Superconducting Quantum Bits
Course Outline and Further Reading
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I) Introduction/Remarks

II) Quantum Computation
   For in-depth and formal discussion of logic, see
   “Quantum Computation and Quantum Information,” Nielsen and Chuang
   Pgs. 129-135 for classical logic gates
   Pgs. 174-177 for single qubit gates
   Pgs. 177- for multiple qubit gates

III) Superconducting Qubits
   A general discussion of superconducting qubits can be found in
   “Implementing Qubits with Superconducting Integrated Circuits,” Devoret and Martinis,

IV) Circuit Engineering and Decoherence
   A good microwave engineering textbook is “Microwave Engineering,” Pozar.
   A seminal paper on the quantum mechanics of electrical circuits is
   “Quantum Fluctuations in Electrical Circuits,” Devoret, Les Houches Session LXIII, 351-
   386 (1995); Download at http://www.eng.yale.edu/qlab/archives.htm
   The relation between decoherence and noise is described in “Decoherence of a

V) C Decoherence (Dielectric Loss from TLS)
   95, 210503 (2005).

VI) 1/f Noise Sources

VII) Tomography

VIII) Coupled qubits

Extra) Superconductivity
   A discussion of the BCS state and Josephson Effect (and exact derivation via Andreev
   bound states) can be found in the conference proceeding
   “Superconducting Qubits and the Physics of Josephson Junctions,” Osbourne and
   Martinis, Les Houches conference proceedings, cond-mat/0402415/.
   A formal and complete discussion of superconductivity is found, for example, in
   “Introduction to Superconductivity,” Tinkham.

Papers by Martinis can be downloaded at
http://www.physics.ucsb.edu/~martinisgroup/#publications
Superconducting Quantum Bits

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Scalable with IC microfabrication:
1) Introduction Remarks

- Quantum Integrated Circuits
  
  Power of IC's; complex comb. of electronics + mechanics
  
  Combine this power with qu. mechanics
  
  (Not just QM behind electronics, ie transistor)
  
  But QM of I's + V's in circuit
  
  Ideas around since 80's ... which new.

- Exp's work! Many ideas
  
  Quantum computation gives end goal.

- What's unique about J.J. approach?
  
  Strong coupling of quantum bits

Normal  \[ \rightarrow \]  (Hard)

\[ \text{weak coupling} \quad \text{good qubit} \]

\[ \text{weak coupling} \quad \text{bad/good coupling} \]

\[ \text{big} \quad \text{bad} \]

\[ \text{explain why} \]

\[ \text{result have to work} \]

\[ \text{hard, m understanding} \quad \text{decoupling = decoherence} \]

\[ \text{if doesn't work, harder figure out, hope these notes will help} \]

- Decoherence determines whether works!

  - This focus of class

  Will get to coupled qubits; relatively simple

  Most examples conventional  (Qubit) - please ask questions other

  - Course

  - This is 1st review of coherence/course

  - Discussion session (H.W., problems)

  - Syllabus
  
  - Paper list

  - Questions?
Quantum Computation

1) Classical Comput. : Easy to build logic so that hardware has two "logic" values 0 or 1.

2) But, Nature allows us to encode inform. as 0 and 1 at same time via QM. Mechanics e.g. \( |\psi\rangle = (|0\rangle + |1\rangle) / \sqrt{2} \)

   \( \Rightarrow \) Can use this to do powerful computations.

Example: \( H \) atom 0 = s state

1 = p state

\( \psi = (|s\rangle + |p\rangle) / \sqrt{2} \) encodes a state not achievable classically.

Note: Difference is basic operation

- Toggle switch - Dissip. implies switching may be imprecise - "Error Correction" built into classical switch.

Quantum System have no (ideally) dissipation. This toggle is floppy/bouncy.

- have to precisely push over switch

\( \Rightarrow \) Control, error correction is a big issue (neglected here).

\[ \text{But: no power dissipate is interesting/useful} \]

since this a limitation to classical logic
Computation Power

Classical

- Power of speed, size
  - 1 vs 2 GHz processor
  - 1x Dual vs Quad core.

- Memory size
  - Well trained by semic, business/advertisement (my sons ~ 8-10 yrs old understood)

Quantum

- 3 qubits

\[ \Psi = (107 + 117) \times (1\overline{0} + 1\overline{1}) \times (107 + 117) \]

\[ = 100007 + 100117 + \ldots + 11117 \]

- 8 combinations

Now do \( \frac{1}{n} \) computation with 8 possible input states!

Parallel computer: Power scales as \( 2^n \) vs \# of qubits

Example:

Qubox 64 \( \rightarrow 2 \times \rightarrow \) Qubox 65

- 64 qubits

\[ \text{Just add 1 bit/ generation!} \]

Problem:

- \( \frac{1}{n} \) computation, but only get \( n \) bits output
  - A "collapse" or inform.

Thus, can't speed up every problem.

Few algorithms invented: Factoring, Optimization (exp.) \( (\sqrt{n} \# \text{ops}) \)
Basic Logic Gates

Classical

**NOT**

\[
\begin{array}{c|c}
\text{in} & \text{out} \\
0 & 1 \\
1 & 0 \\
\end{array}
\]

**AND**

\[
\begin{array}{c|c|c}
\text{in 1} & \text{in 2} & \text{out} \\
0 & 0 & 0 \\
0 & 1 & 0 \\
1 & 0 & 0 \\
1 & 1 & 1 \\
\end{array}
\]

**Buffers**

\[
\begin{array}{c|c|c}
\text{in} & \text{out} \\
0 & 1 \\
1 & 0 \\
\end{array}
\]

**OR Gate**

\[
\begin{array}{c|c|c}
\text{in 1} & \text{in 2} & \text{out} \\
0 & 0 & 0 \\
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 1 \\
\end{array}
\]

+ wires = Any gate

Any Gate:

1. T. T.
2. \( \overline{0} \) take 1's
3. Make \( n \)-input AND
4. \( \overline{0} \) input OR's

Example: XOR (also known as CNOT)

\[
\begin{array}{c|c|c}
\text{in} & \text{out} \\
0 & 1 \\
1 & 0 \\
\end{array}
\]

\[
\begin{array}{c|c}
\text{control} & \text{flip} \\
0 & 0 \\
1 & 1 \\
\end{array}
\]
Quantum Logic Gates

(1) Big Diff. is Q.M. reversible; implies
   No Fan-Out of logic wires — as classical

(2) Need to keep track of every qubit in
    circuit;
    time sequence of ops in a wire

(3) Single qubit operation (detailed later, quantum)

(4) Coupled (2 qubit)

   CNOT (controlled NOT)

   Has classical equiv.

   Note this gate is also applicable for superpositions
   (Q.M.)
Quantum Circuit: Many qubits

(5)

\[ \text{describes seg. of operations like a circuit schematic} \]

1. Single qubit rotations \( \left( 2^{\pi/2} \right. \text{ transitions} \)

2. CNOTS (arbitrary pair of qubits) \( \left( 2^{\pi/4} \text{ transitions} \right) \)

= Arbitrary logic (like classical theorem)

[Other 2 qubit gates possible, \text{CNOT conventional, class. analog}]

\[ \text{classical equivalents} \]

\[ \text{Quantum Equivs} \]

\[ \text{need to defined complex 1 qubit operators} \]

\( \text{next more rich than} \) \text{BUF NOT} \]

\[ H: 0 \rightarrow 0 + 1/\sqrt{2} \]

Conclusion: At this time, basic physics -- focus on 1 and 2 qubit gates
Single Qubit (2-state): Spin in $\vec{B}$

(1) 

$$H = -\frac{\mu_B}{2} \vec{B} \cdot \vec{B}$$

$$= -\frac{\mu_B}{2} \left[ \left( \begin{array}{cc} 0 & 0 \\ 0 & 1 \end{array} \right) B_z + \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right) B_x \right]$$

Pauli matrices: $\sigma_z$, $\sigma_x$, $\sigma_y$

$$= \frac{\mu_B}{2} \left[ \begin{array}{cc} B_x & B_x - i B_y \\ B_x + i B_y & B_z \end{array} \right]$$

(2) Can be solved by standard linear algebra. Consider $B_z$ field only (easy).

$$E = \pm \frac{\mu_B}{2} B_z$$

$$\mu_B B_z = E_{10}$$

$10\rangle$, $11\rangle$ are eigenstates

$$| \langle 10 | + | \langle 11 | \rangle \rangle = 1$$

(3) Any state in $B_z$:

$$\Psi = a | 10 \rangle + b | 11 \rangle$$

$$= \cos(\frac{\theta}{2}) | 10 \rangle + i \sin(\frac{\theta}{2}) e^{i \phi} | 11 \rangle$$

Explicitly puts in O.M. time dependence.

(4) Geometric representation in Bloch sphere

$$| 10 \rangle + i | 11 \rangle \sqrt{2}$$

(5) Q. state describe by point. Rotated around $z$ axis by $e^{-iE_{10}t/\hbar}$.

(6) Or, rotating frame of $E_{10}/\hbar$ static direction. This most useful?
Eigenstates for an arbitrary $B_\perp$

1. Solve the HW problem.

2. Eigenenergies
   \[ E_{10} = \mu B \sqrt{B_x^2 + B_y^2 + B_z^2} \]
   \[ \theta \perp B \]
   
   Eigenstates
   \[ \theta = \theta' \]
   \[ \phi = \phi' \]

- Parallel to $B$ field

(Makes sense since $10$, $11$ $\parallel$ $\perp B_z$.)
Useful Example: Eigenvalues For Varying $B_z$, Fixed $B_x$.

1. Solve as H.W. problem

2. $E_x = \mu \beta B_x$, $E_z = \mu \beta B_z$

3. $E = \pm \sqrt{E_x^2 + E_z^2}$

4. Avoided level $\chi$-in
   $\Delta E = 2 |E_x|$

5. Slow chg $\delta B E_z$, stay on $E_-$ band
   Adiab. chg from $10^7 \rightarrow 11^7$.

6. Fast chg. ("Landau-Zener tunnel")
   $10^7 \rightarrow 10^7 \); unchanged.
1st Qubit Gate (Z): Rotation of state

(1)\[ B_z \]

Changes $E_{10}$ by $\Delta E_{10}(\tau)$

$\Psi = \alpha|0\rangle + \beta e^{-i\frac{\Delta E_{10}(\tau)}{\hbar}}|1\rangle$

(2) Phase term:

$e^{-i\frac{\Delta E_{10}(\tau)}{\hbar}} = e^{-i\frac{\Delta E_{10}(\tau)}{\hbar}} = e^{-i\frac{\Delta E_{10}(\tau)}{\hbar}}$

= \text{const.}

= \frac{\Delta E_{10}(\tau)}{\hbar}$

Rotates Bloch vector around $\hat{z}$ axis; prop to $\hbar$, $-\Pi_0$

(3) IF $\phi_z = \Pi (180^\circ)$ then call rotation

axis $\hat{z}$ in circuit language.

(4) Question:

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(nothing)

(5) note $|10\rangle = 10\rangle$, so it's not a NOT gate!

(6) Note: Gate looks like geometric rotation of Bloch vector.

(This is why B.V. is so useful!)

(B Field $\Rightarrow$ Rotation)
$X, Y$ rotations

(1) Guess $0 < 1$, need photon radiation

$$B = B_0 \hat{z} \rightarrow B_x \cos(\omega t + \phi) \hat{x}$$

$$H = \begin{bmatrix} -E_{10}/2 & E_x \cos(\omega t + \phi) \\ E_x \cos(\omega t + \phi) & E_{10}/2 \end{bmatrix}$$

(2) To proceed, need mathematics for rotating frame

Let $u' = U u$

$$U = \begin{pmatrix} 1 & 0 \\ 0 & e^{i \omega t} \end{pmatrix}$$

$$\psi' = \left( \begin{array}{c} a \\ be^{-i E_{10} t / 2} \end{array} \right) \rightarrow \psi = \left( \begin{array}{c} a \\ be^{i E_{10} t / 2} \end{array} \right)$$

$C$ removed rapidly rotating term.

$U$ is unitary transformation (basis change)

$$U^* U = U U^* = \mathbb{I}$$

Compute how basis change $U$ affects $H$

(3) S.E.

$$i \hbar \frac{\partial}{\partial t} \psi = H \psi$$

$$i \hbar \frac{\partial}{\partial t} (U \psi) = U H U^* \psi$$

$$U \left( i \hbar \frac{\partial}{\partial t} U^* \right) \psi + i \hbar U^* \frac{\partial}{\partial t} \psi = 0 \Rightarrow U H U^* \psi$$

$$\Rightarrow i \hbar \frac{\partial}{\partial t} \psi = [U H U^* - i \hbar U^* \frac{\partial}{\partial x}] \psi$$

$$= H' \psi$$

new $H'$ in rotating frame.
Rotating Wave Approximation

\[ H' = U H U^+ - i \hbar U (\partial U^+ / \partial t) \]

\[ = \begin{pmatrix} 1 & E_x \cos(\omega t + \phi) e^{i \omega t} & 0 \\ 0 & E_x / 2 & 0 \\ 0 & 0 & -i \omega t \end{pmatrix} - \hbar \begin{pmatrix} 0 & 0 & 0 \\ E_x / 2 & 0 & 0 \\ 0 & 0 & -\hbar \omega \end{pmatrix} \]

(No physics to offset in diagonal E)

Rapidly rotating field
(Averages to 0, Rotating wave approx.)

Effective field in x-y plane
1. $\phi = \pi/2$

$H = \begin{pmatrix} 0 & iE x/2 \\ -iE x/2 & 0 \end{pmatrix}$

$\hat{B}$ Field in $\hat{y}$ dir, state will precess around $\hat{y}$ axis (like pre. around $\hat{x}$ axis for $B_z$ Field)

$|10\rangle \rightarrow (|10\rangle + i|11\rangle)/\sqrt{2} \rightarrow |11\rangle \rightarrow (|0\rangle - i|1\rangle)/\sqrt{2} \rightarrow |10\rangle$

This looks like NOT operation for pulse of $E_x$ of right duration

$M_r = \pi$ rotation $= -\frac{\pi}{\hbar}$

$-\frac{\pi}{\hbar} - \frac{\pi}{\hbar} = (-\text{overall phase}) \rightarrow |11\rangle$ (ignoring no physics)

$Y|10\rangle = |11\rangle$; $Y|11\rangle = -|10\rangle$

2. $\phi = 0$

$|10\rangle \rightarrow |10\rangle - i|11\rangle \rightarrow |11\rangle \rightarrow \ldots$

3. $\hat{H} \neq E_{10}$; Diag. element not canceled.

$\hat{B}$ field out of plane, $|10\rangle$ not rotated to $|11\rangle$ state "off resonance"

4. On reson., in non rotating frame; Circles around $|10\rangle$ from $|10\rangle \rightarrow |11\rangle$. 
5. Vary $\omega, \phi$, rotate any axis.

Ampl. $\times$ Time = rotation amount.

6. By applying multiple pulses, can construct

any rotation from $Z$ "basis" pulses

(e.g., $\hat{x} + \hat{y}$) for arbitrary angle

Example:

$X_{180} Y_{180} = Z_{180}$

$\begin{array}{c}
\hat{x}_{180}
\end{array}
\begin{array}{c}
\hat{y}_{180}
\end{array}
= 1

\begin{array}{c}
\hat{z}_{180}
\end{array}$

7. Non-commuting Operations

$X_{90} Y_{90} \neq Y_{90} X_{90}$

8. Practically, sweep $\omega$ + Ampl. x Time + observe

change of state to calibrate.

9. Give example of Rabi, T., Ramsey

10. First Exposure to Imperfect World

Why not oscillate between 0 and 1?

Bloch vector resets to 0 state (randomly with prob. $\frac{1}{2}$)

Prob. sum of states

$\hat{x}$ can be understood as avg. of Bloch sphere

→ Decreases length in Bloch sphere

→ Oscillates smaller amplitude

→ See with multiple guses
Simple Qubit Gates

1. Rabi: 
   - Gate sequence: \( \tau \)
   - Probability 1 state

2. Ramsey: 
   - Gate sequence: \( \tau \)
   - Probability 1 state

3. More complex due to slightly detuned oscillations.