Symmetry and Aesthetics in Contemporary Physics

CS-10, Spring 2016

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CLASS 9:

SYMMETRY: THE SEARCH CONTINUES

Looking for deeper symmetries and spontaneous symmetry breaking

$$S = \int dx \sqrt{g} \left[\frac{1}{G} R + \frac{1}{g^2} F^2 + \overline{\psi} \not{D} \psi + \left(D\varphi \right)^2 + V\left(\varphi\right) + \overline{\psi} \varphi \psi \right]$$

mmmwah!

p. 111: To say that physics possesses a certain symmetry, is to say that the *<u>Action</u>* is invariant under the transformation associated with that Symmetry.

Action: path of stable energy, invariant to rotations, path of maximum proper time

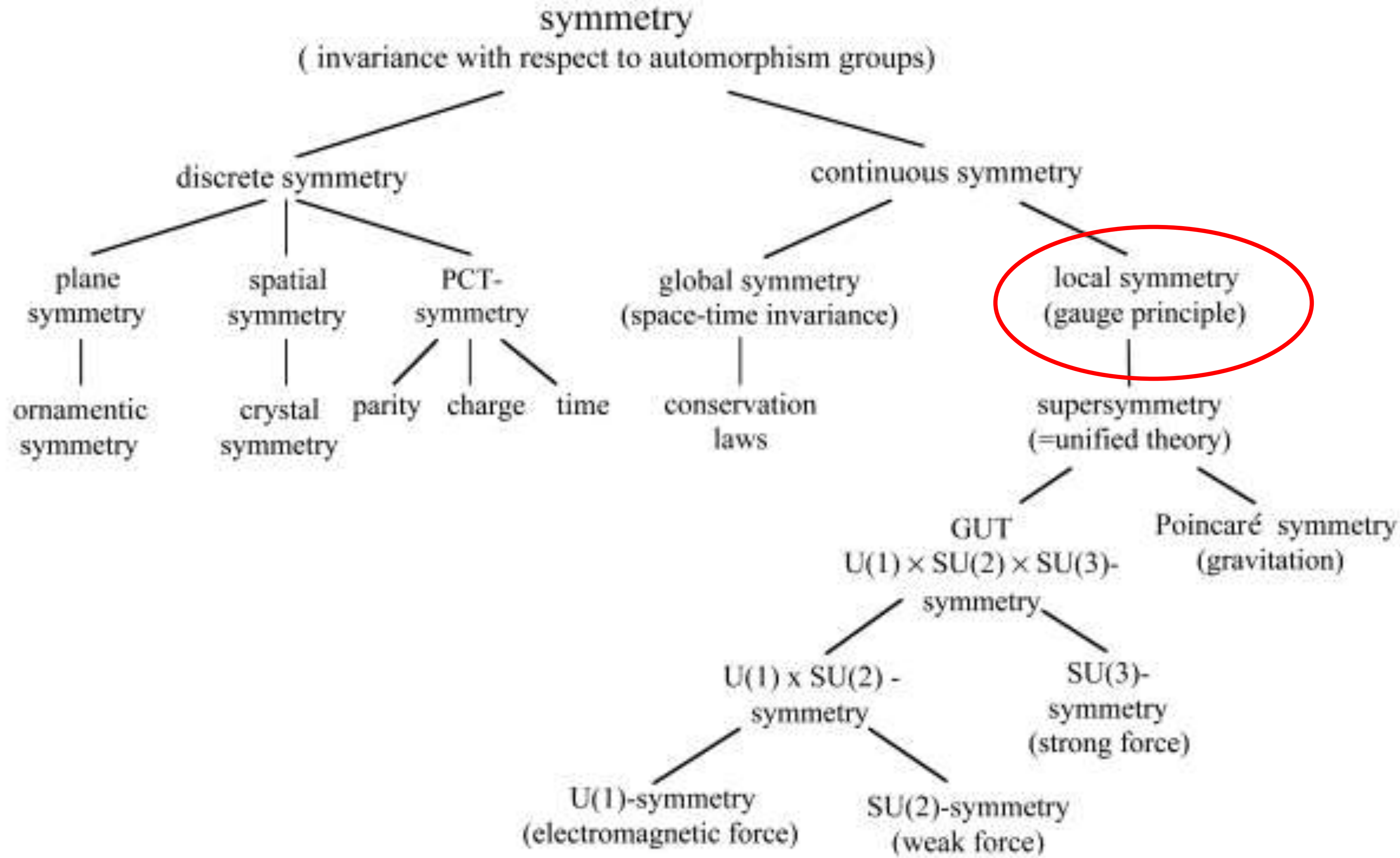


Figure 9. Classification of symmetry

A gauge theory is a type of field theory in which the Lagrangian is invariant under a continuous group of local transformations – i.e., depend on spacetime.

- When the symmetry group depends on spacetime, it is called a *local symmetry*.
- The continuous symmetry that depends on spacetime is called a *gauge group*.
- The transformation that depends on spacetime is called a *gauge transformation*.

Yang-Mills Theory: a gauge theory in which a *field* is defined everywhere in space, mediated by the exchange of *virtual particles*

$$\Delta E \Delta t \leq \hbar$$

…but first, a short excursion into SO(n) and SU(n):

Special Orthogonal Groups of order n: SO(n) are defined:

$O^T O = 1$ and det $O = 1$ and has n(n-1)/2 degrees of freedom

The group SO(n) consists of rotations in n-dimensional Euclidean space, represented by n-dimensional tensors. SO(n) represents GLOBAL symmetries that are independent of spacetime.

SO(2):
for a counter-clockwise rotation:

$$R(\theta) = \begin{pmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{pmatrix}$$

SO(3):
for 3 dimensions, just add the z-axis:

$$R(\varphi) = \begin{pmatrix} \sin\varphi & \cos\varphi & 0 \\ -\cos\varphi & \sin\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Rotations in spacetime are SO(4)

DEFINE:

$$\gamma = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$\beta = \frac{v}{c}$$

$$or, v = \beta c$$

$$x' = \gamma(x - vt)$$

$$t' = \gamma\left(t - \frac{\beta x}{c}\right)$$

Lorentz Transformation: the rule that translates between inertial reference frames in spacetime

For motion along the x-axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ t' \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta c \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{\gamma\beta}{c} & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix}$$

Global symmetry in Minkowski spacetime (no mass, no gravity)

SU(n): Special Unitary groups of order n

complex **unitary matrices (det U = 1)**

SU(n) represent LOCAL symmetries that DO depend on local variations in spacetime.

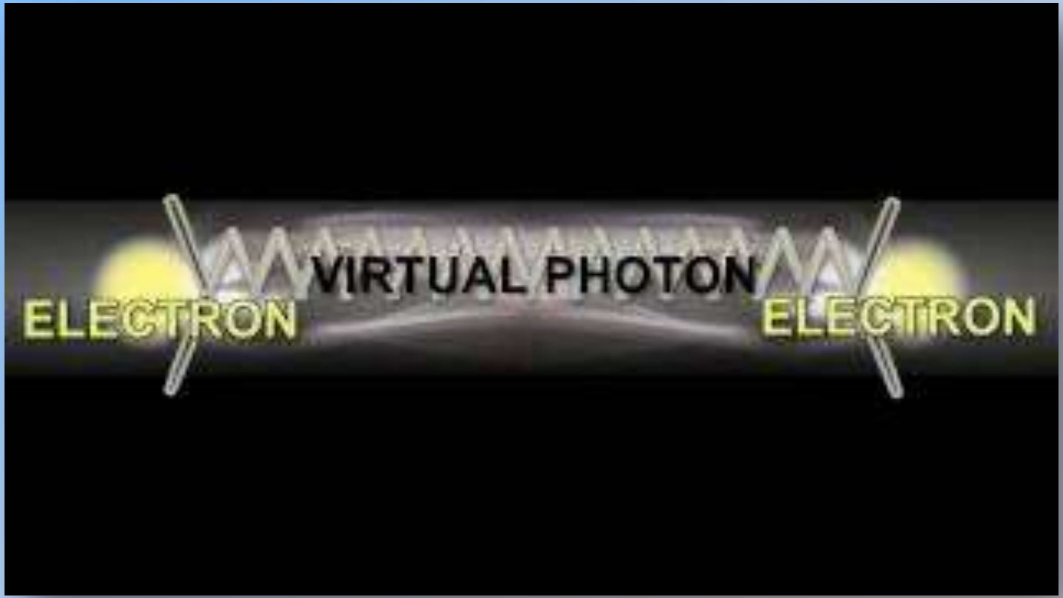
Examples: GR and symmetries in particle physics which depend on local FIELDS, i.e. ***gauge theories***

Examples of Gauge Theories

Quantum Electro-Dynamics

QED has the gauge group = U(1) E&M.
The number of gauge fields is dim(U(1)EM) = 1.
This gauge field is the photon.
It couples to charged leptons and quarks.

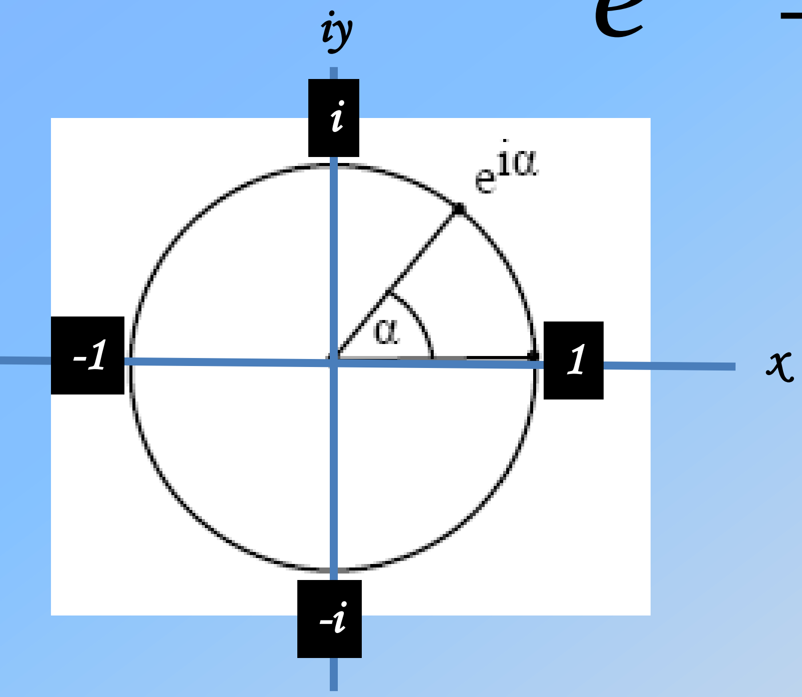
Does SSB occur: No. So the photon remains massless.



U(1) is the group of all possible phase multiplications $e^{i\alpha}$

$$\Psi(x) \to e^{i\alpha}\Psi(x) \quad ; \qquad \overline{\Psi}(x) \to e^{-i\alpha}\overline{\Psi}(x)$$

$$e^{i\alpha} = \cos\alpha + i\sin\alpha$$



Examples of Gauge Theories : Quantum Chromo-Dynamics

QCD has the gauge group $SU(3)_{color}$. A gauge transformation is $U \in SU(3)_{color}$. QCD offers a new way of thinking about matter.

Every quark field of flavor f, say f (x), has an associated color of red, green or blue. Define

$$\chi^f(x) = \begin{pmatrix} \Psi^f_{\mathbf{red}}(x) \\ \Psi^f_{\mathbf{green}}(x) \\ \Psi^f_{\mathbf{blue}}(x) \end{pmatrix}$$

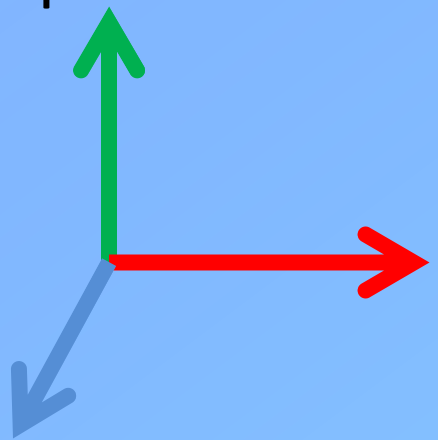
Construct the gauge-invariant Lagrangian:

$$\mathcal{L} = \overline{\chi}^f (i\gamma^\mu D_\mu - m)\chi^f$$

This Lagrangian must be invariant to 'rotations' in SU(3).

There are 3^2-1 = 8 degrees of freedom, which are the 8 gluon fields.

quark color space

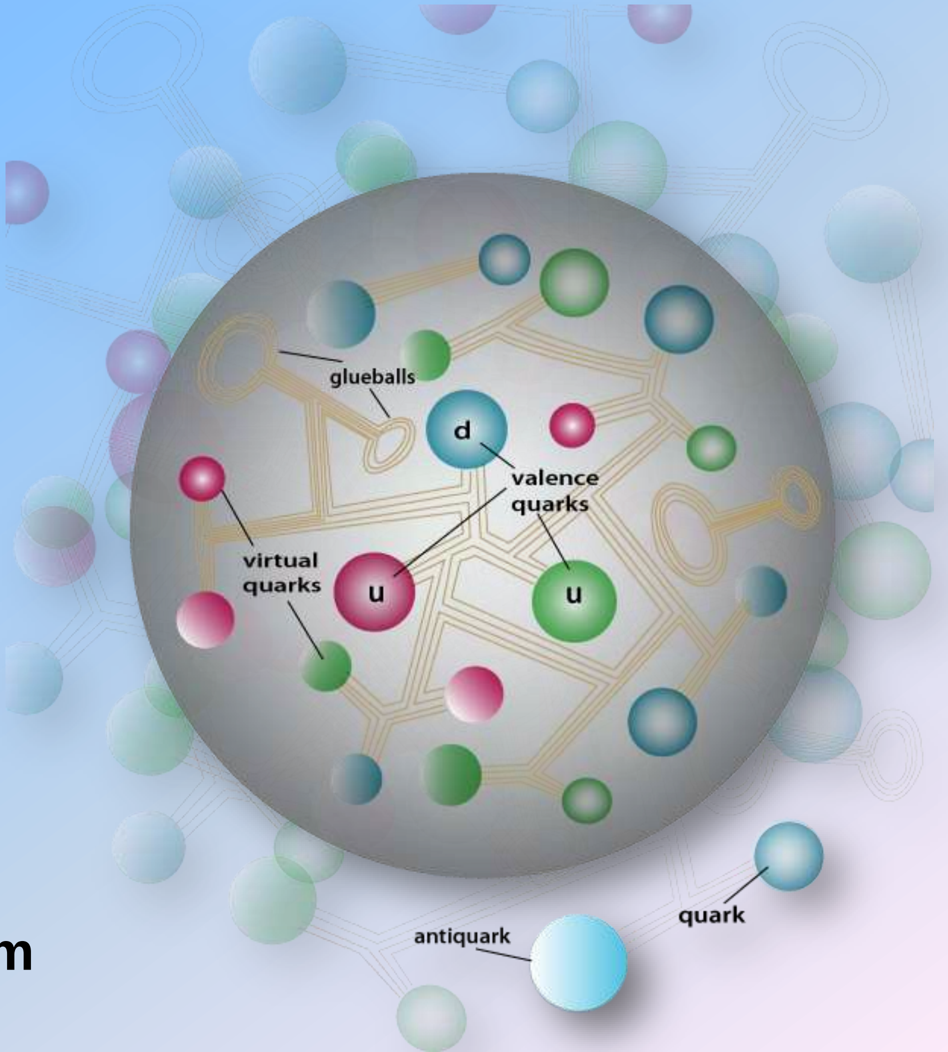


$$r = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$g = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

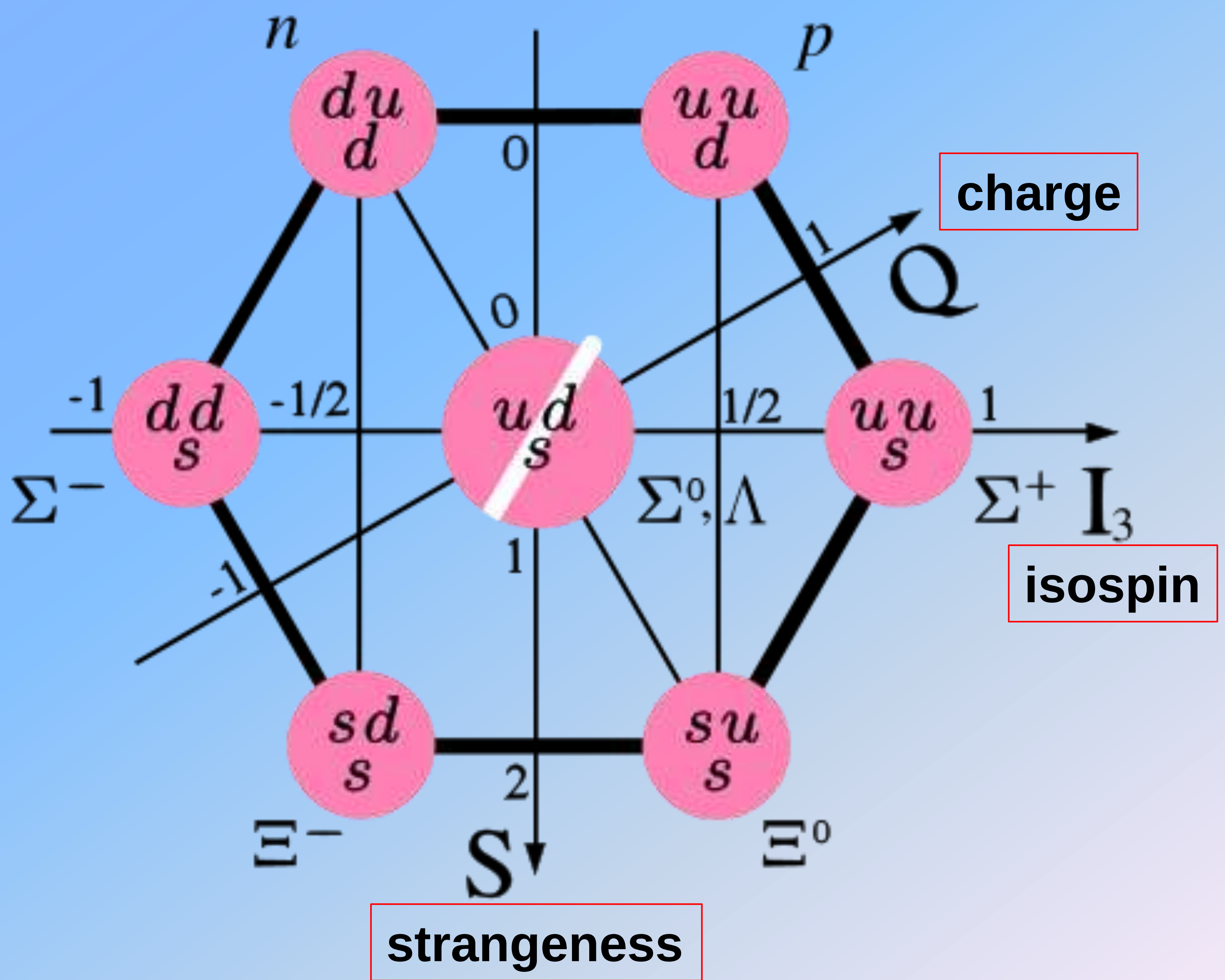
$$b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

color symmetry of quarks is an exact symmetry : each quark can be transformed into a different ‘color’ quark

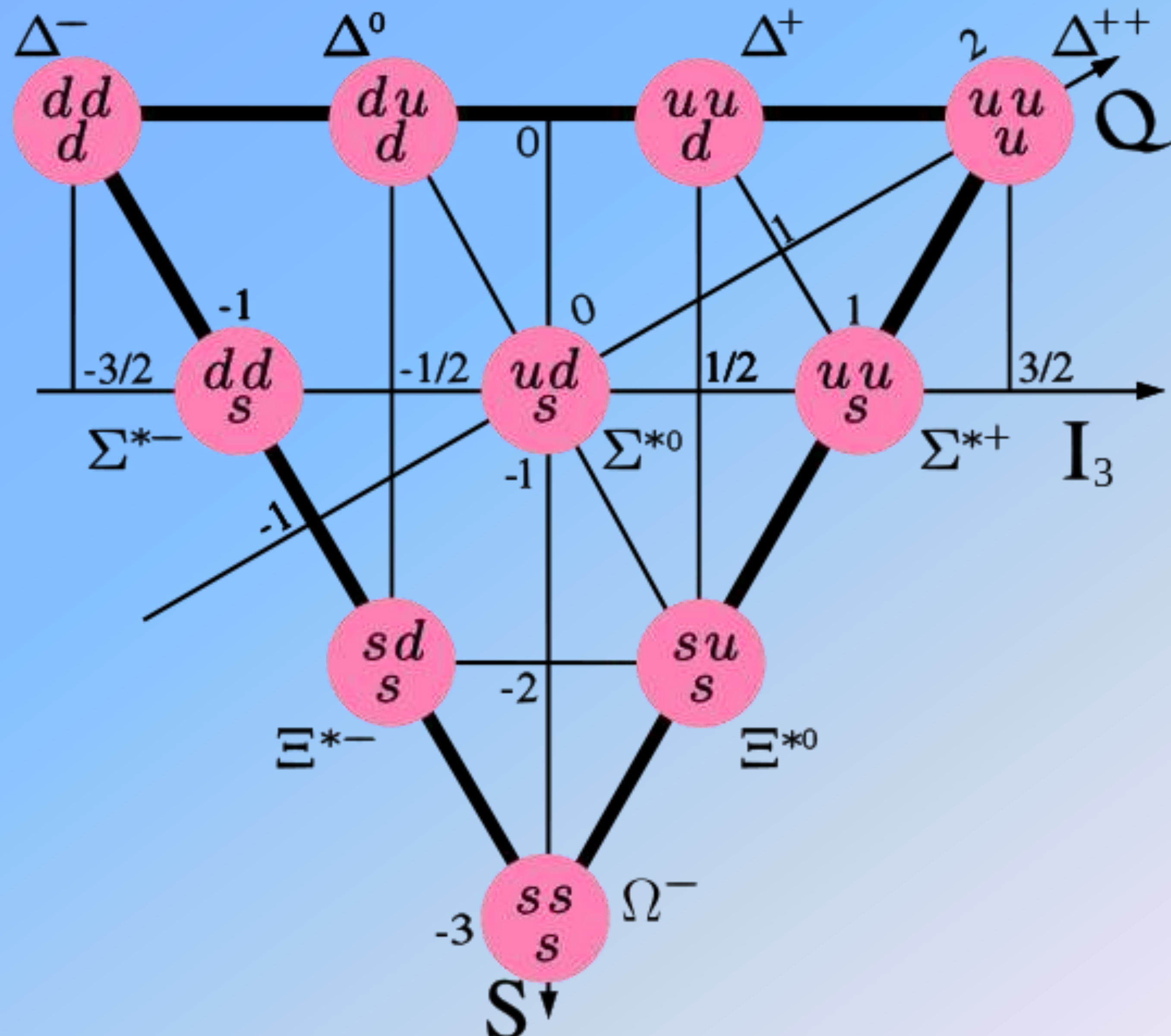


Only around 2% of the mass of the proton comes from the three valence quarks. The rest comes from the gluon field and virtual quarks.

“Eightfold Way” representation of the spin ½ baryons



almost symmetries of the spin 3/2 baryons



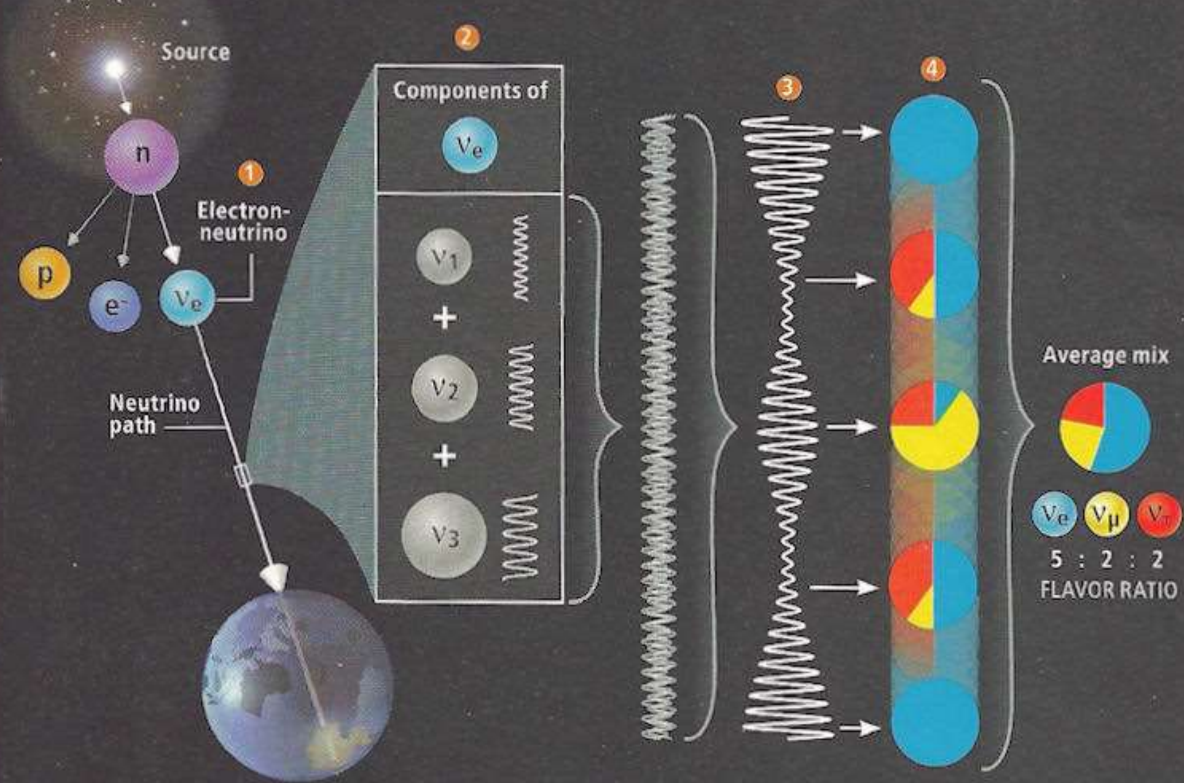
The Electro-Weak theory has the gauge group $SU(2)_{left} \times U(1)_{hypercharge}$. The number of gauge fields is equal to dim(SU(2)left × U(1)hypercharge) = 4. The gauge fields are the W^a_μ , B_μ, a = 1, 2, 3. Does SSB occur? Yes.

After SSB, the massive gauge fields are called $W^\pm_\mu$,Z^0_μ while the massless gauge field is called the photon.

The $W^\pm_\mu$ couple to left handed matter causing flavor changing processes like beta decay, the Z^0_μ couples to all particles and the photon couples to charged matter.

FLAVOR OSCILLATIONS

When created or detected, a neutrino has a specific flavor. For instance, the beta decay of a neutron creates an electron-neutrino (1). This neutrino has no specific mass but is a mix of all three possibilities—represented by a sum of three waves with different wavelengths (2). As the neutrino propagates, the waves become misaligned, so they no longer add up to the original flavor but to some mix of all three flavors (3). The mix varies as the neutrino travels (4). Here the average mix is 5:2:2—which means a detector has a five-ninths chance of seeing it as an electron-neutrino and a fourth-ninths chance as a muon- or a tau-neutrino.



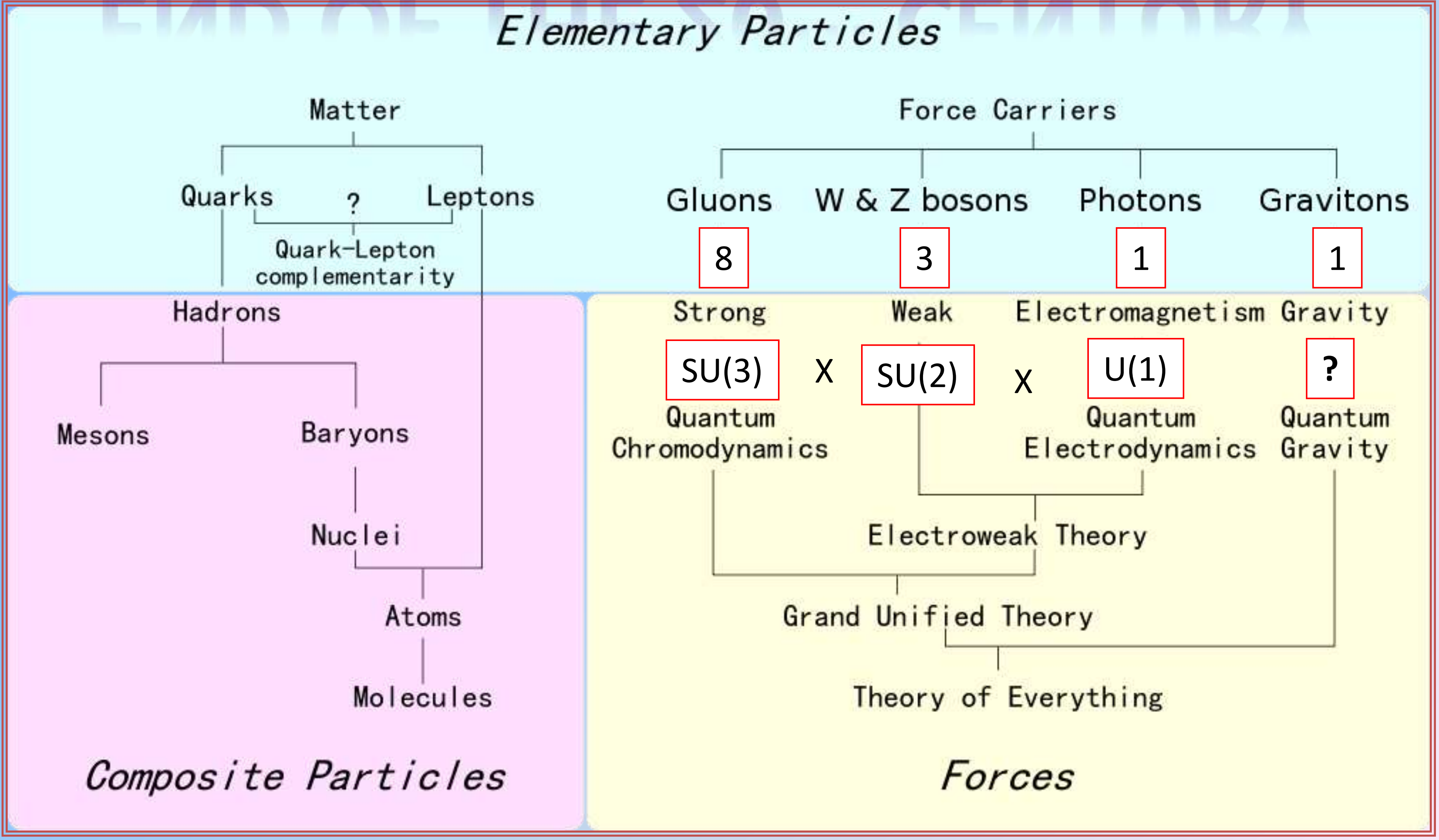
FLAVOR MIXES

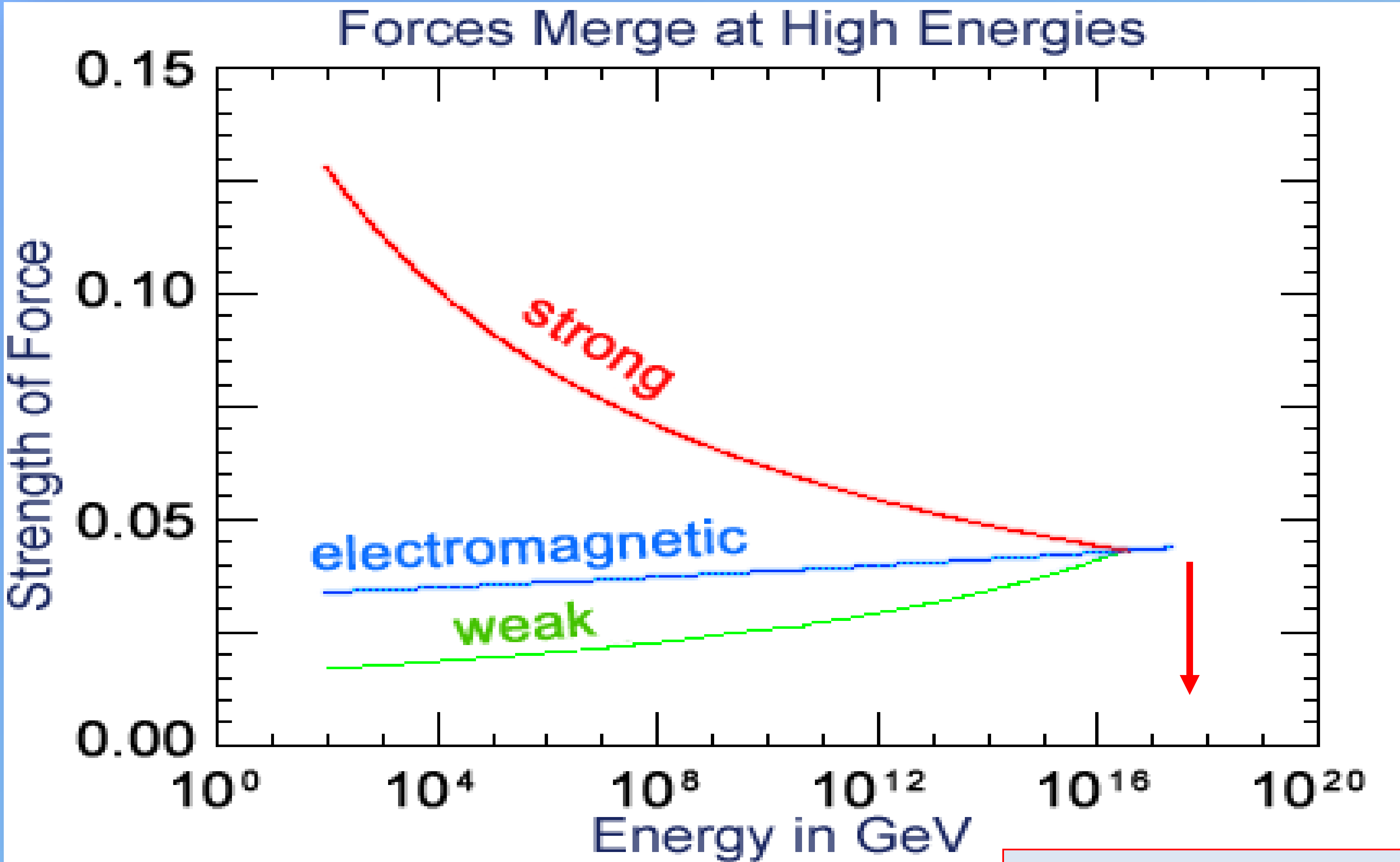
Astrophysical processes produce distinctive flavor mixes, which astronomers can deduce by accounting for the metamorphosis. Muon- and tau-neutrinos always arrive in equal proportions, a consequence of their intrinsic symmetry.

Source	Ratios at Source	Ratios at Earth
Neutron decay	$1\nu_e:0\nu_\mu:0\nu_\tau$	$5\nu_e:2\nu_\mu:2\nu_\tau$
Pion decay (complete)	1:2:0	1:1:1
Pion decay (incomplete)	0:1:0	4:7:7
Dark matter decay (example)	1:1:2	7:8:8
Spacetime foam	Any	1:1:1
Neutrino decay (ν_1 lightest)	Any	4:1:1
Neutrino decay (ν_3 lightest)	Any	0:1:1

and flavor oscillations among neutrinos

THE STANDARD MODEL AT THE END OF THE 20TH CENTURY

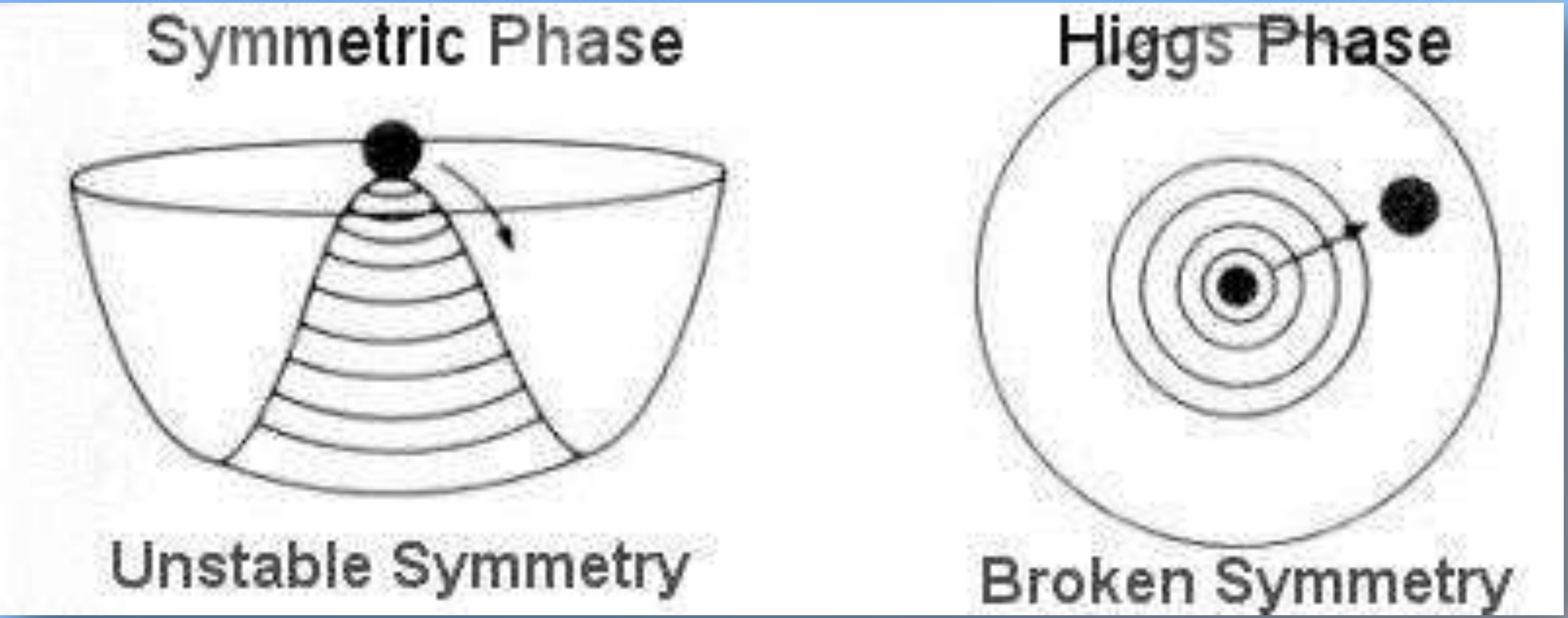




GUT scale: t ~ 10 $^{-43}$ sec,
E ~ 10^{19} GeV

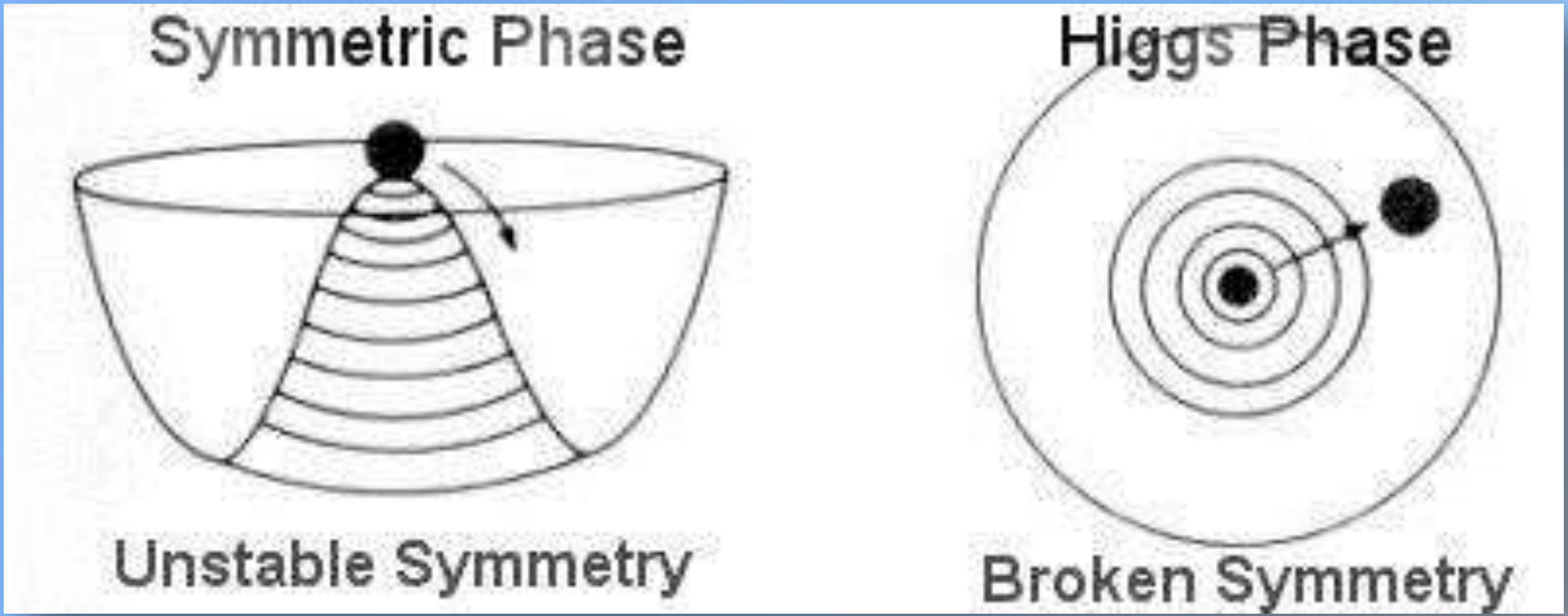
The four different forces are now seen as broken symmetries in a low-energy universe, reduced from a higher symmetry at higher energies in the first instants after “creation.”

Almost symmetries: A symmetric theory can have asymmetric consequences. For example, the equations of a ball and the wheel of a roulette are symmetric with respect to the rotation axis, but the ball always keeps lying in an asymmetric position.



The symmetric state is not the state of minimum energy, i.e., the ground state, and in the process of evolving towards the ground state, the intrinsic symmetry of the system has been broken.

http://universe-review.ca/R15-12-QFT21.htm



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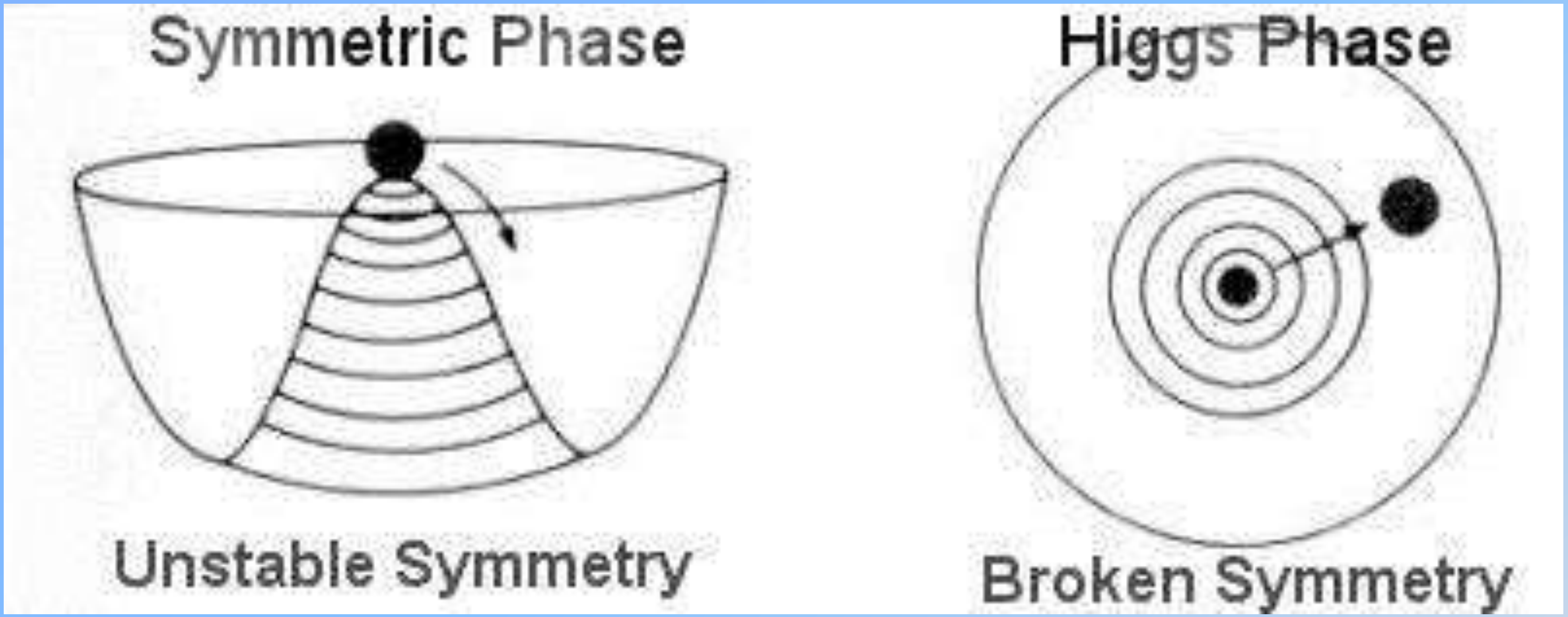
A small perturbation will cause the rotational symmetry to be broken and the system to assume the ground state configuration.

<u>*Nothing*</u> **is unstable!**

- Professor Rocky Kolb
Fermilab, 1995
personal
communication



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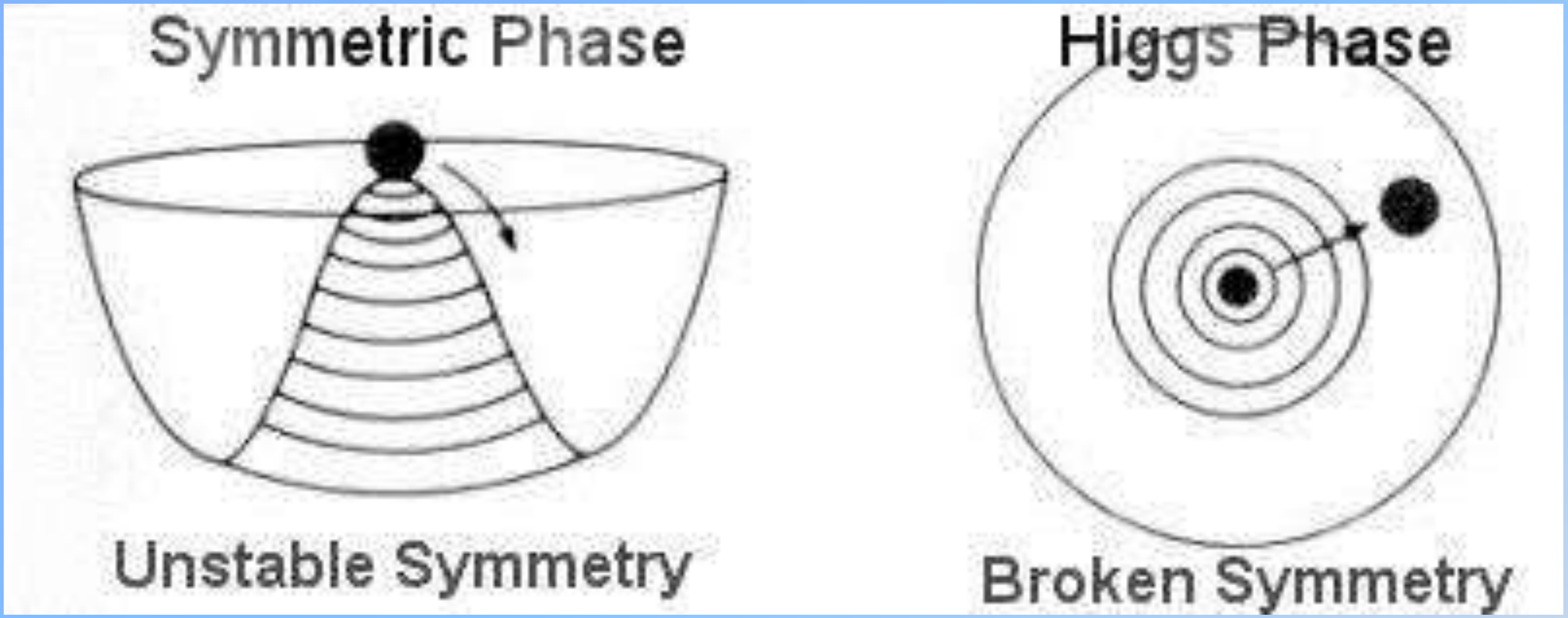
When the symmetry of a physical system is broken in this way, it is often referred to as "spontaneous symmetry breaking“ (SSB).



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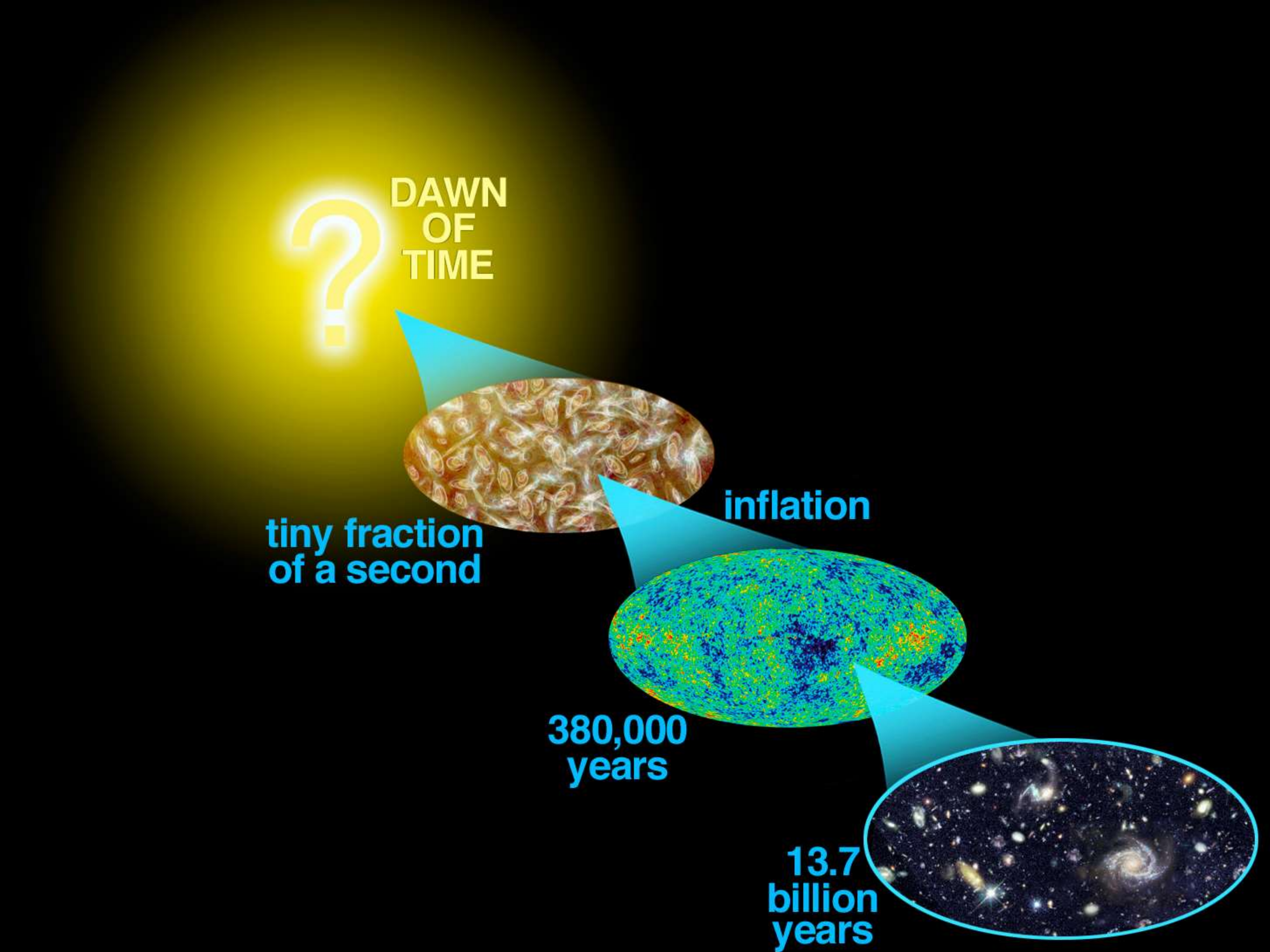
And this is how our current cosmological model explains how the universe came into existence!



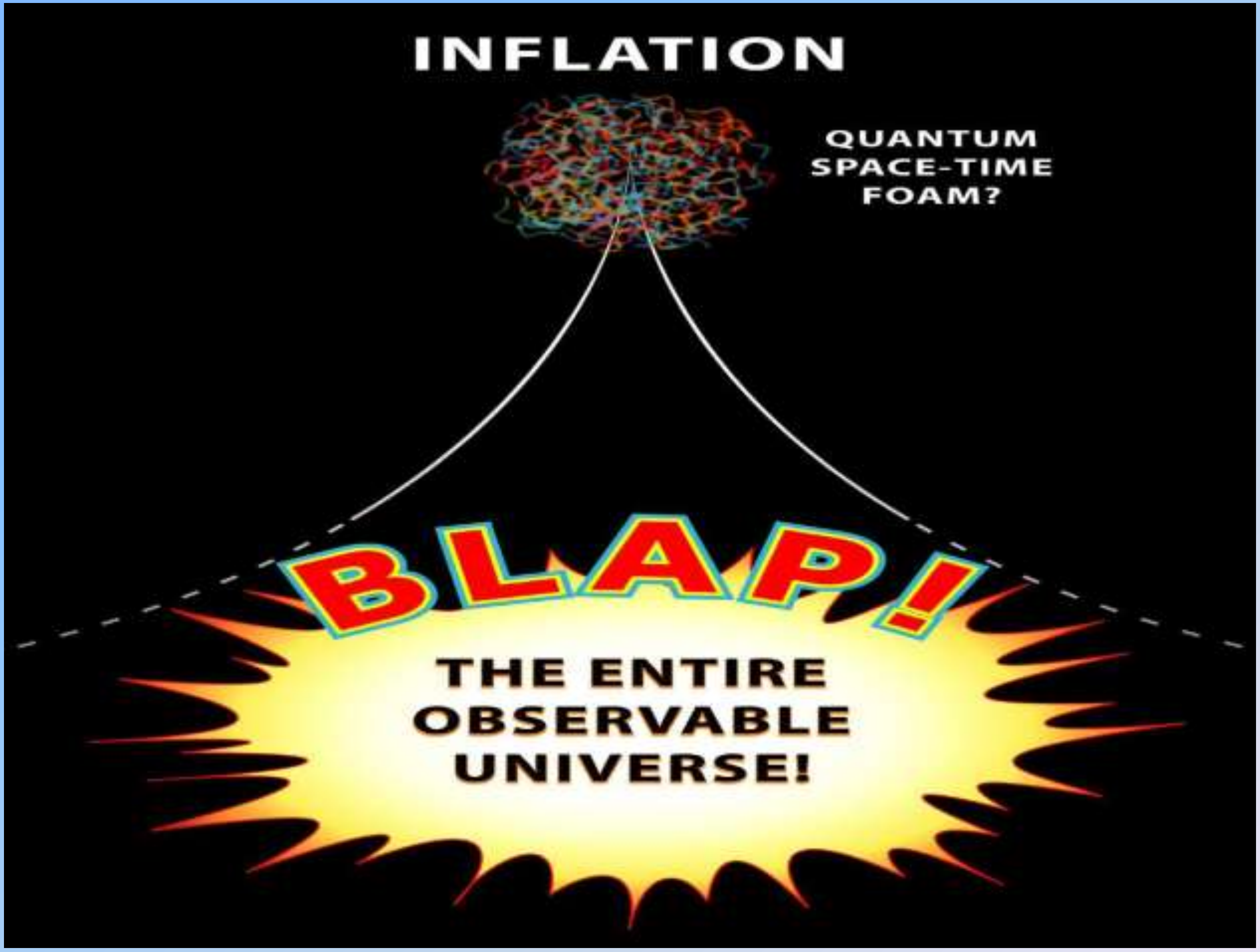
DAWN
OF
TIME
inflation
tiny fraction
of a second
380,000
years
13.7
billion
years

t=0 – "The Big Bang"
Is there a "before"?

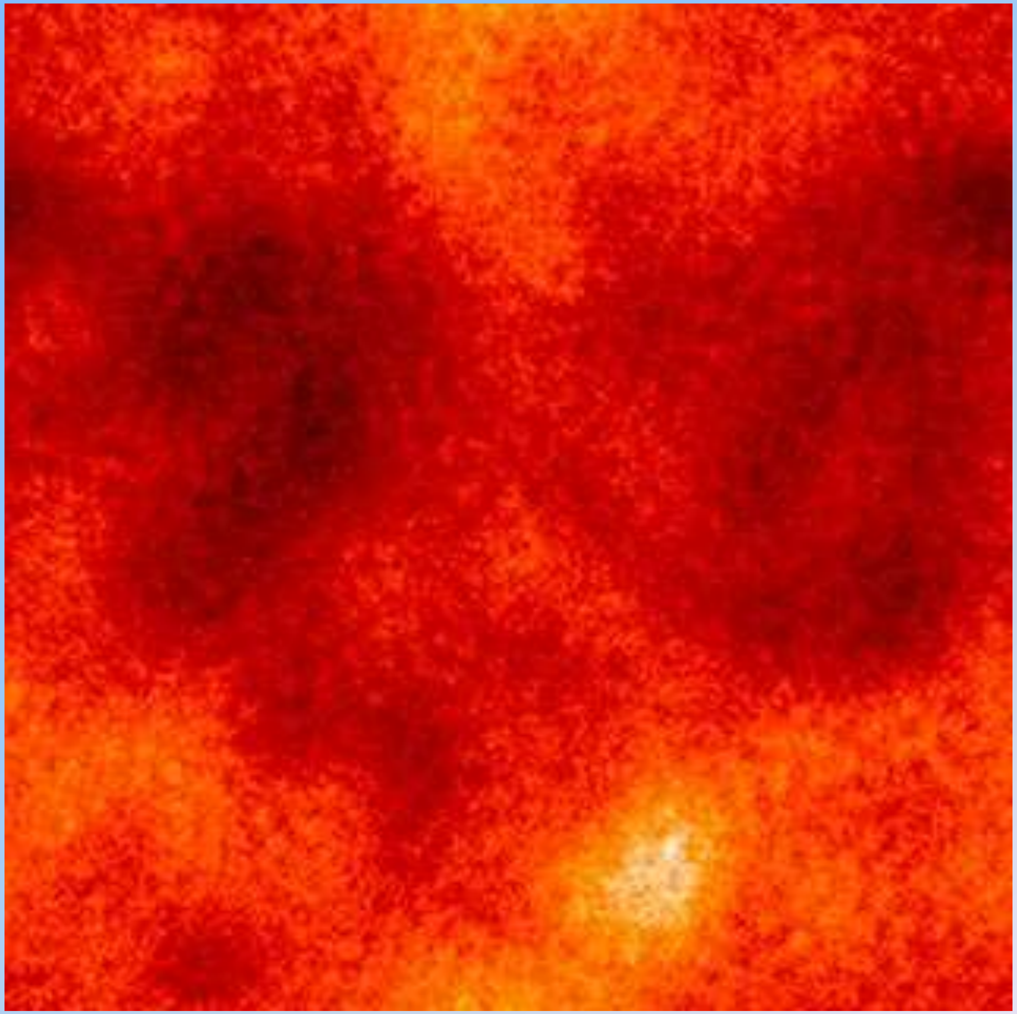
http://planck.cf.ac.uk/timeline/universe/bigbang

Quantum Space Time -> Inflation - Hyperexpansion?

in 10^{-35} sec there was 10^{60} Times Expansion



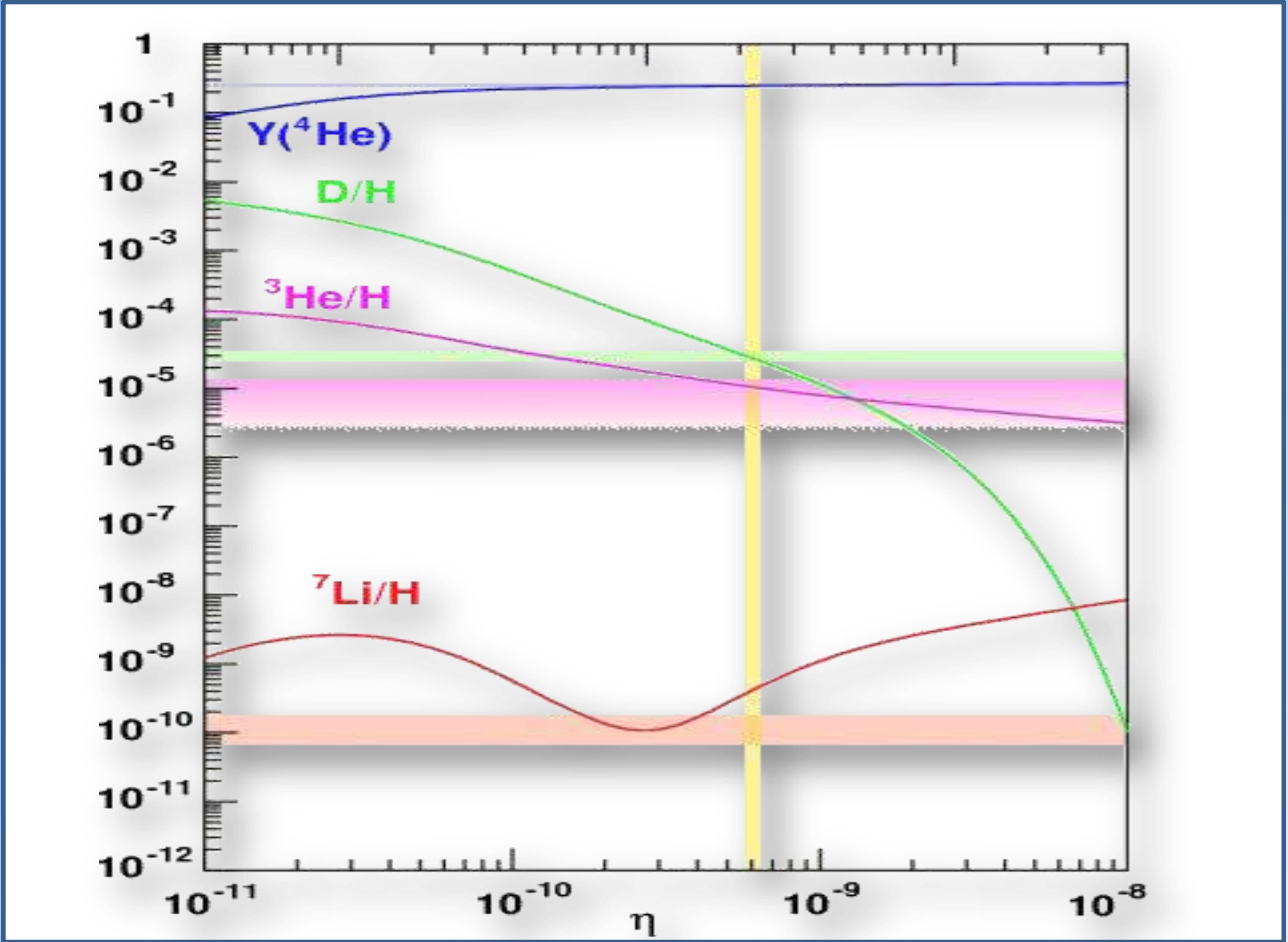
First Three Minutes –
Baryogenesis and Nucelosynthsis



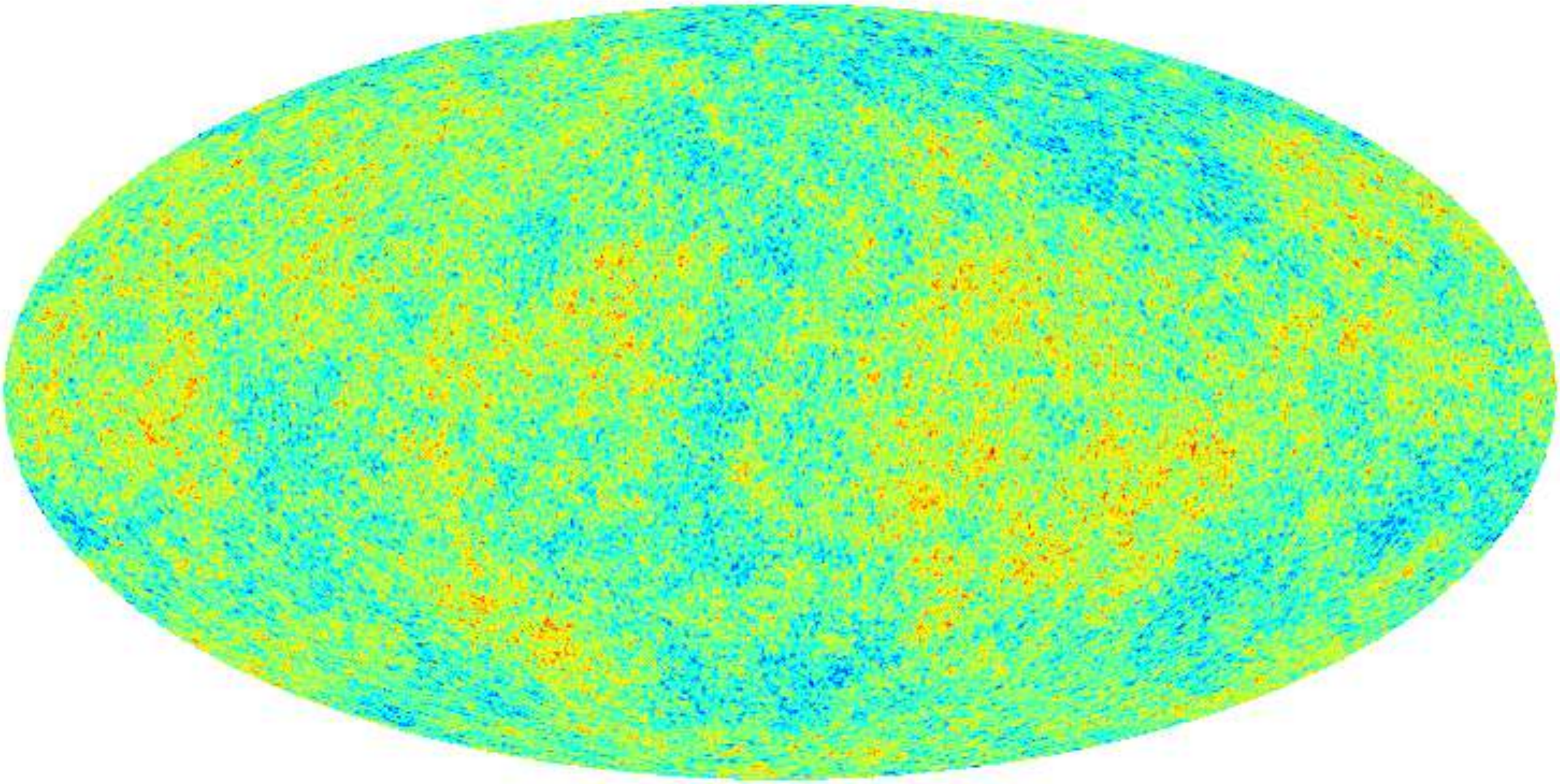
Nucleosythesis depends on Proton to Photon ratio ή

(you are alive because there are no stable mass 5 or 8 elements)

Vertical axis is mass ratio



400 Kyr – Ionized to Neutral – Thomson to Rayleigh Scattering
Opaque to Clear - “The CMB”



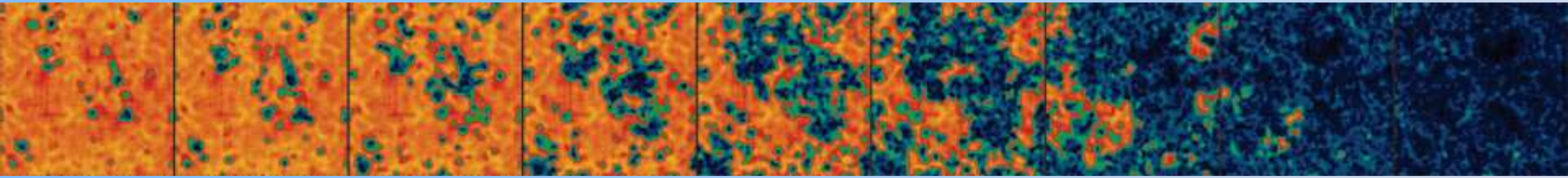
400 Kyr to 400 Myr - The “Dark Ages”

The Universe is largely Neutral but no stars yet – Baryonic collapse in progress

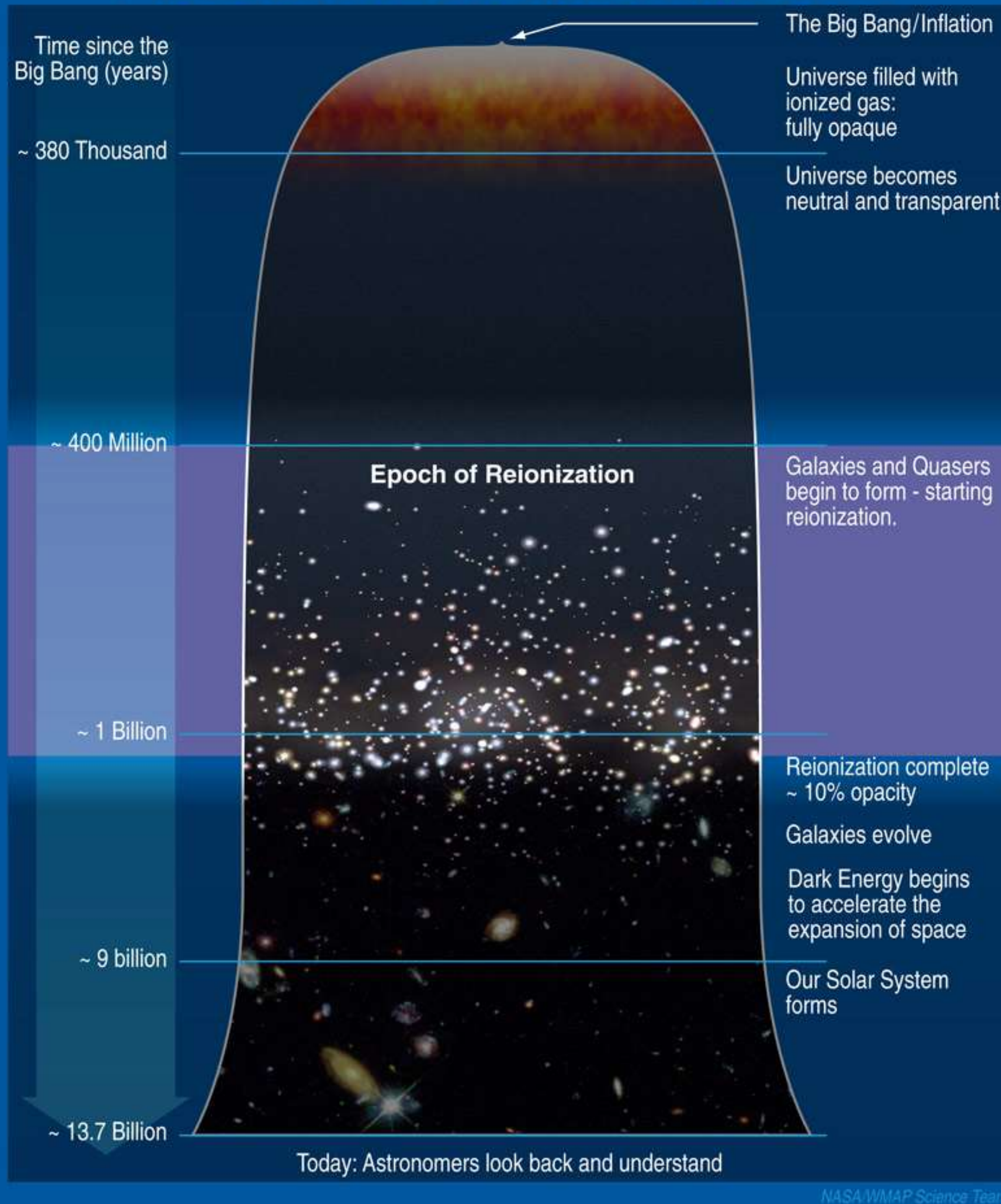
simulation rendition– WMAP team

The Universe goes from an Ionized Plasma to Neutral to Ionized
Approx 0.4 Gyr
The First Stars Reionize the Universe

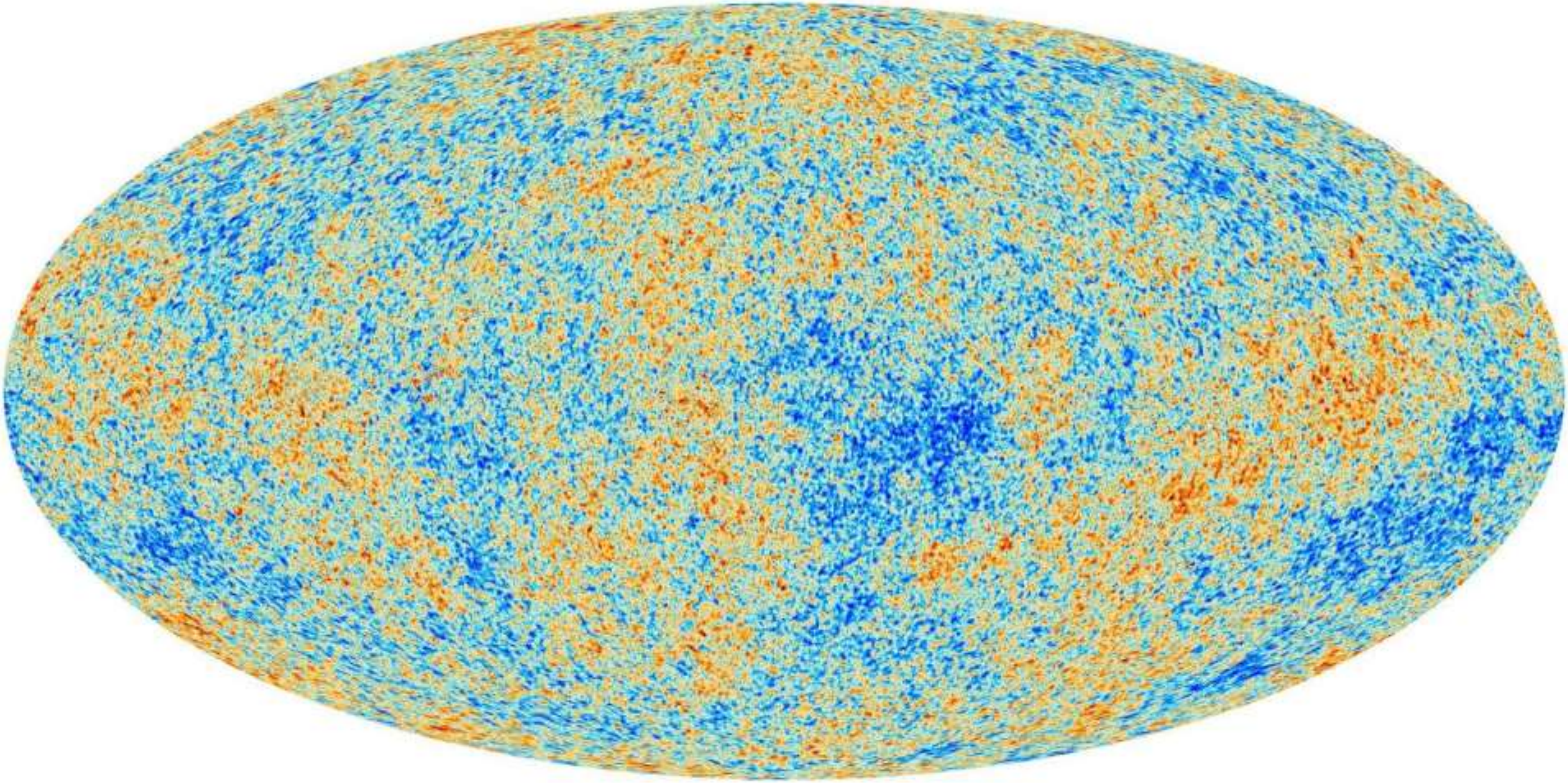
B. Ciardi – Nature 2006 - simulation



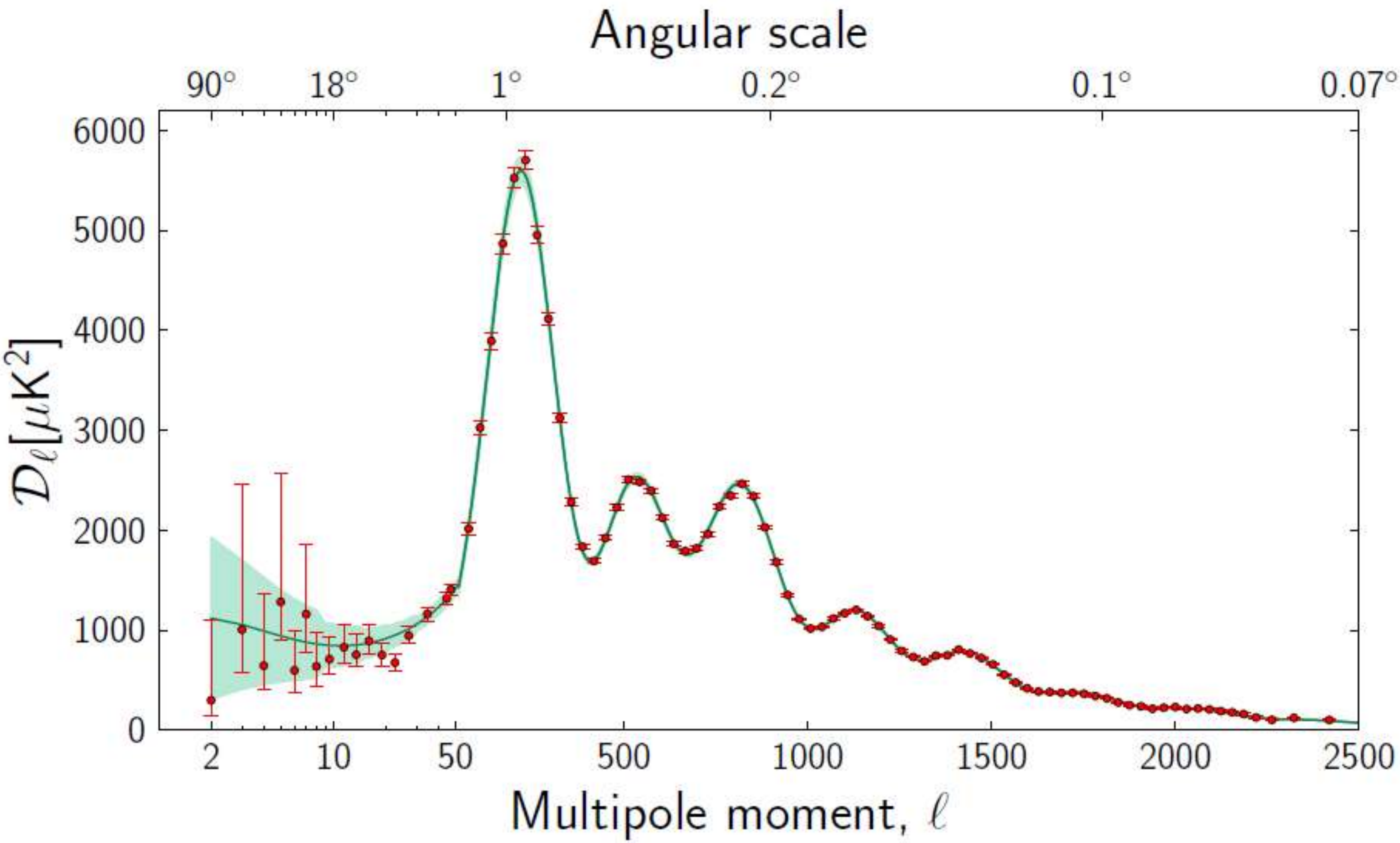
First Stars and Reionization Era



Foreground removed CMB fossil light from the edge of the visible universe



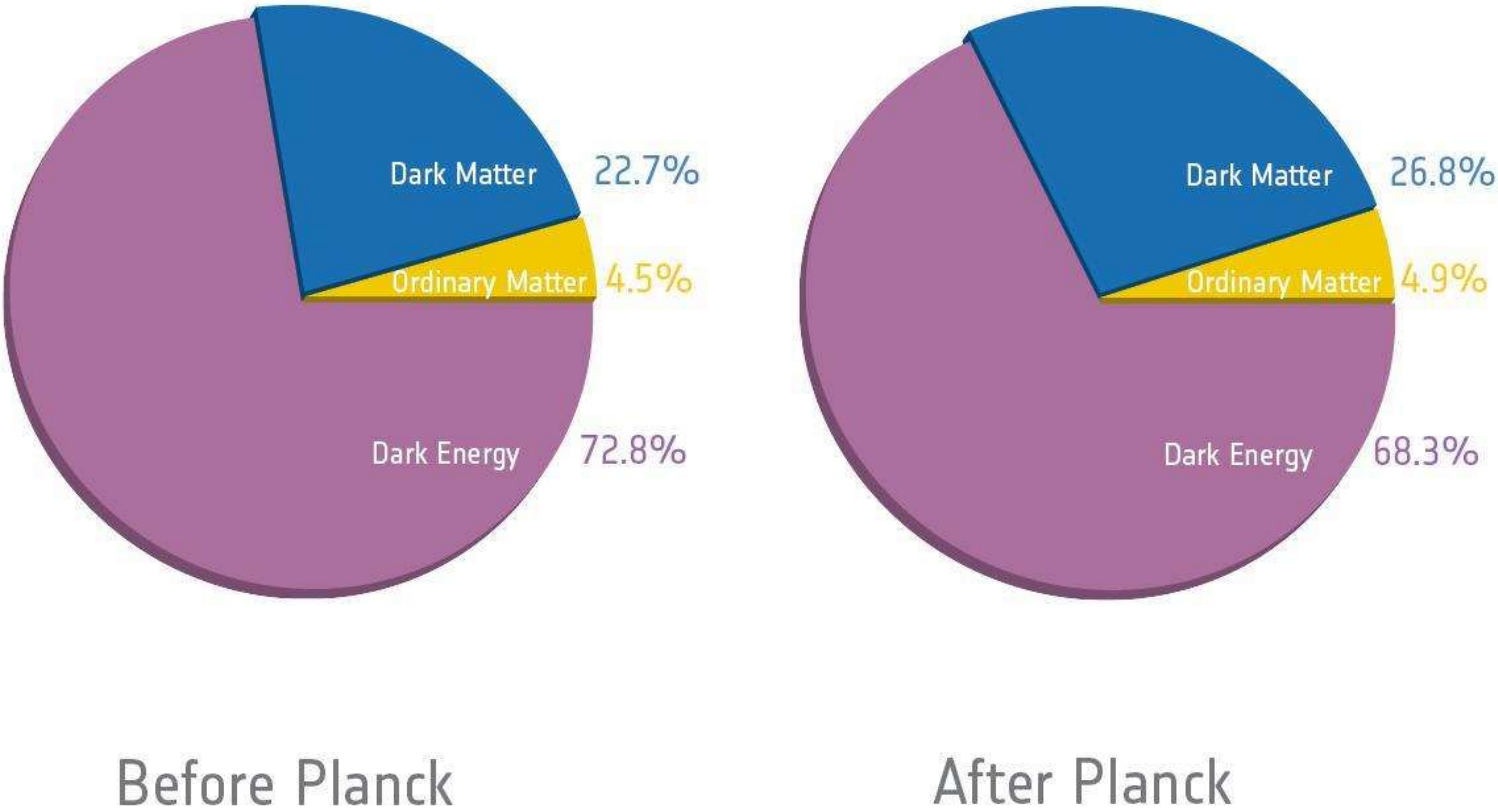
CMB temperature power spectrum



Parameter	*Planck* (CMB+lensing) Best fit	68 % limits	*Planck*+WP+highL+BAO Best fit	68 % limits
$\Omega_b h^2$	0.022242	0.02217 ± 0.00033	0.022161	0.02214 ± 0.00024
$\Omega_c h^2$	0.11805	0.1186 ± 0.0031	0.11889	0.1187 ± 0.0017
$100\theta_{MC}$	1.04150	1.04141 ± 0.00067	1.04148	1.04147 ± 0.00056
τ	0.0949	0.089 ± 0.032	0.0952	0.092 ± 0.013
n_s	0.9675	0.9635 ± 0.0094	0.9611	0.9608 ± 0.0054
$\ln(10^{10} A_s)$	3.098	3.085 ± 0.057	3.0973	3.091 ± 0.025
Ω_Λ	0.6964	0.693 ± 0.019	0.6914	0.692 ± 0.010
Ω_m	0.3036	0.307 ± 0.019		
σ_8	0.8285	0.823 ± 0.018	0.8288	0.826 ± 0.012
z_{re}	11.45	$10.8^{+3.1}_{-2.5}$	11.52	11.3 ± 1.1
H_0	68.14	67.9 ± 1.5	67.77	67.80 ± 0.77
$10^9 A_s$	2.215	$2.19^{+0.12}_{-0.14}$		
$\Omega_m h^2$	0.14094	0.1414 ± 0.0029		
$\Omega_m h^3$	0.09603	0.09593 ± 0.00058		
Y_P	0.247785	0.24775 ± 0.00014		
Age/Gyr	13.784	13.796 ± 0.058	13.7965	13.798 ± 0.037
z_*	1090.01	1090.16 ± 0.65		
r_*	144.58	144.96 ± 0.66		
$100\theta_*$	1.04164	1.04156 ± 0.00066	1.04163	1.04162 ± 0.00056
z_{drag}	1059.59	1059.43 ± 0.64		
r_{drag}	147.74	147.70 ± 0.63	147.611	147.68 ± 0.45
k_D	0.13998	0.13996 ± 0.00062		
$100\theta_D$	0.161196	0.16129 ± 0.00036		
z_{eq}	3352	3362 ± 69		
$100\theta_{eq}$	0.8224	0.821 ± 0.013		
$r_{drag}/D_V(0.57)$	0.07207	0.0719 ± 0.0011		

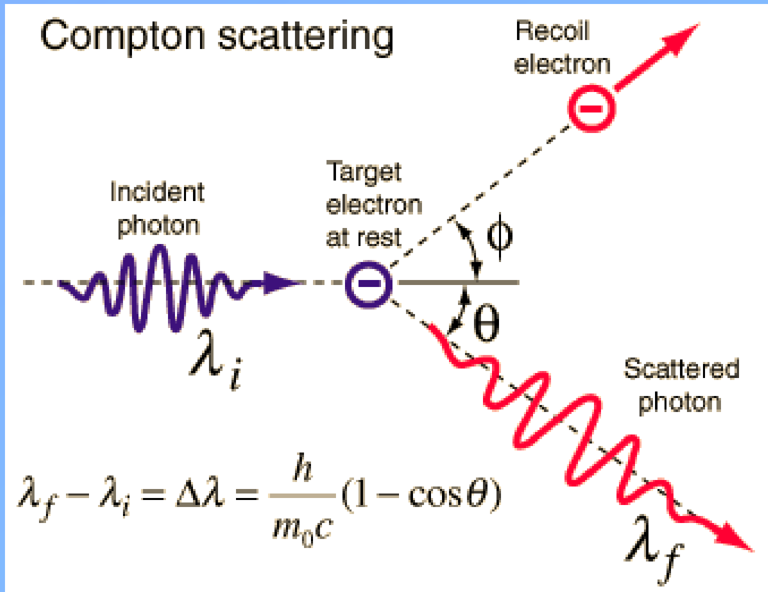
26 parameters!

composition of the universe



what we thought **what we now think**

http://www.esa.int/esatv/Videos/2013/03/Planck_reveals_an_almost_perfect_Universe/George_Efstathiou_Professor_of_Astrophysics_University_of_Cambridge_English_The_theory_of_the_expansion_of_the_Universe

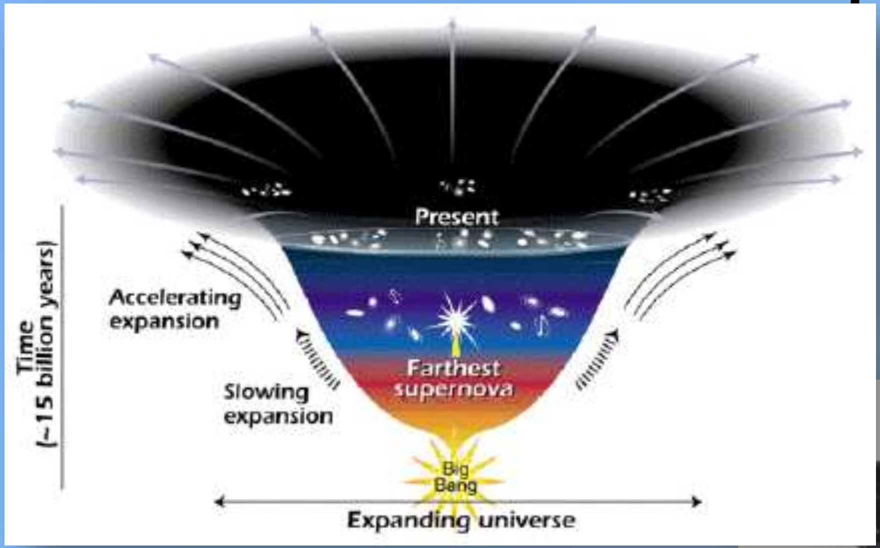


The Planck Scale:
When the entire universe was contained within its own Compton wavelength

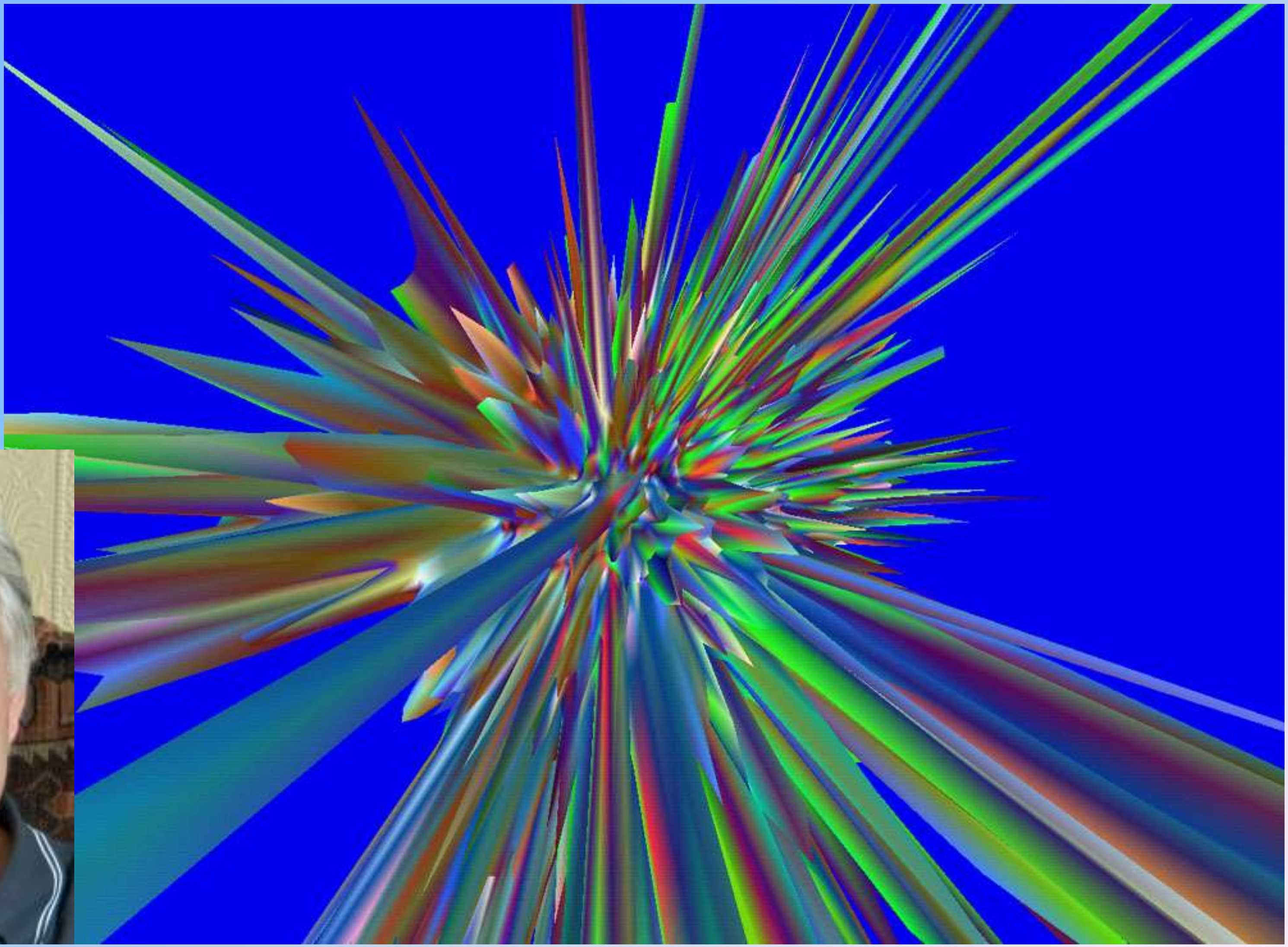
The Planck length:	$\left(\frac{\hbar G}{c^3}\right)^{1/2}$	$= 1.6 \times 10^{-35}$ metres,
The Planck mass:	$\left(\frac{\hbar c}{G}\right)^{1/2}$	$= 2.1 \times 10^{-8}$ kilograms,
The Planck time:	$\left(\frac{\hbar G}{c^5}\right)^{1/2}$	$= 5.4 \times 10^{-44}$ seconds,
The Planck energy:	$\left(\frac{\hbar c^5}{G}\right)^{1/2}$	$= 1.2 \times 10^{19}$ GeV.

global symmetries! do not depend on spacetime

One Universe or Many
Perhaps Infinite



Discussion of Andrei Linde: Self-Replicating Multiverse



Parting thought: Our universe seems to be always on the edge of a black hole…

$$R_S = \frac{2GM}{c^2}$$

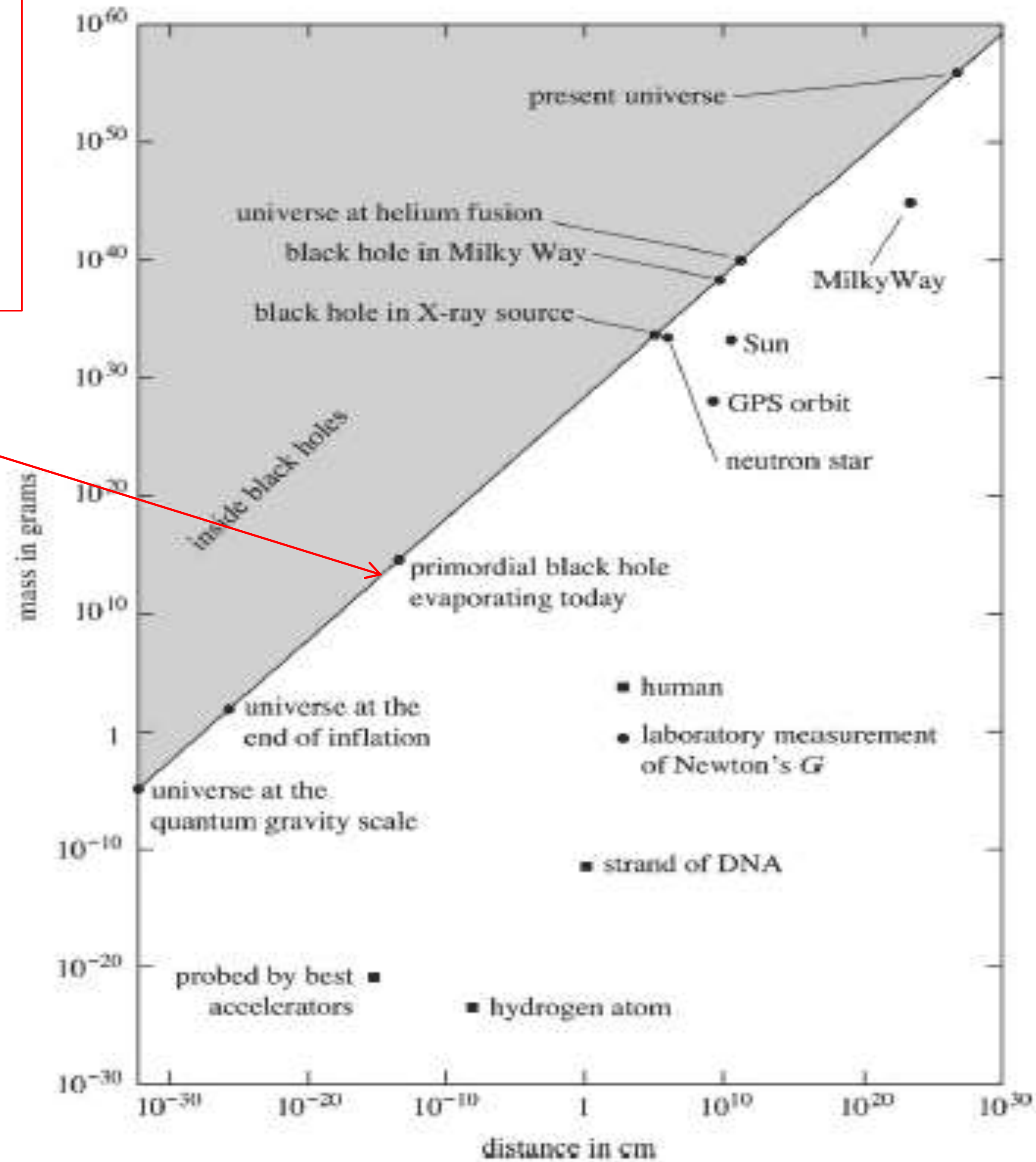


figure from Professor Jim Hartle's book on Gravity

Brian Greene explains String Theory: The next symmetry?