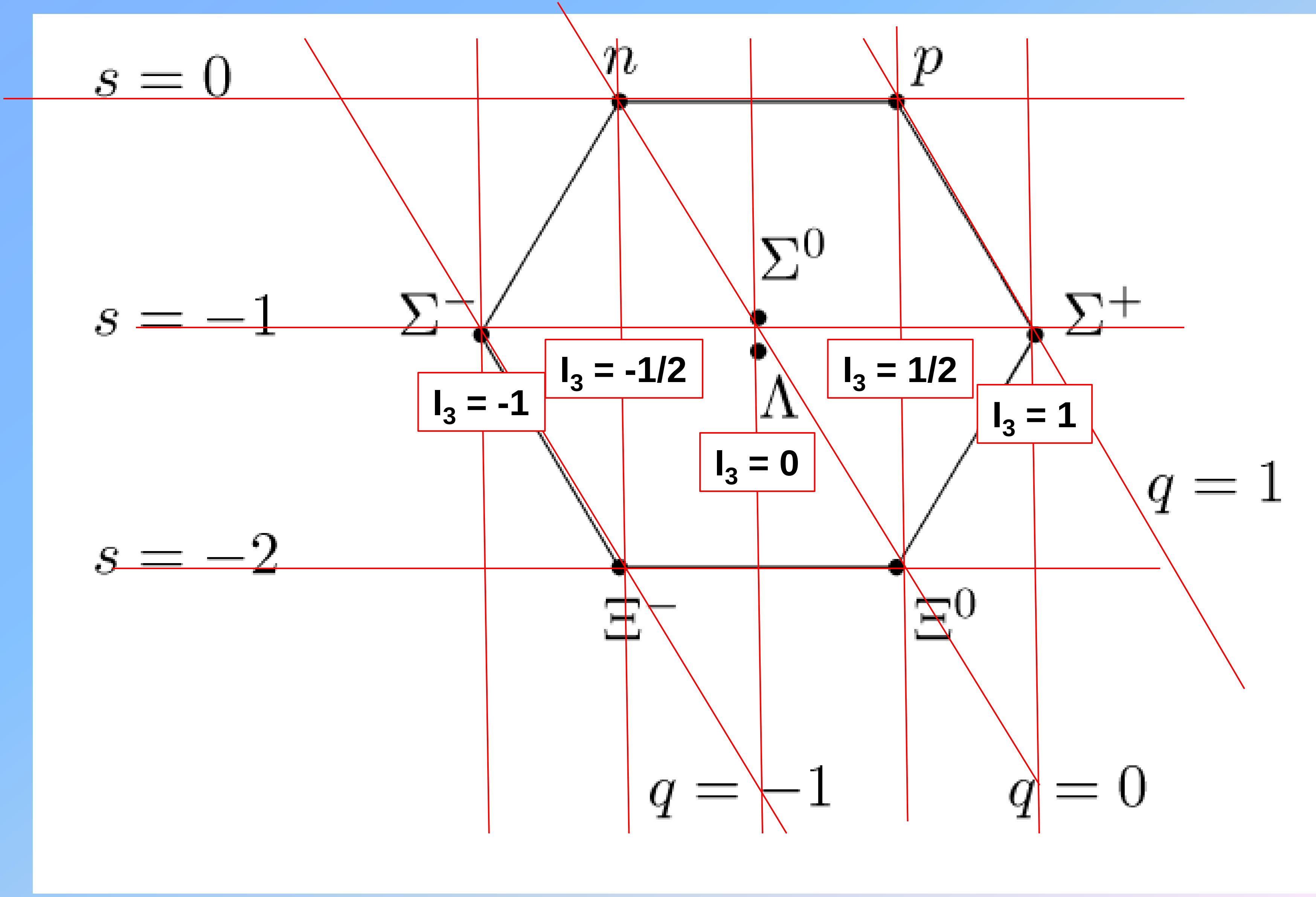
$$b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

color symmetry of quarks is an exact symmetry : each quark can be transformed into a different 'color' quark with same mass, same spin, same isospin

Quark wave function, ψ =
(space term) x (spin term) x (flavor term)x (color term).

Isospin

This led to the "eightfold way" - *almost* symmetries of the spin ½ baryons



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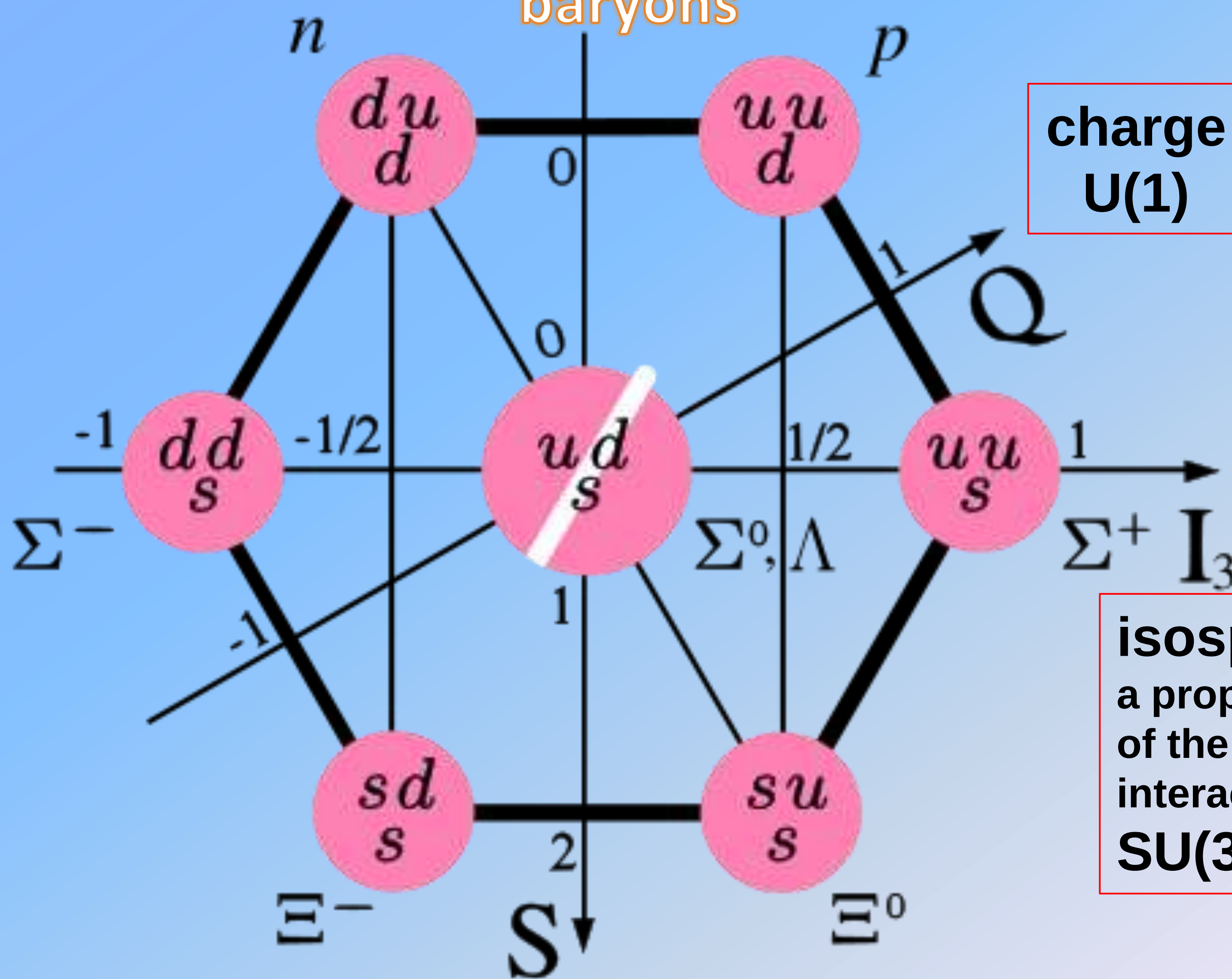
The octets and other arrangements are representations of this group.

The "eightfold way" : *almost* symmetries of spin ½ baryons

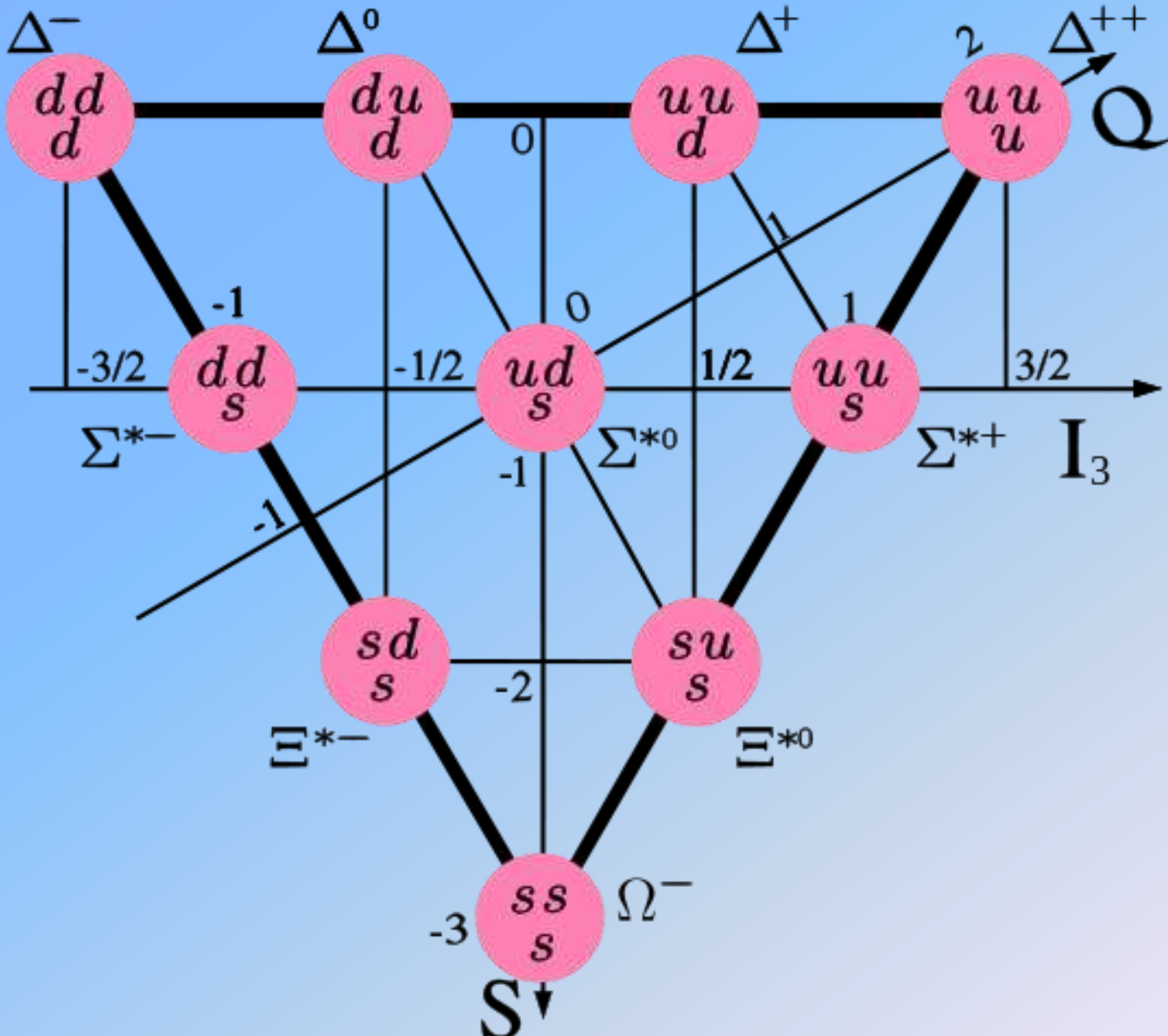


charge
U(1)

isospin
a property of the strong interaction
SU(3)

strangeness

almost symmetries of the spin 3/2 baryons



SUMMARY:

(Real) Special Orthogonal Groups: SO(n) have generators $$\frac{n(n-1)}{2}$$

(Complex) Special Unitary Groups: SU(n) have generators $$n^2 - 1$$

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SO(2): $$\frac{2(2-1)}{2}=1$$ **degree of freedom (rotation about z-axis)**

SO(3): $$\frac{3(3-1)}{2}=3$$ **degrees of freedom (rotation about x, y, or z axes)**

SUMMARY:

(Complex) Special Unitary Groups: SU(n) have generators $n^2 - 1$

SU(2): 2^2-1 = 3 degrees of freedom → **3 force-carrying bosons weak force**

SU(3): 3^2 – 1 = 8 degrees of freedom → **eight gluons Strong Force**

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(Real) Special Orthogonal Groups: SO(n) have $\frac{n(n-1)}{2}$ generators

(Complex) Special Unitary Groups: SU(n) have $n^2 - 1$ generators

SO(2): 2(1)/2 = 1 degree of freedom (rotation about z-axis)

SO(3): 3(2)/2 = 3 degrees of freedom (rotation about x, y, or z axes)

SU(2): 2^2-1 = 3 degrees of freedom → 3 force-carrying bosons
weak force

SU(3): 3^2 – 1 = 8 degrees of freedom → eight gluons
Strong Force

U(1): 1 degree of freedom → one photon
electromagnetic force

THE STANDARD MODEL AT THE END OF THE 20TH CENTURY