Superconducting Quantum Bits
Course Outline and Further Reading

John Martinis, UC Santa Barbara

17th Jyvaskyla Summer School, August 2007

I) Introduction/Remarks

II) Quantum Computation

For in-depth and formal discussion of logic, see
"Quantum Computation and Quantum Information," *Nielsen and Chuang*
Pgs. 129-135 for classical logic gates
Pgs. 174-177 for single qubit gates
Pgs. 177- for multiple qubit gates

III) Superconducting Qubits

A general discussion of superconducting qubits can be found in "Implementing Qubits with Superconducting Integrated Circuits," *Devoret and Martinis*, Quantum Information Processing **vol. 3** (2004).

IV) Circuit Engineering and Decoherence

A good microwave engineering textbook is "Microwave Engineering," *Pozar*.
A seminal paper on the quantum mechanics of electrical circuits is "Quantum Fluctuations in Electrical Circuits," Devoret, Les Houches Session LXIII, 351-386 (1995); Download at http://www.eng.yale.edu/qlab/archives.htm
The relation between decoherence and noise is described in "Decoherence of a Superconducting Qubit from Bias Noise," *Martinis et al,* Phys. Rev. **B** 67, 094510 (2003).

V) C Decoherence (Dielectric Loss from TLS)

"Decoherence in Josephson Qubits from Dielectric Loss," *Martinis et al,* Phys. Rev. Lett. **95**, 210503 (2005).

VI) 1/f Noise Sources

VII) Tomography

VIII) Coupled qubits

Extra) Superconductivity

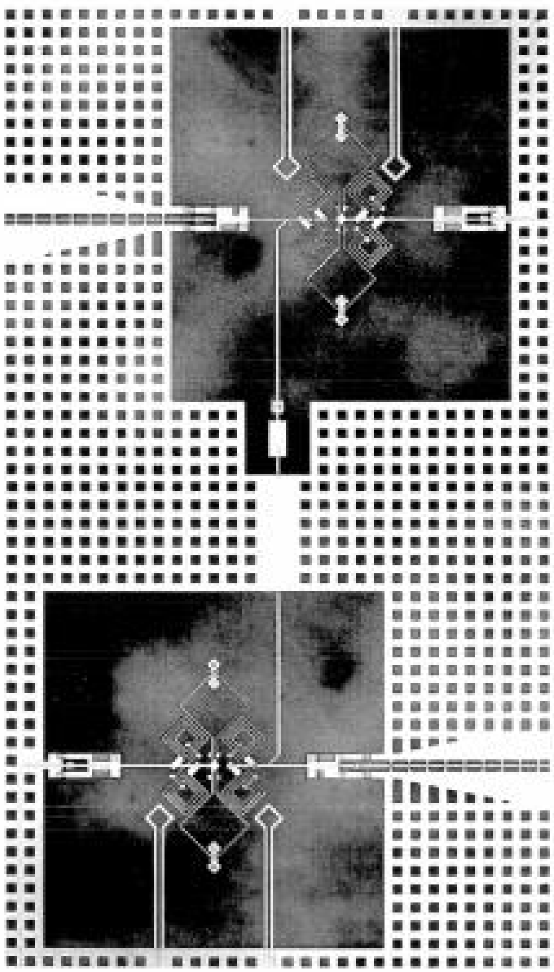
A discussion of the BCS state and Josephson Effect (and exact derivation via Andreev bound states) can be found in the conference proceeding "Superconducting Qubits and the Physics of Josephson Junctions," *Osbourne and Martinis*, Les Houches conference proceedings, cond-mat/0402415/.
A formal and complete discussion of superconductivity is found, for example, in "Introduction to Superconductivity," *Tinkham.*

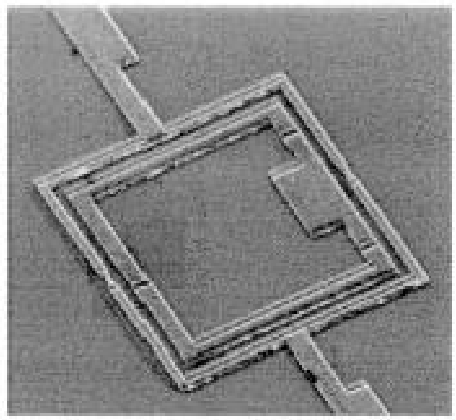
Papers by Martinis can be downloaded at
http://www.physics.ucsb.edu/~martinisgroup/#publications

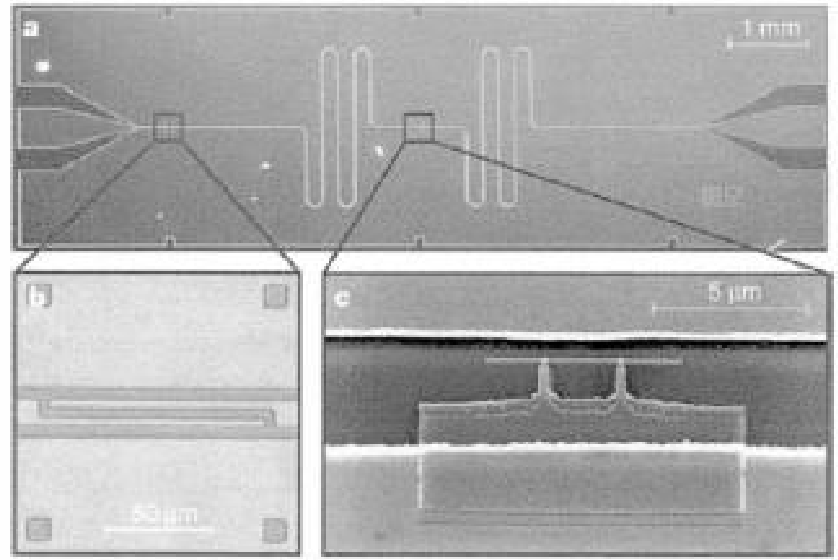
Superconducting Quantum Bits

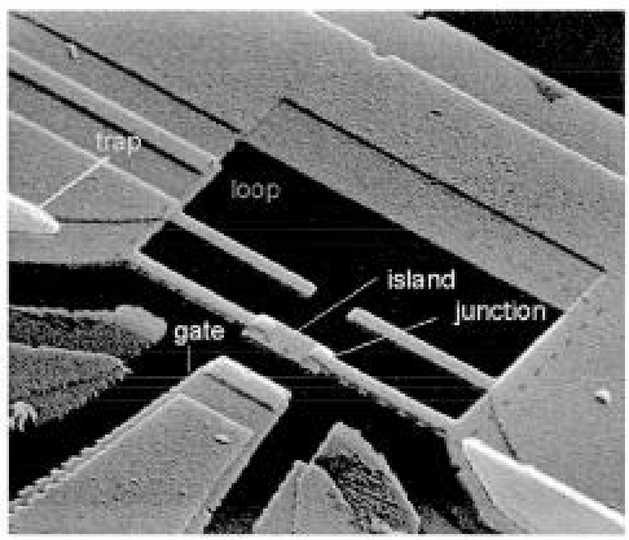
John Martinis, UC Santa Barbara

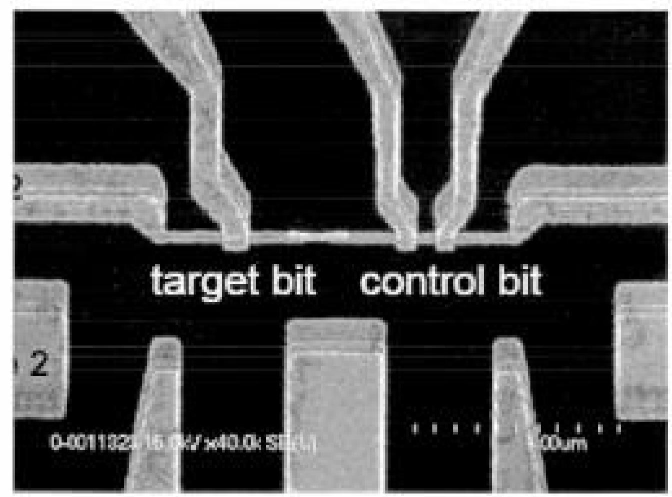
Scalable with IC microfabrication:











1) Introduction / Remarks

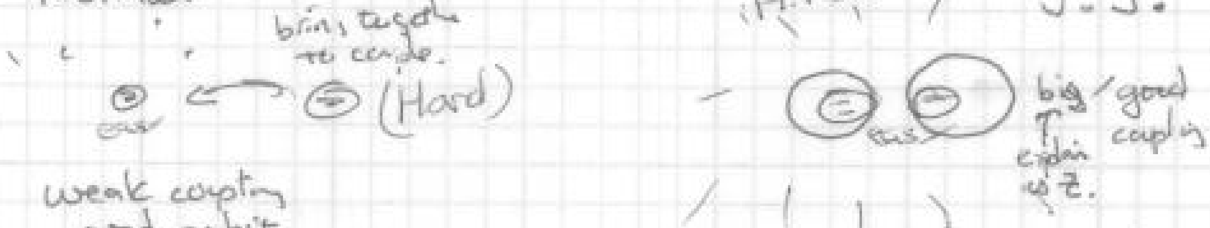
Show Picts

- Quantum Integrated Circuits
  Power of IC's; complex comb. of electronics μfab
  Combine this power with qu. mechanics
  (Not just QM behind electronics, ie transistor)
  But QM of I's + V's in circuit
  Ideas around since 80's ... whats new.

- Exp's work! Many ideas
  Quantum Computation gives end goal.

- What's unique about JJ approach?
  Strong coupling of quantum bits

Normal | HARD / J. J.



weak coupling
good qubit

weak couple
~~big~~ $Z/377\Omega \gg 1$

strong c/ But have to decouple
(~~Z~~ 377Ω) from rest of world

⇒ Result, have to work
hard on understanding decoupling = decoherence

→ If doesn't work; hard to figure out; hope these notes will help

What I want my students/colleagues to know.

- Decoherence determines whether works!
  Thus focus of class / Whether field eventually dies.
  Will get to coupled qubits; relatively simple.

Course: - Mostly examples our exps (know best) - please ask questions other systems.
Spend time on basics, easy to understand more complex exps.

- This is 1st review of coherence / course
  Please ask questions!
  Will help define subject, how will be taught in future

- Discussion session (H.W. problems)
  Syllabus
  Paper list

- Questions?

Quantum Computation

1) Classical Comput. : Easy to build Logic so that hardware has two "logic" values 0 <u>or</u> 1

2) But, Nature allows us to encode inform. as 0 <u>and</u> 1 at same time via Qu. Mechanics.
eg [$\psi = (|0\rangle + |1\rangle)/\sqrt{2}$]
$\Rightarrow$ Can use this to do powerful computation's

Example: H atom $0 =$ s state
$1 =$ p state
$\psi = (|s\rangle + |p\rangle)/\cancel{\sqrt{2}}$ ecodes a state not achievable classically

Note: ~~Difference is basic operation~~

Toggle switch – Dissip implies switching may be imprecise – "Error Correction" built into classical switch.

Quantum System have no (ideally) dissipation. This toggle is floppy/bouncy, have to precisely push over switch
$\rightarrow$ Control, error correction is a big issue (neglected here).

* But, no power dissip is interesting/useful since this a limitation to class. logic

Computation Power

Classical

Power $\propto$ speed, size

(1 vs 2 GHz processor

1x Dual vs Quad core.

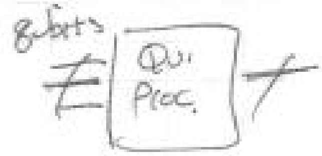
Memory size

⋮

• Well trained by semic. business/advertisement

(my sons ~ 8-10yrs old understood)

Quantum



3 qubits

$$\psi = (|0\rangle + |1\rangle)_{1} * (|0\rangle + |1\rangle)_{2} * (|0\rangle + |1\rangle)_{3}$$

$$= \underbrace{|0\,0\,0\rangle_{1\,2\,3} + |0\,0\,1\rangle_{1\,2\,3} + \dots \quad |1\,1\,1\rangle_{1\,2\,3}}_{8 \text{ combinations}}$$

Now do $\underline{\underline{1}}$ computation with 8 possible input states!

Parallel computer; Power scales as 2^{n} ← # of qubits

Example:

Qubox 64 → 2x → Qubox 65

64 qubits

Just add 1 bit/generation!

Problem:

// computation, but only get n bits output

"A "collapse" of inform.

Thus, can't speed up every problem.

Few algorithms invented; Factoring (exp.), Optimiz ($\sqrt{n}$ # ops)

Basic Logic Gates

Classical

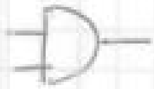
NOT 

in	out
0	1
1	0

examples ⇒ 

Buffer

0 → 0
1 → 1

AND 

in	out
0 0	0
0 1	0
1 0	0
1 1	1

00	out
0 0	0
0 1	1
1 0	1
1 1	1

OR gate

+ wires = Any Gate

Any Gate:

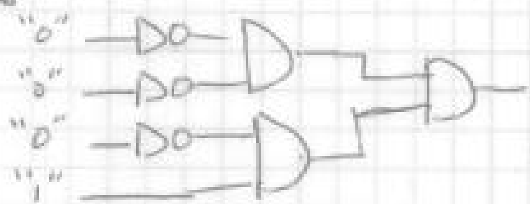
1

in	out
0000	0
0001	1
0010	1
0011	0
⋮	

T. T.

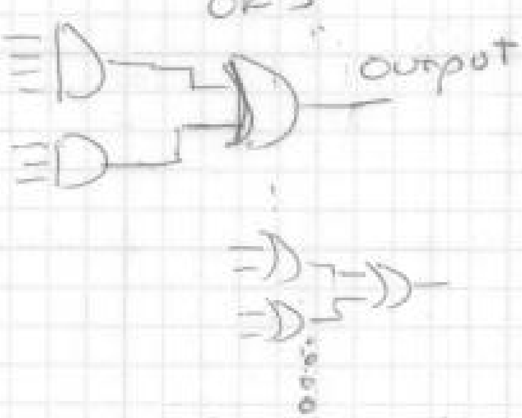
2 } take 1's

3 Make n-input AND



1 iff 0001

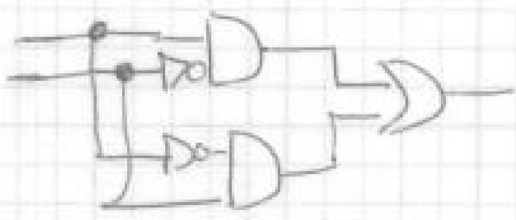
4 n input OR's



Example: XOR (also know as CNOT)

in	out
0 0	0
0 1	1
1 0	1
1 1	0

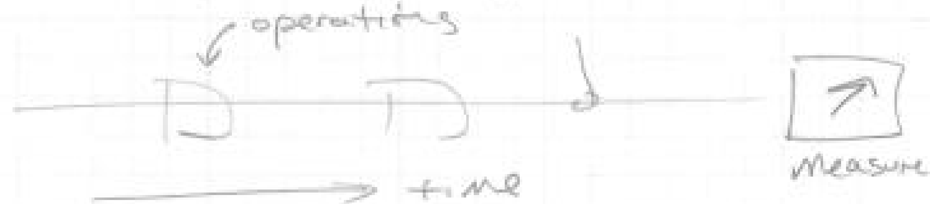
↑ control flip



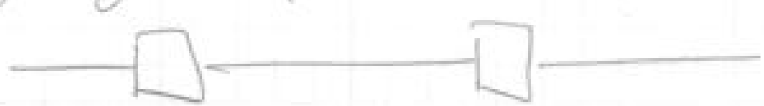
Quantum Logic Gates

(1) Big Diff. is QM reversable; implies
No direct Fan-Out of logic wires
as classical

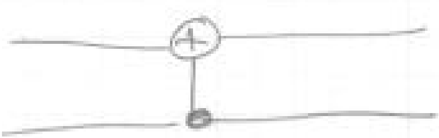
(2) Need to keep track of every qubit in
circuit;
time sequence of oper's in wire



(3) Single qubit operations (detailed later, quantum)

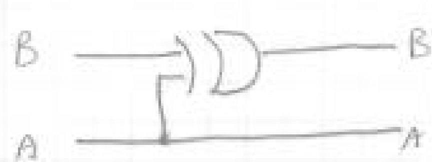


(4) Coupled (2 qubit)



CNOT (controlled NOT)

Has classical Equiv.



1 ⟷ 1
mapping; thus reversable
(so Q.M. possible)

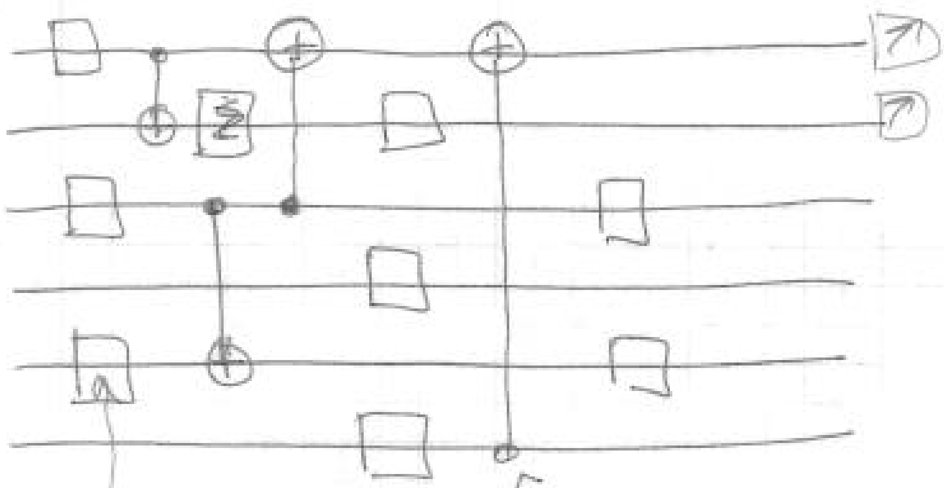
Note this gate is
also applicable
for superpositions
(Q.M.)

IN	OUT
0 0	0 0
0 1	0 1
1 0	1 1
1 1	1 0
A B	A B'

(Class; need AND 2 bit gate
QM; need XOR 2 qubit gate)

Quantum Circuit ; Many qubits

(5)



describes seq. of operations; like a circuit schematic.

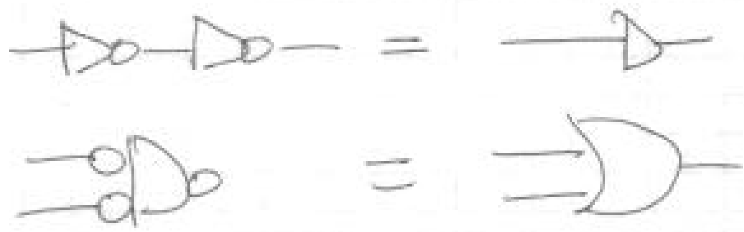
→ time

(A) Single qubit rotations ($2^n/2$ transitions) (H.W.)

\# (B) CNOTS (arbitrary pair of qubits) ($2^n/4$ transitions) (H.W.)

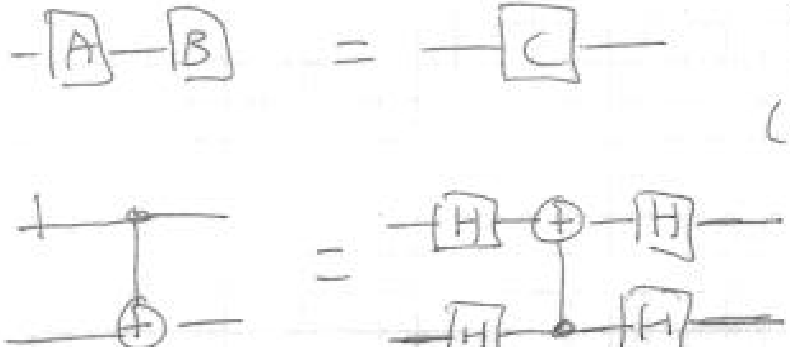
= Arbitrary logic. (Like classical theorem)

[Other 2 qubit gates possible, CNOT conventional, class. analog]



classical equivalents

Quantum Equiv's



need to defined complex 1 qubit operators (next, more rich than BUF NOT)

$$H: \begin{cases} 0 \to 0 + 1 / \sqrt{2} \\ 1 \to 0 - 1 / \sqrt{2} \end{cases}$$

Conclusion: At this time, basic physics — focus on 1 and 2 qubit gates

Single Qubit (2-state): Spin in $\overline{B}$

(1) $$H = -\frac{\sigma_B}{2}\,\overline{\sigma}\cdot\overline{B}$$

$$= -\frac{\mu_B}{2}\left[\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}B_z + \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}B_x + \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}B_y\right]$$

Pauli matrices → σ_z σ_x σ_y

$$= \frac{\mu_B}{2}\begin{bmatrix} B_z & B_x - iB_y \\ B_x + iB_y & -B_z \end{bmatrix}$$

(2) Can be solve, standard linear algebra
Conside B_z Field only (easy).

—— $|1\rangle$

$\mu_B B_z = E_{10}$

—— $|0\rangle$

$$E = \pm \frac{\mu_B}{2} B_z$$

$|0\rangle$, $|1\rangle$ are eigenstates

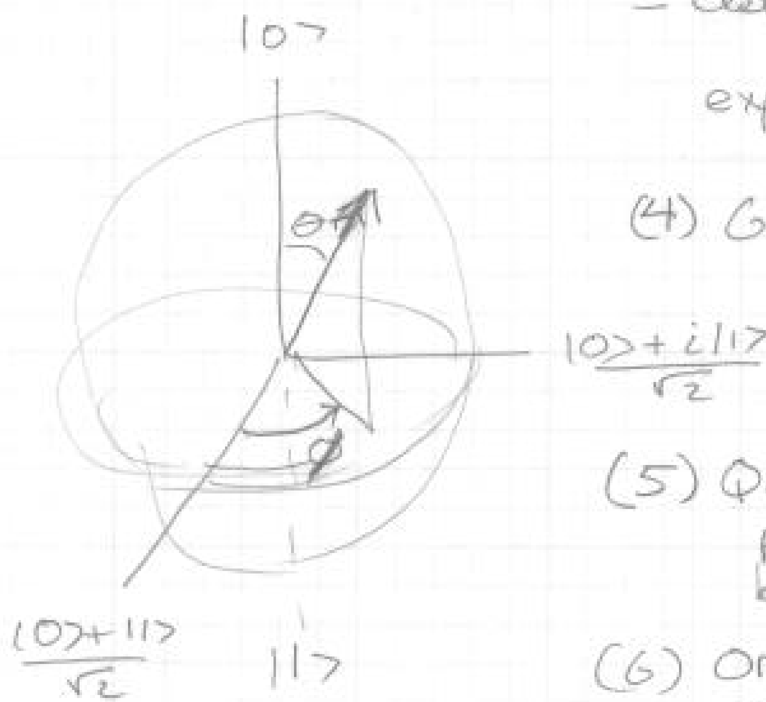
$|a|^2+|b|^2=1$

(3) Any state $\psi = a|0\rangle + b|1\rangle$

$$= \cos\left(\frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\theta}{2}\right)e^{i\phi}\,e^{-E_{10}t/\hbar}\,|1\rangle$$

explicitly puts in Q.M. time dependence.

(4) Geometric Represent. in Bloch Sphere

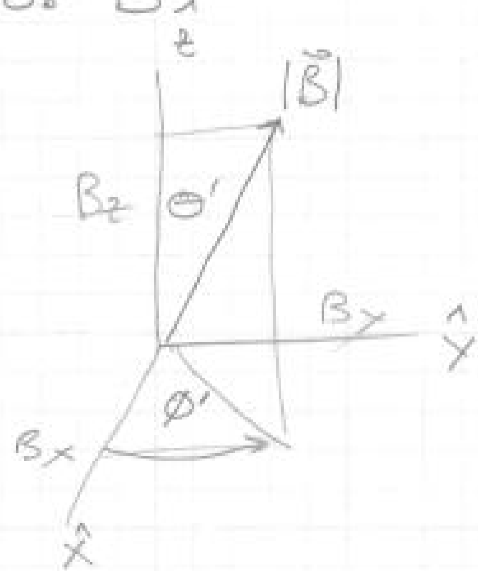


(5) Q. state describe by point. Rotates around z axis by $e^{-E_{10}t/\hbar}$.

(6) Or, rotating frame of $E_{10}/\hbar$, static direction. This most useful!

Eigenstates for arb. $\vec{B}$

(1) Solve in HW problem.



(2) Eigenenergies

$$E_{1,0} = \mu_B \sqrt{B_x^2 + B_y^2 + B_z^2}$$

$$\propto |B|$$

Eigenstates

$$\theta = \theta'$$

$$\phi = \phi'$$

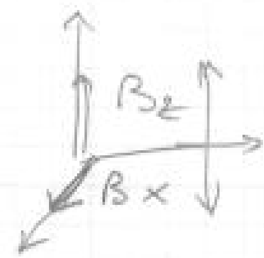
$\Rightarrow$ Parallel to $\vec{B}$ Field!

(Makes sense since $|0\rangle, |1\rangle \parallel$ to B_z).

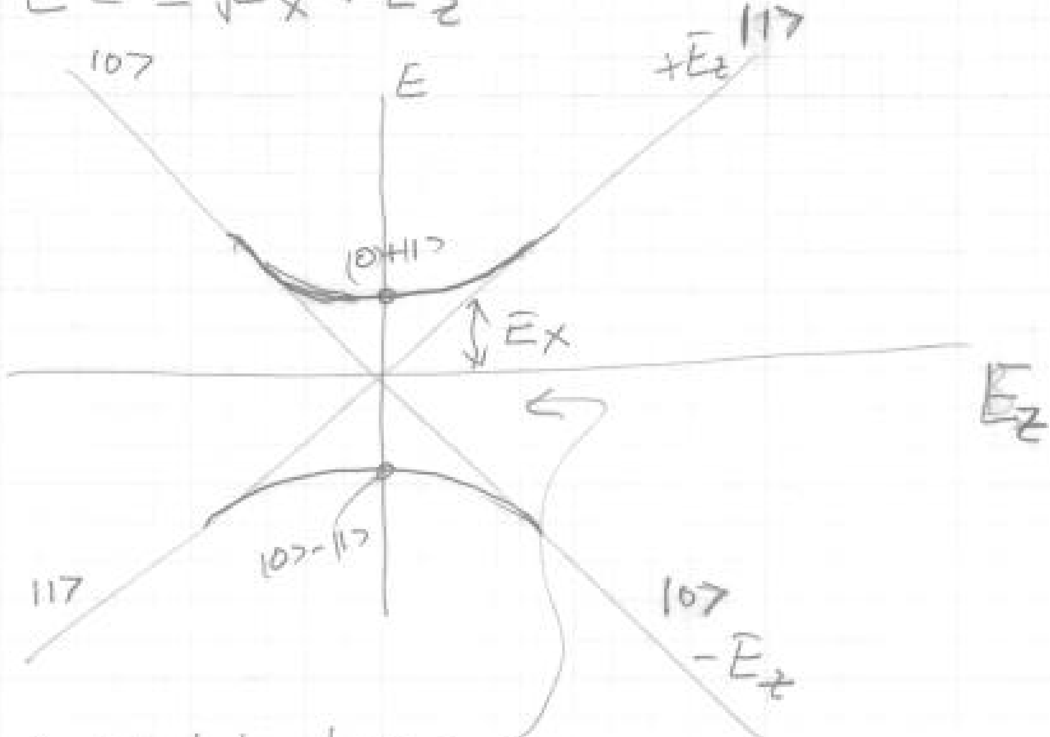
Useful Example: Eigenvalues for Varying B_z, Fixed B_x.

(1) Solve as H.W. problem

(2) $E_x = \mu_B B_x$, $E_z = \mu_B B_z$



(3) $E = \pm\sqrt{E_x^2 + E_z^2}$

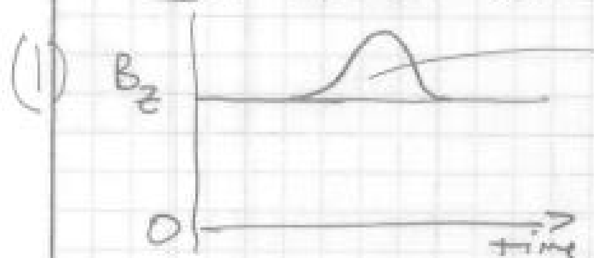


(4) Avoided level X-in
$\Delta E = 2|E_x|$

(5) Slow chg of E_z; stay on E_- band
Adiab. chg from $|0\rangle$ to $|1\rangle$.

(6) Fast chg. (Landau-Zener tunnel)
$|0\rangle \to |0\rangle$; unchanged.

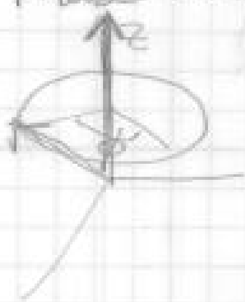
1st Qubit Gate (Z); Rotation of state

(1) 

Changes E_{10} by $\Delta E_{10}(t)$

$$\psi = a|0\rangle + b\, \underbrace{e^{i\int E_{10}(t)dt/\hbar}}|1\rangle$$

(2) Phase term: $= e^{-i\int E_{10}(t)dt/\hbar}$



$$= \underbrace{e^{-iE_{10}t/\hbar}}_{\text{const. ihg.}} \; \underbrace{e^{-i\int \Delta E_{10}(t)dt/\hbar}}$$

$$= \left[e^{-iE_{10}t/\hbar}\right] \underbrace{e^{-i\phi_z}}$$

Rotates Bloch vector around $\hat{z}$ axis; prop to field, time

(3) If $\phi_z = \pi$ (180°) then call rotation

axis

$-\boxed{Z_\pi}-$ in circuit language

magnitude [dropped if π (sometimes $\pi/2$)]

(4) Question:

$-\boxed{Z_\pi}-\boxed{Z_\pi}- \; = \; ——$ (nothing)

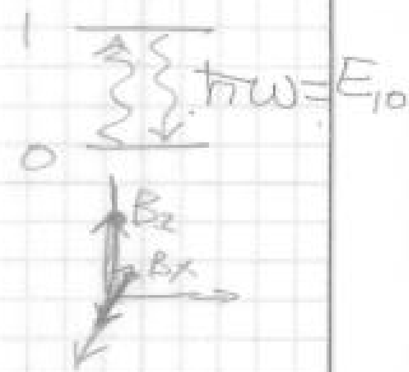
(5) note $Z|0\rangle = |0\rangle$, so its <u>not</u> a NOT gate!

(6) Note: Gate looks like geometric rotation of Bloch vector.

(This is why B.V. is so useful!)

(B field ⇒ Rotation)

$\hat{X}, \hat{Y}$ rotations



(1) Guess $0 \leftrightarrow 1$; need photon radiation

$$B = B_z\,\hat{z} - B_x \cos(\omega t + \phi)\,\hat{x}$$

$$H = \begin{bmatrix} -E_{10}/2 & E_x \cos(\omega t + \phi) \\ E_x \cos(\omega t + \phi) & E_{10}/2 \end{bmatrix}$$

(2) To proceed, need mathematics for rotating frame

$$\underset{\text{new}}{\psi'} = \underset{\text{transf}}{U}\,\underset{\text{old}}{\psi} \qquad U = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\omega t} \end{pmatrix} \quad (\hbar\omega = E_{10})$$

if $\psi = \begin{pmatrix} a \\ b e^{-iE_{10}t/\hbar} \end{pmatrix} \begin{matrix} |0\rangle \\ |1\rangle \end{matrix}$ $\quad U\psi = \begin{pmatrix} a \\ b\,\underbrace{e^{-iE_{10}t/\hbar}\, e^{i\omega t}}_{=1} \end{pmatrix}$

← removed rapidly rotating term.

(U is unitary transf ; A basis change
$U^\dagger U = U U^\dagger = I$)

Compute how basis change U affects H

(3) S.E. $\quad i\hbar \frac{\partial}{\partial t}\psi = H\psi$

$$i\hbar \frac{\partial}{\partial t}(U^\dagger \psi') = H U^\dagger \psi'$$

$$U\, i\hbar\left(\frac{\partial}{\partial t}U^\dagger\right)\psi' + i\hbar\, U U^\dagger \frac{\partial}{\partial t}\psi' = U H U^\dagger \psi'$$

$$i\hbar \frac{\partial}{\partial t}\psi' = \left[U H U^\dagger - i\hbar\, U\left(\frac{\partial}{\partial t}U^\dagger\right)\right]\psi'$$

$$= H'\,\psi'$$

new H in rotating frame.

Rotating Wave Approximation

$$H' = UHU^{+} - i\hbar U(\partial U^{+}/\partial t)$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & e^{i\omega t} \end{pmatrix} \begin{pmatrix} -E_{10}/2 & E_x\cos(\omega t+\phi) \\ E_x\cos(\omega t+\phi) & E_{10}/2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\omega t} \end{pmatrix} - i\hbar \begin{pmatrix} 1 & 0 \\ 0 & e^{i\omega t} \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & -i\omega e^{-i\omega t} \end{pmatrix}$$

$$= \begin{pmatrix} -E_{10}/2 & E_x\cos(\omega t+\phi)e^{-i\omega t} \\ E_x\cos(\omega t+\phi)e^{i\omega t} & (E_{10}/2)e^{-i\omega t}e^{i\omega t} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & -\hbar\omega \end{pmatrix}$$

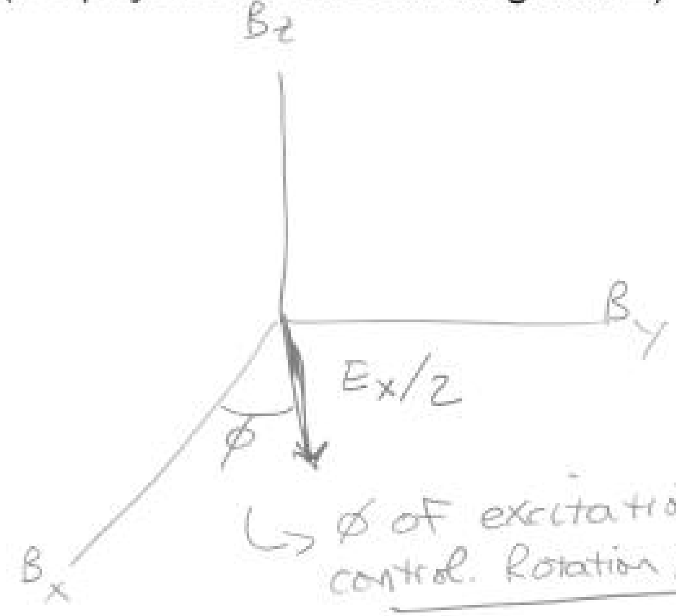
$$= \begin{pmatrix} 0 & E_x\cos(\omega t+\phi)e^{-i\omega t} \\ E_x\cos(\omega t+\phi)e^{i\omega t} & 0 \end{pmatrix}$$

(No physics to offset in diagonal E)

$$= \frac{E_x}{2}\begin{pmatrix} 0 & e^{i\phi} \\ e^{-i\phi} & 0 \end{pmatrix} + \cancelto{0}{\frac{E_x}{2}\begin{pmatrix} 0 & e^{-i2\omega t-i\phi} \\ e^{i2\omega t+i\phi} & 0 \end{pmatrix}}$$

Effective field
In x-y plane

Rapidly rotating field
(Averages to 0,
Rotating wave approx.)

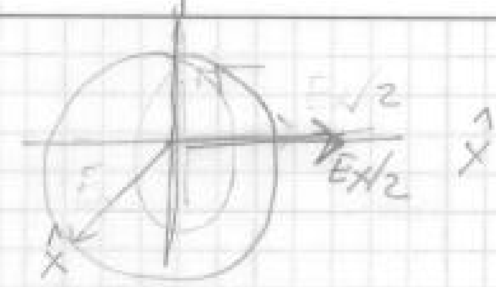


↳ φ of excitation control. Rotation Axis!

Comments

(1) $\phi = \pi/2$

$$H = \begin{pmatrix} 0 & iE_x/2 \\ -iE_x/2 & 0 \end{pmatrix}$$



$\vec{B}$ Field in $\hat{y}$ dir, state will precess around $\hat{y}$ axis (like prec. around $\hat{z}$ axis for B_z Field)

$|0\rangle \to (|0\rangle + |1\rangle)/\sqrt{2} \to |1\rangle \to (|0\rangle - |1\rangle)/\sqrt{2} \to |0\rangle$

This looks like NOT operation for pulse of E_x of right duration

 $\Rightarrow \pi$ rotation $\equiv$ $-\boxed{Y_\pi}-$

$-\boxed{Y_\pi}-\boxed{Y_\pi}-$ $=$ —— (− overall phase 1 qubit no physics)

$Y|0\rangle = |1\rangle$; $Y|1\rangle = -|0\rangle$

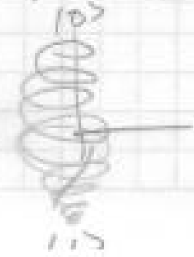
(2) $\phi = 0$ $\quad |0\rangle \to |0\rangle - i|1\rangle \to |1\rangle \to \ldots$

$-\boxed{X}-$

(4) $\hbar\omega \neq E_{10}$; Diag element not canceled

B Field out of plane, $|0\rangle$ <u>not</u> rotated to $|1\rangle$ state "OFF resonance"

(3) On reson, in <u>non rotating frame</u>; Circles around from $|0\rangle$ to $|1\rangle$.



Summary / Impt. comments.

⑤ Vary $\underline{\omega}$, $\underline{\phi}$; rotate $\underline{\text{any}}$ axis.

Ampl + Time = rotation amount.

⑥ By Apply multiple pulses, can construct arb. rotation from 2 "basis" pulses (eg. $\hat{X}_\theta$ + $\hat{Y}_\theta$) arb angle.

examp $X_{180} Y_{180} = Z_{180}$

$$-\boxed{X_{180}}-\boxed{Y_{180}}- = |Z_{180}|$$

axis

rotation amount

Notation

⑦ Non-commuting Operations

$$X_{90} Y_{90} \neq Y_{90} X_{90}$$

⑧ Practically, sweep ω + Ampl. time + observe change of state to calibrate.

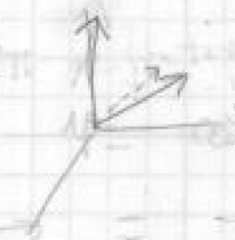
⑨ Give example of Rabi, T_1, Ramsey,

show Data

⑩ $\underline{\text{1st}}$ Exposure to Imperfect World

Why not osc. between 0 and 1 ?

Bloch vector resets to 0 state (randomly with prob)



Prob. sum of states

* Can be understood as $\underline{\text{avg.}}$ of B.V

→ Decreases length in Bloch sphere

→ Osc's smaller amplitude

Simple Qubit Gates