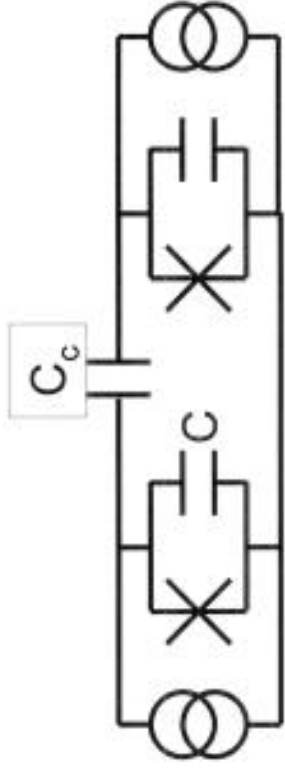


Coupled Qubits

Straightforward to implement:
simple coupling
tunable
fast readout
simultaneous measurement

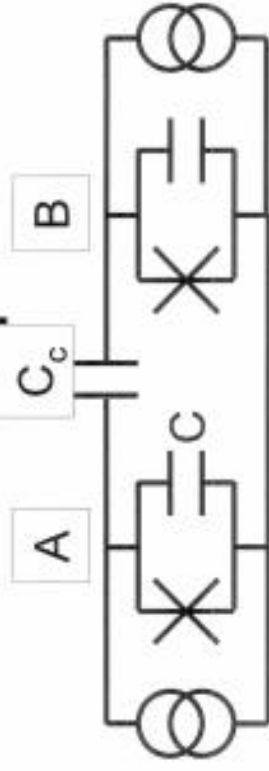


$$S/2 = \frac{C_c}{C} \hbar \omega_{10} (|01\rangle\langle 10| + |10\rangle\langle 01|)$$

On Resonance:

$$\frac{10}{00} \dots \frac{01}{11}$$

Coupled Qubits: Spectroscopy

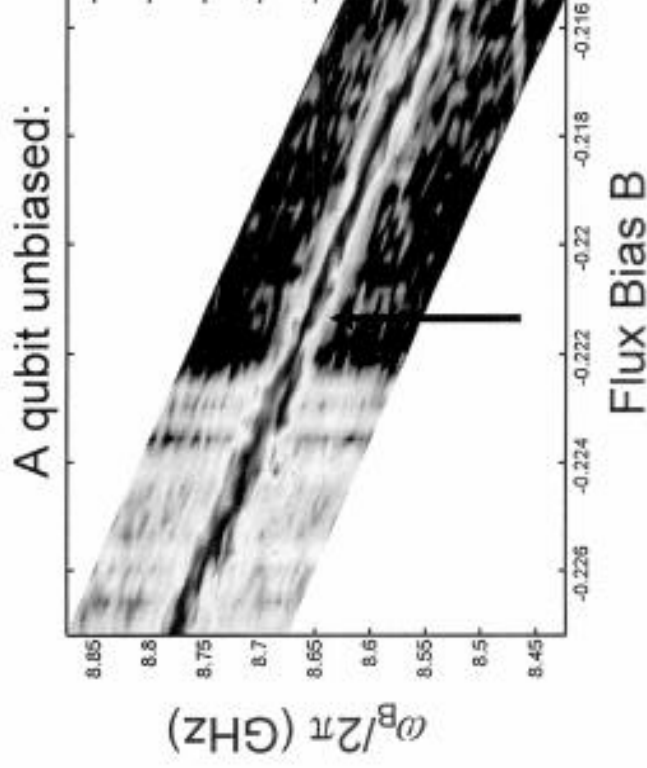
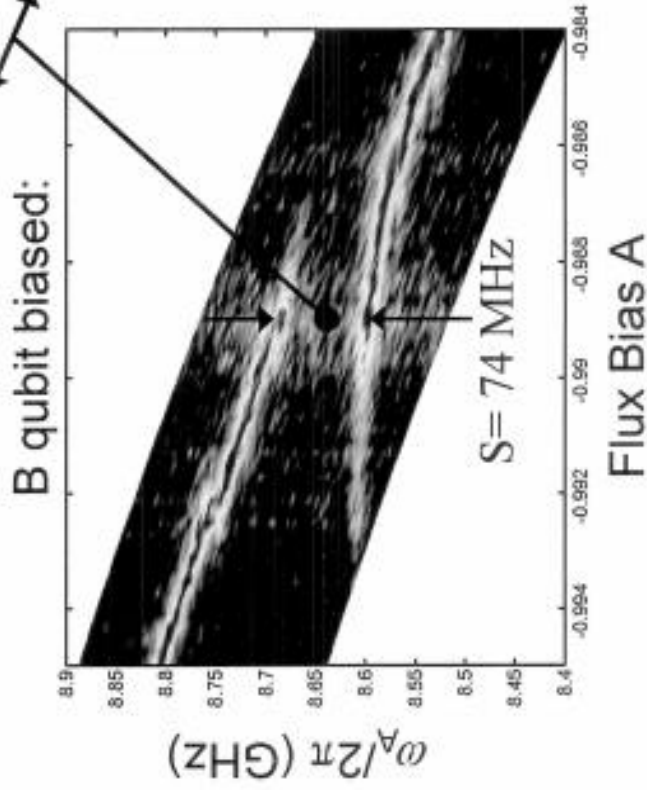
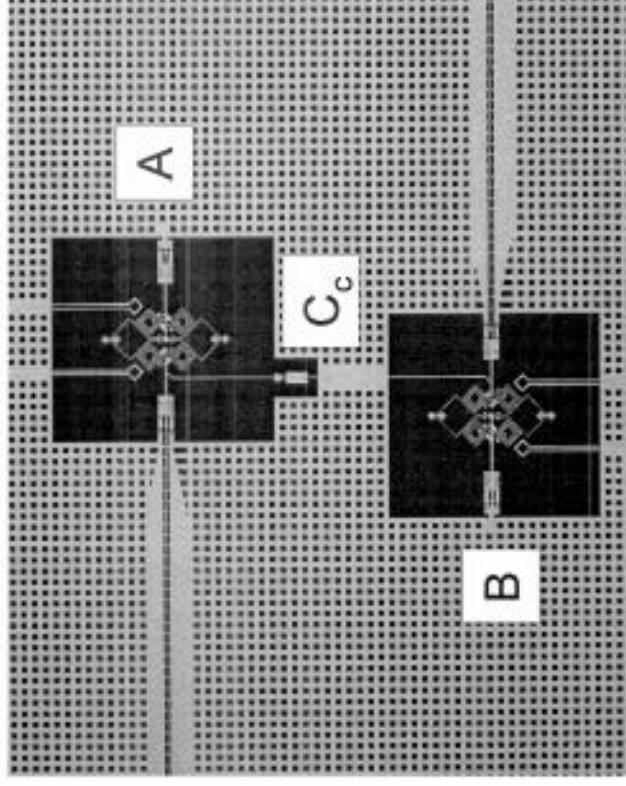


$$H_{int} = \frac{C_c}{C} \hbar \omega_{10} (|01\rangle\langle 10| + |10\rangle\langle 01|)$$

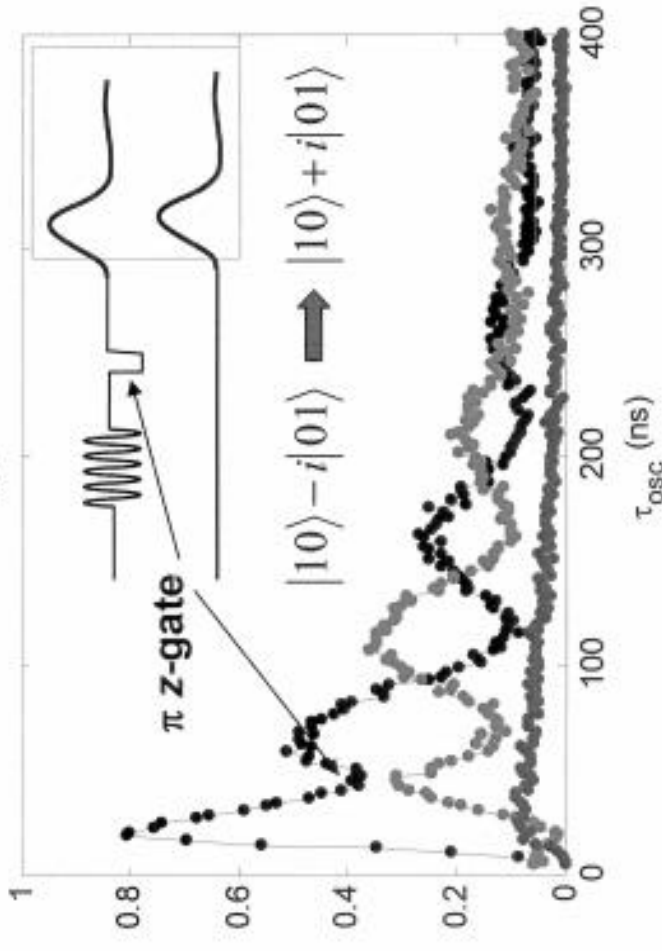
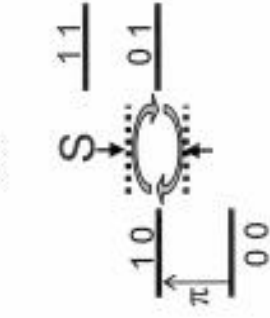
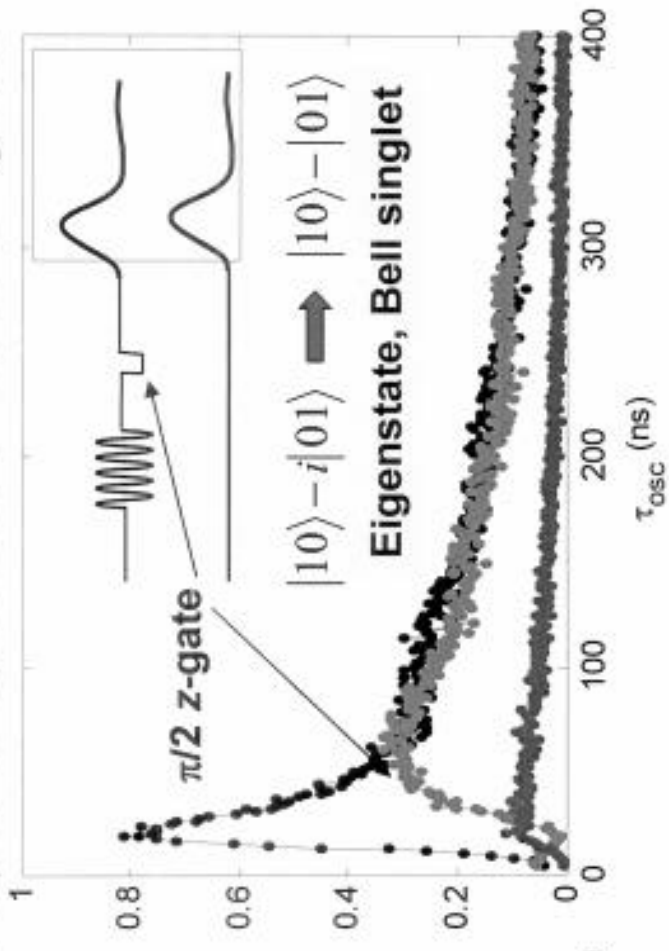
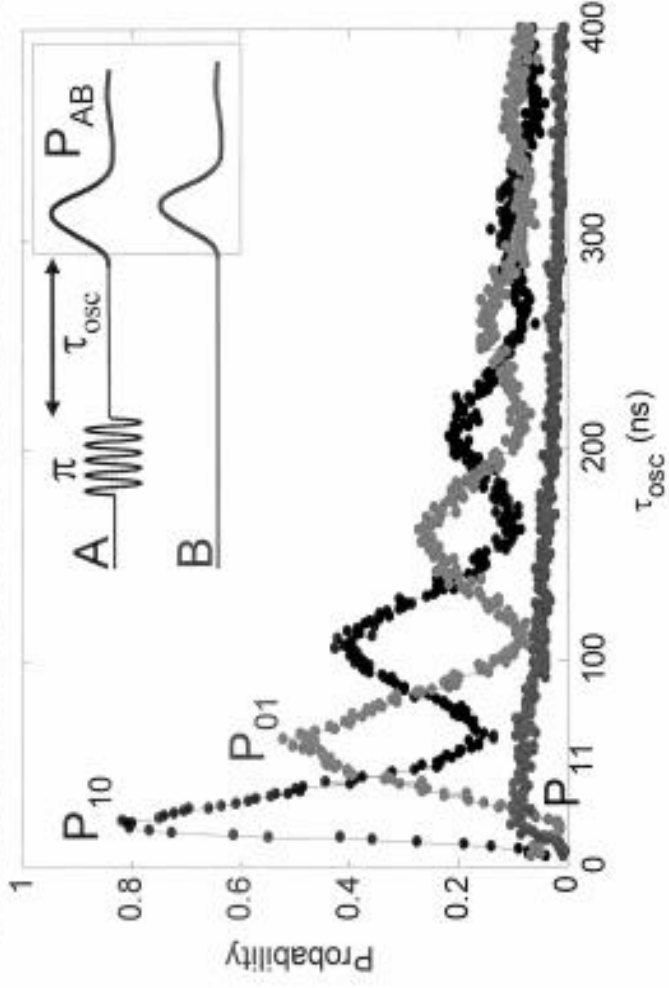
Resonant:

$$\frac{10}{00} \dots \frac{01}{11}$$

Moves with bias on B



Simultaneous Measure of Coupled Qubits: i-SWAP gate



$$\begin{aligned}
 |\Psi\rangle &= \frac{1}{2}(|10\rangle + |01\rangle) + \frac{1}{2}(|10\rangle - |01\rangle) e^{-iS\tau_{\text{osc}}} \\
 &= \frac{1}{2} \cos(S\tau_{\text{osc}}/2) - i|01\rangle \sin(S\tau_{\text{osc}}/2)
 \end{aligned}$$

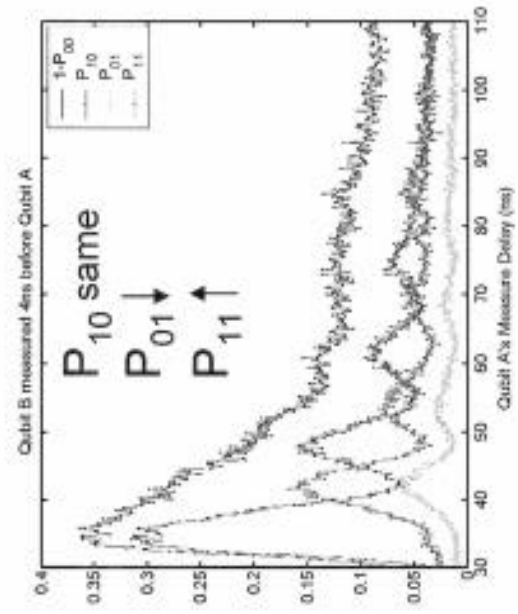
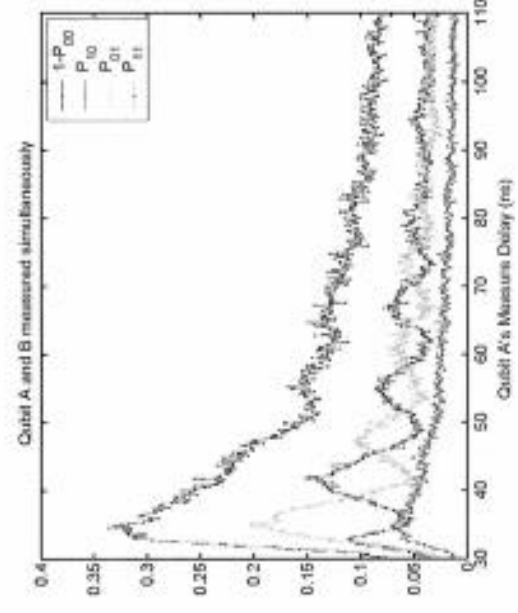
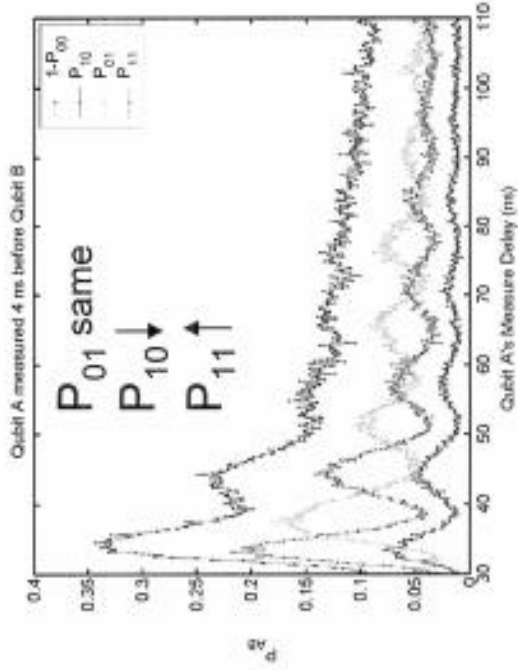
i-SWAP gate

Cross Coupling when Measurement is Delayed

Fixed Coupling:



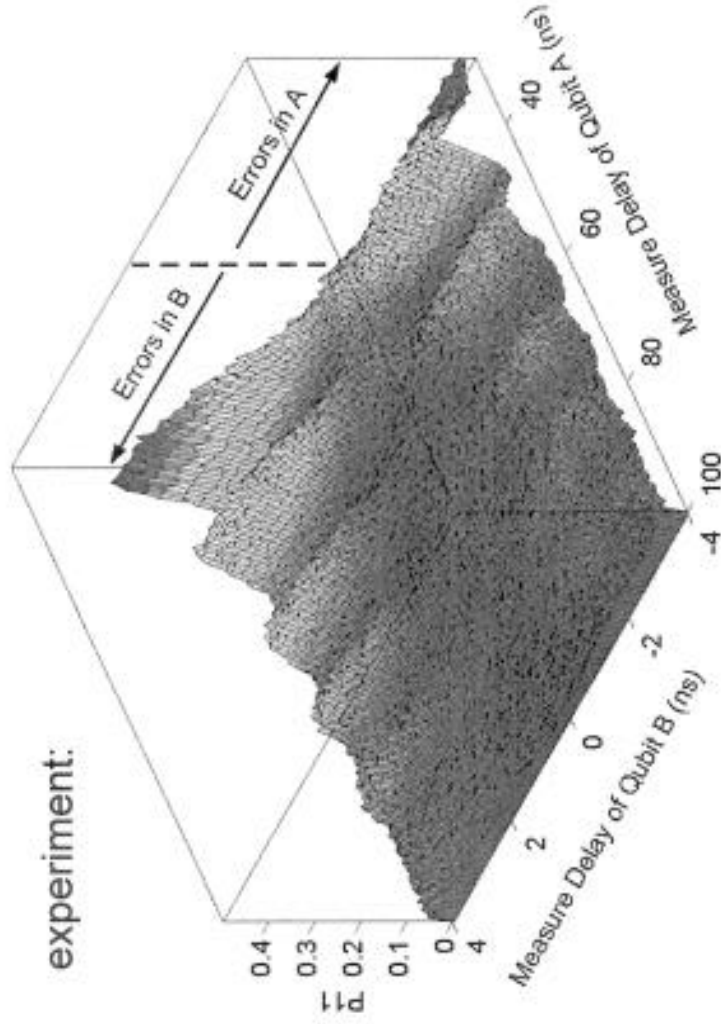
Qubit gate easy to make,
 During measurement coupling still on!
 Measurement of 1 state dissipates energy



When measure 1 state:
 pumps energy into 2nd qubit,
 producing 0 -> 1 transition

Time Scale of Measurement Crosstalk

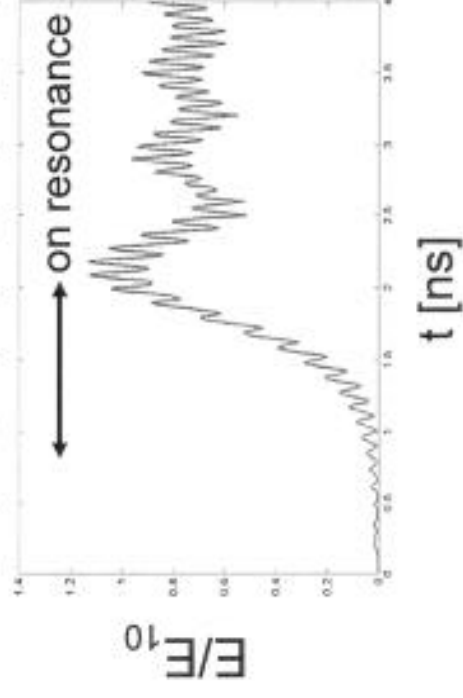
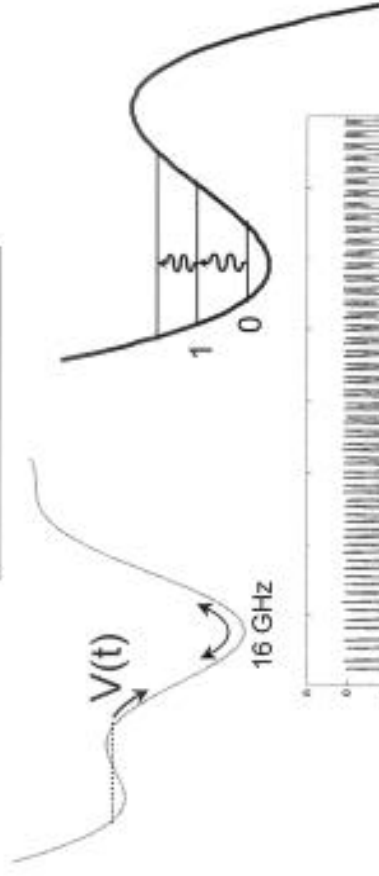
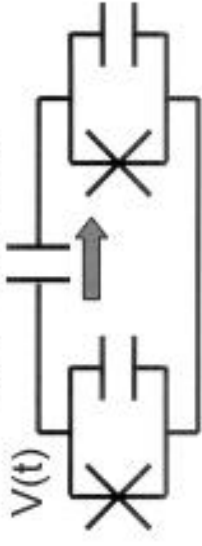
experiment:



Small crosstalk for misalignment <1 ns

theory:

$$I(t) = C_x \frac{dV}{dt}$$



Density Matrix

outer prod. of wavefun

$$\rho = \sum p_i |\psi_i\rangle\langle\psi_i|$$

\uparrow Classical prob sum over pure states

eg. $\psi = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$; coher. state

$$\rho_{\text{coh.}} = \frac{1}{2} (|01\rangle + |10\rangle) (\langle 01| + \langle 10|)$$

$$= \frac{1}{2} (|01\rangle\langle 01| + |10\rangle\langle 10|)$$

diag.

$$+ |01\rangle\langle 10| + |10\rangle\langle 01|$$

off diag.

eg. $\frac{1}{2}$ prob $|01\rangle$; $|10\rangle$

$$\rho_{\text{incoh.}} = \frac{1}{2} (|01\rangle\langle 01| + |10\rangle\langle 10|)$$

$$\rho = \begin{pmatrix} & \langle 00| & \langle 01| & \langle 10| & \langle 11| \\ \downarrow & & & & \\ & & \frac{1}{2} & & \\ & & & \frac{1}{2} & \\ & & & & & \\ & & & & & \end{pmatrix} \begin{matrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{matrix}$$

off diag (= diag)

\Rightarrow coh. state.

ρ is Hermitian

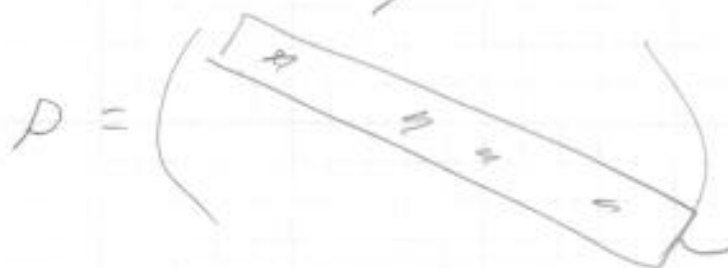
$$\rho = \begin{pmatrix} \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{pmatrix}$$

6 complex = 12 real.

3 real #s

15 real #s describe state!

Meas. of ρ



In z -basis,
Meas. These 3 \neq 's
($\Sigma = 1$; normalized).

How to meas. off diag \rightarrow Need to
rotate them in with X_{90}, Y_{90} pulses
(meas in X, Y basis) as with 1 qubit.

9 experiments $U_{\text{rotate}} = (I, X, \text{or } Y)^{(1)} \otimes (I, X, \text{or } Y)^{(2)}$

$\rho' = U_{\text{rot}} \rho U_{\text{rot}}^{\dagger}$; then meas. diagonal elements.

Issue: It would be nice if could rotate only
a off diag into diag; but get linear sum
of off diag. So have to do Math to invert.

9 experiments; 3 diag elements \Rightarrow 27 linear equations
of all 15 ρ elements.

Overconstrained (good!).

Use least squares fit;

In MATLAB, 1 line of code!

1 qubit: 3 expts \rightarrow 3 ρ elements

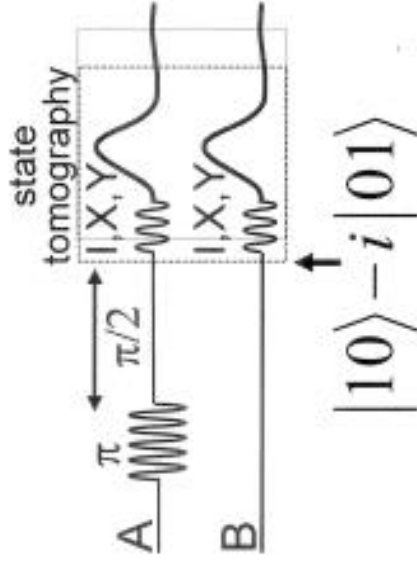
2 qubits: 9 expts \rightarrow 15 ρ elements; overconstrained

\vdots

8 qubits: Reduce # of experiments,
Matrix inversion is hardest (Innsbruck)

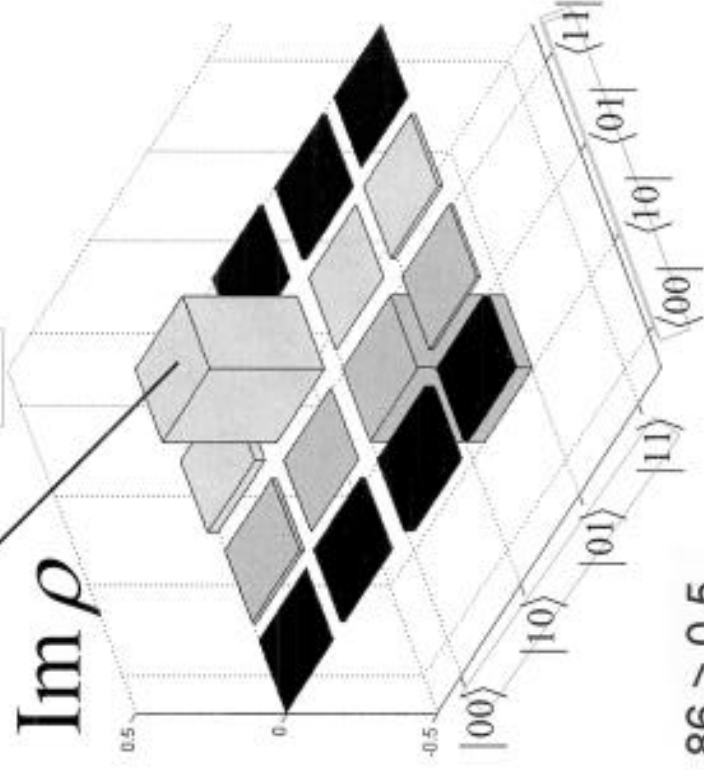
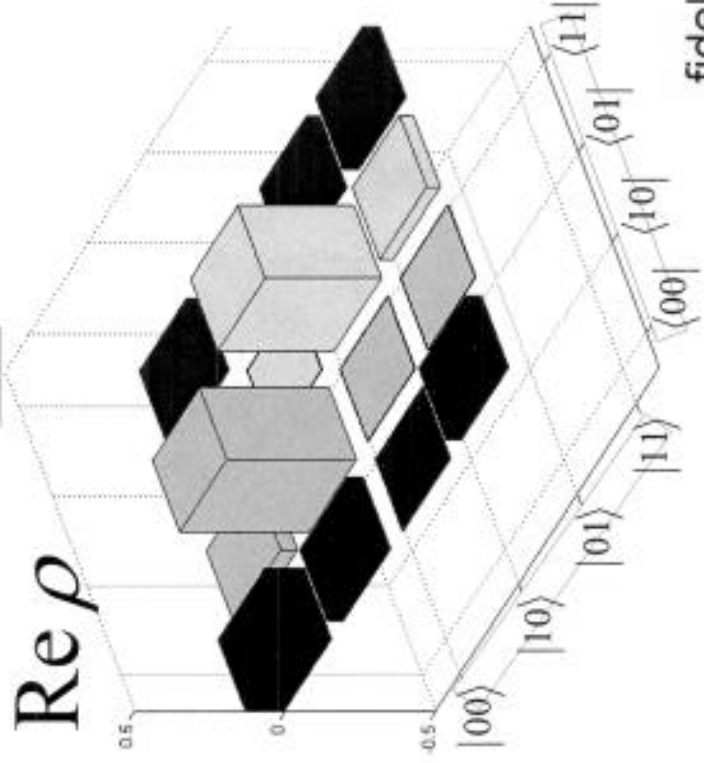
$$U_I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad U_X = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}; \quad U_Y = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

Tomography: Direct Proof of Entanglement



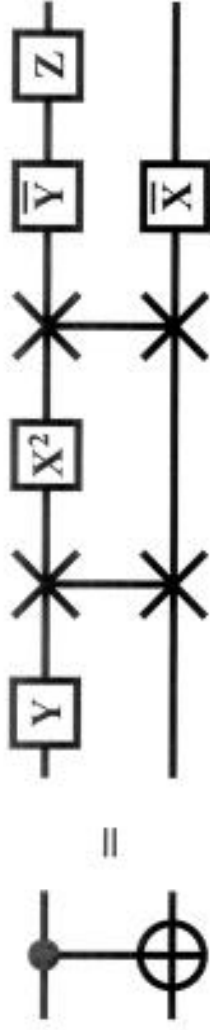
$$\rho_{\text{incoh}} = (1/2)(|10\rangle\langle 10| + |01\rangle\langle 01|)$$

$$\begin{aligned} \rho_{\text{coh}} &\equiv (|10\rangle - i|01\rangle)(\langle 10| + i\langle 01|) \\ &= (1/2)(|10\rangle\langle 10| + |01\rangle\langle 01| \\ &\quad + i|10\rangle\langle 01| - i|01\rangle\langle 10|) \end{aligned}$$



fidelity = 0.86 > 0.5
expect = 0.87

CNOT Gate

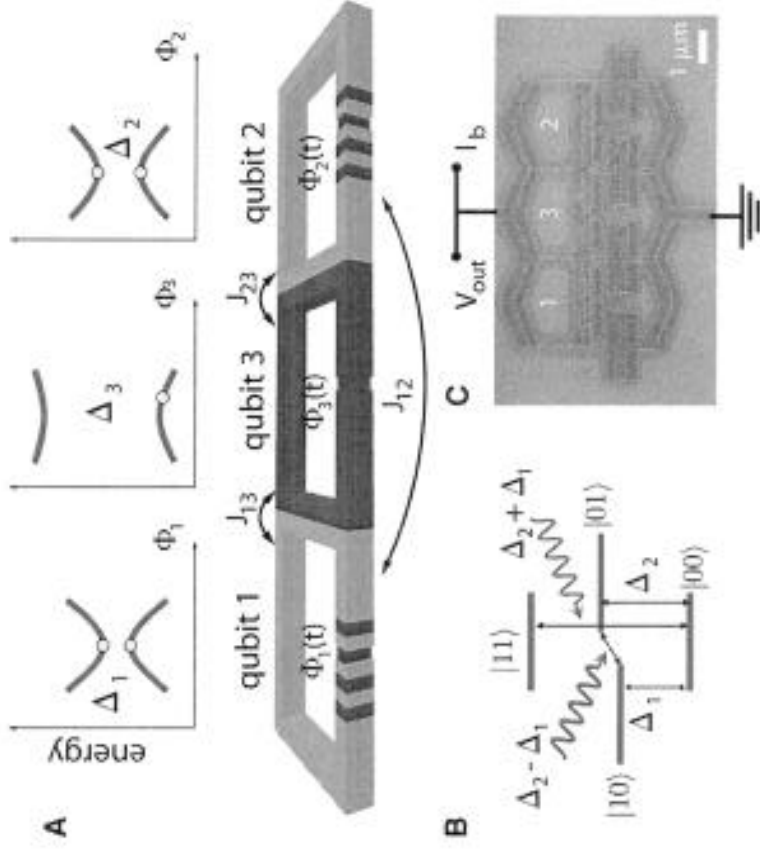


\sqrt{i} -SWAP is a universal gate

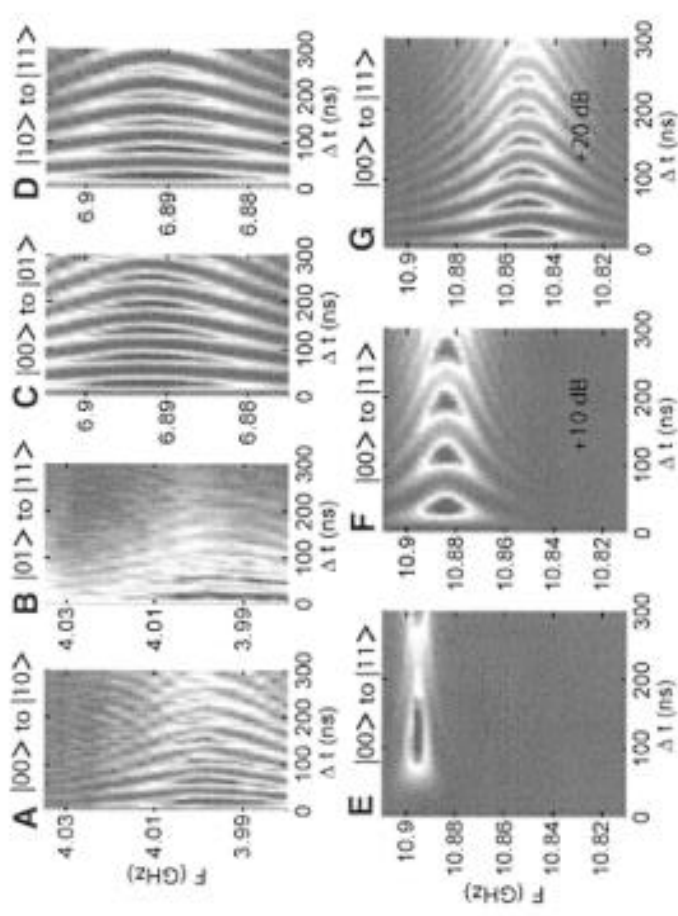
(General qubit coupling uses this sequence !)

Degeneracy Point: Parametric Coupling (NEC)

Coupling more difficult at degeneracy point -
Must modulate M with time to couple



Common μwave drive



On/off ratio = 20
20 ns gate time
Visibility ~ 40%