Quantum Logic Gates in Superconducting Qubits

John M. Martinis
Department of Physics
University of California, Santa Barbara 93111
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INTRODUCTION

Successful operation of a quantum computer will require unprecedented control of quantum systems. The basic qubit operations, quantum logic gates, are described by the linear Schrodinger equation: the "analog" nature of quantum state evolution makes these logic gates fundamentally sensitive to imperfections in control and loss of energy. In contrast, conventional digital logic can correct errors due to built-in gain and non-linearity. In a quantum computer, these imperfections fortunately can be removed with error-correction protocols, which work as long as the probability for the production of errors is small enough.

The performance specifications for error correction depend on details of the quantum computer architecture. Rough estimates for conventional gate-based architectures give limits below $\sim 10^{-4}$ [1], whereas more recent proposals based on surface codes may allow errors in the 10^{-2} range [2].

Much research in superconducting qubits has been directed towards improving the coherence of qubits and demonstrating quantum logic gates, both for single and coupled qubits. I am optimistic that quantum gates can eventually meet performance requirements needed for error correction.

Here, I focus on several important issues concerning the high-level design of quantum logic gates. In particular, I will review the need to effectively turn on and off coupling interactions between qubits to produce scalable controlled-not (CNOT) gates. This is an important topic for superconducting qubits, since they typically use fixed coupling elements set by fabrication.

TRANSITION LOGIC GATES

To illustrate some design issues, I first discuss a simple example of logic gates defined by inducing transitions between quantum states that are selected by their transition frequency. Figure 1a shows the energy-level diagram for two uncoupled qubits with frequency ω_1 and ω_2. A single qubit gate is generated by applying a pulse of microwaves, at an excitation frequency ω_1 for changing the first qubit state, or frequency ω_2 for the second. As the two qubits are not coupled, the energy level diagram has equal transition frequencies for pairs of states, given by the dashed and dotted arrows.

Although single qubit gates are simple to generate when they are uncoupled, it is not possible to also have CNOT logic. As illustrated in Fig. 1b, the CNOT gate must swap the state amplitudes between $|10\rangle$ and $|11\rangle$ while other state amplitudes remain unchanged. A simple solution would be to produce a π-pulse transition between these two state, as shown in Fig. 1a. However, this transition frequency is degenerate with that for the states $|00\rangle \leftrightarrow |01\rangle$, and such action would produce a pair of transitions that are, of course, equivalent to a single qubit gate.

This problem can be overcome by adding coupling between the two qubits, such that the four transition frequencies are all different [3]. Now, an applied π-pulse at the $|10\rangle \leftrightarrow |11\rangle$ frequency gives a CNOT since all other transitions are off resonant. However, this solution adds significant complexity: now a *pair* of frequencies have to be applied to make the single qubit transitions, and their pulse strength has to be matched carefully in order to maintain symmetric transition strengths and produce

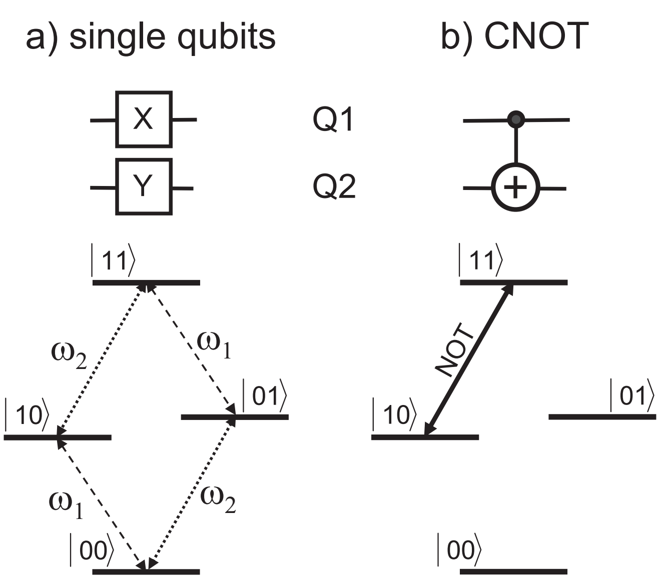

FIG. 1: Single and CNOT logic gates, in the transition picture, for two uncoupled qubits. Plotted is the qubit energy (vertical) for the four possible states. (a) Here, the transition frequency ω_1 for the first qubit (dashed lines) is the same for the pair of transitions $|00\rangle \leftrightarrow |10\rangle$ and $|01\rangle \leftrightarrow |11\rangle$. There is a similar pair (dotted lines) at frequency ω_2 for the second qubit. (b) The CNOT logic gate must swap only the states $|10\rangle$ and $|11\rangle$, which cannot be accomplished because of the degeneracy with the transition frequency $|00\rangle \leftrightarrow |01\rangle$.

what looks like a single qubit logic operation.

Although such transition logic is often discussed in the literature as "CNOT gates for quantum computation", in reality they probably cannot be used in a real quantum computer because they are not *scalable* gates. What happens to these logic gates as the number of qubits n increases? First, the number of transition frequencies that one must track grows as 2^n, which implies that simply calibrating the qubit system has exponential overhead. The exponentially large number of transition frequencies implies there can be significant problems with frequency overlap and crowding. The number of applied frequencies for single and CNOT gates grows as 2^{n-1} and 2^{n-2}, respectively, so that such gates become increasing more complex to generate and control accurately. Even at a modest size of $n = 8$, keeping track of 128 transition frequencies for a single qubit logic gate would certainly be taxing. The fundamental problem is that exponential growth of classical resources is needed to control logic built with transition frequencies, which makes the system unscalable for constructing a real quantum computer.

TUNABLE FREQUENCY LOGIC

A solution to this scaling problem is to turn the qubit coupling on and off. Although adjustable couplers have been demonstrated in superconducting qubits [4, 5], the circuitry is somewhat complex, and the more favored approach is to use fixed capacitive coupling and a tunable qubit frequency. Here, circuit design is simpler, and the interaction strength can be effectively turn up and down by tuning the qubits into and out of resonance. In Fig. 1a, "on coupling" would correspond to adjusting the qubit frequencies $\omega_1 = \omega_2$ so that the $|10\rangle$ and $|01\rangle$ states are resonant, with the resulting swapping [6] between the two gates combined with single qubit gates [7, 8] producing a CNOT.

The figures of merit for tunable logic is given through the fixed coupling energy g and the off detuning energy Δ. When the qubits have zero detuning, the swapping frequency [6] is given by $2g/\hbar$, whereas when off resonance the effective interaction energy between the qubits is given by the dispersive interaction [9], which scales as g^2/Δ. In this simple picture, the off to on strength is the ratio of these energies $(g^2/\Delta)/g = g/\Delta$. Note here that the effective coupling is never entirely turned off; with typical values for the phase qubit given by $g/h = 30\,\mathrm{MHz}$ and $\Delta/h = 300\,\mathrm{MHz}$, the off-coupling detuning is not large enough to neglect.

There are presently two general ways to design such gates. At UCSB, we use direct swapping transitions between the $|10\rangle$ and $|01\rangle$ states, as well as between the $|11\rangle$ and $|20\rangle$ states for controlled-phase and CNOT [10]. The Yale group uses a dispersive interaction between these states [11], which requires about 5 times larger g than for coupling interaction so that the gates can remain adiabatic in similar gate times; the off coupling is similar since they use greater detuning.

Logical errors from finite dispersion g^2/Δ can be understood in a simple model. Defining the transition energy between the ground and excited state of qubit 1 as E_1, this energy depends on whether qubit 2 is in the state $\Psi_2 = |0\rangle$ or $|1\rangle$

$$E_1 = \begin{cases} E_c & \text{for } \Psi_2 = |0\rangle \\ E_c + g^2/\Delta & \text{for } \Psi_2 = |1\rangle \end{cases} , \quad (1)$$

where E_c is a constant. The dispersion energy of qubit 1 depends on the state of the coupled qubit 2. Defining a rotating frame for qubit 1 at frequency E_1/h, the time dependence of an equal superposition of ground and excited states is

$$\Psi_1(t) = \frac{1}{\sqrt{2}} \times \begin{cases} |0\rangle + |1\rangle & \text{for } \Psi_2 = |0\rangle \\ |0\rangle + e^{-itg^2/\hbar\Delta}|1\rangle & \text{for } \Psi_2 = |1\rangle \end{cases} , \quad (2)$$

which shows a dependence on the state of qubit 2. For uncoupled qubits ($\Delta \to \infty$), the state of qubit 1 does not depend at all on the state of qubit 2: for finite Δ, the phase change in the second equation can be understood as a phase error from non-zero coupling. Note that this phase error is zero at time $t = 0$, increases quadratically with time, and then has a maximum magnitude of unity at $tg^2/\hbar\Delta = \pi$.

Although small at first, the potentially large magnitude from this phase error implies that small dispersive coupling cannot be ignored. It can be neglected for present experiments because the simple algorithms do not need to store data for long times, but as algorithm complexity grows, this error growing as t^2 will be increasingly important.

REFOCUSSING

The phase error of Eq. (2) may be reduced through refocussing techniques [1], as developed for nuclear magnetic resonance (NMR). The basic idea is to periodically change the state of a qubit with a π-pulse so that the phase accumulation is effectively balanced out between the $|0\rangle$ and $|1\rangle$ states. For best cancelation, the phase reversal should occur halfway during the qubit storage time.

Although simple for two qubits, refocussing becomes increasing complex as the number of qubits increase, since refocussing should optimally be placed between every pair of qubits that are coupled together. As the number of pairs scales as $n!$, significant control overhead is expected for large n. For example, in the experiment to factor 15 in NMR, most of the time of the algorithm

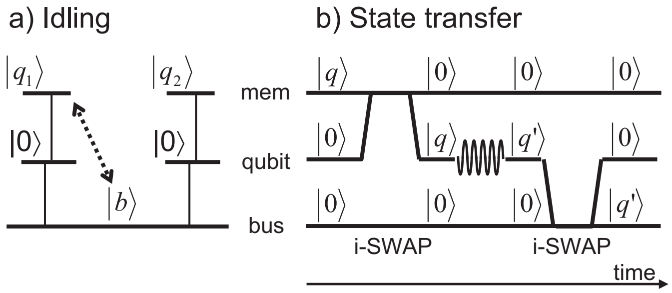

FIG. 2: Quantum Von Neumann architecture and RezQu protocol. (a) Plot of qubit and resonator frequency (vertical) for two qubits and their memory resonator, along with a bus resonator. Qubits and resonators are coupled (vertical thin lines), and qubit frequency may move up and down to the bus and memory frequencies. Only small (4th-order) residual coupling remains between the memory $|q_1\rangle$ and bus $|b\rangle$ states (dashed line). (b) Plot of qubit frequency versus time showing memory to qubit transfer, single qubit logic via a microwave pulse, and qubit to bus transfer.

was spent turning off the coupling between spins via refocussing [12]. It will be interesting to see if this technique will be a scalable solution for a large number of qubits.

QUANTUM VON NEUMANN ARCITECTURE AND REZQU PROTOCOL

Another solution to the off-coupling problem uses what we call the quantum von Neumann architecture [13], as illustrated in Fig. 2. Here each qubit is coupled to a memory resonator as well as the resonator bus, the latter providing inter-qubit coupling. The qubit and resonator bus has residual coupling as described previously, so a state in the resonator bus will disperse the state of each qubit, causing phase errors.

This source of error is minimized in the von Neumann architecture by storing the qubit state in the memory resonator. In the RezQu (resonator zero-qubit) protocol, the qubit state is swapped into the memory resonator when it is not being manipulated by single or coupled qubit logic. When a quantum state is stored in memory, the qubit is in the ground state $|0\rangle$. Although there is a frequency shift in the memory qubit and the bus from the qubit ground state, no error is generated since the state is known and always produces a constant shift.

Phase errors remain between the memory and bus resonators, but they are quite small because they proceed through a 4th order process that produces a virtual state in the qubit by memory-qubit coupling, then a frequency shift in the bus through qubit-bus coupling. It is interesting to note that this coupling from memory to bus goes to zero when the qubit frequency is placed halfway between the memory and bus frequencies [14].

An additional advantage of this architecture is the use of resonators as qubit memories, since these elements presently have the longest coherence time. Additionally, resonators require no control signals, so they are inexpensive in terms of control lines and electronics. Using memory resonators with closely tuned frequencies might also minimize the difficulty of accurately tracking differential phases between the many qubits.

CONCLUSIONS

Now that long coherence times have been demonstrated in superconducting qubits, achieving high-fidelity gates is an important topic for future research. In this article, I have discussed two fundamental issues that confront the designer of quantum logic: scalability and qubit errors from residual off-coupling. As solutions are available, either with refocussing or using the RezQu architecture, much progress is expected in the next few years.

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