UCSB Astro Journal Club

John McCann

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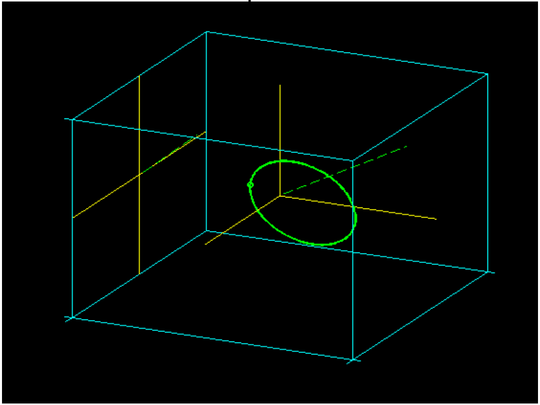

THE ROLE OF THE KOZAI-LIDOV MECHANISM IN BLACK HOLE BINARY MERGERS IN GALACTIC CENTERS

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ABSTRACT

In order to understand the rate of merger of stellar-mass black hole binaries (BHBs) by gravitational wave (GW) emission it is important to determine the major pathways to merger. We use numerical simulations to explore the evolution of BHBs inside the radius of influence of supermassive black holes (SMBHs) in galactic centers. In this region the evolution of binaries is dominated by perturbations from the central SMBH. In particular, as first pointed out by Antonini and Perets, the Kozai-Lidov (KL) mechanism trades relative inclination of the BHB to the SMBH for eccentricity of the BHB, and for some orientations can bring the BHB to an eccentricity near unity. At very high eccentricities, GW emission from the BHB can become efficient, causing the members of the BHB to coalesce. We use a novel combination of two N-body codes to follow this evolution. We are forced to simulate small systems to follow the behavior accurately. We have completed 400 simulations that range from ~ 300 stars around a 10^3 $M_\odot$ black hole to ~ 4500 stars around a 10^4 $M_\odot$ black hole. These simulations are the first to follow the internal orbit of a binary near a SMBH while also following the changes to its external orbit self-consistently. We find that this mechanism could produce mergers at a maximum rate per volume of ~ 100 Gpc^{-3} yr^{-1} or considerably less if the inclination oscillations of the binary remain constant as the BHB inclination to the SMBH changes, or if the binary black hole fraction is small.

Sections

I. Introduction

II. Timescales

  i. Mutual Inclination

III. Numerical Simulations

  i. Methodology

  ii. Codes

  iii. Initial Conditions

IV. Results

  i. Detection Rate

  ii. Eccentric Mergers

V. Other Considerations

VI. Conclusions

Objects of Interest

- Gravitational waves via stellar–mass black hole binaries (BHB) mergers has been confirmed; but the question of merger rates remain
- Population synthesis models have predicted volumetric rates of 0.1—300 $\mathrm{Gpc}^{-3}\ \mathrm{yr}^{-1}$.
- Recent aLIGO results constrain the models to volumetric rates of 2—400 $\mathrm{Gpc}^{-3}\ \mathrm{yr}^{-1}$.
- Our focus is BHB near supermassive black holes (SMBH), where secular effects can become significant

The final AU problem†

- Peters (1964) derived time for (our simplification equal mass) binaries to merge due to gravitational wave radiation

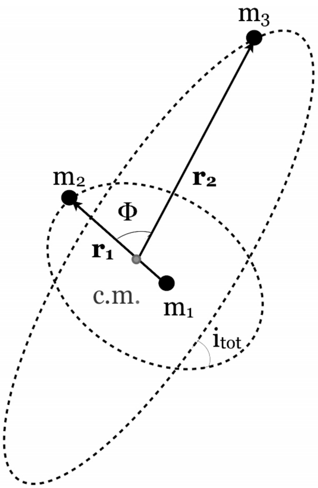
$$t_{gr} \approx 1.2 \times 10^{14} \left(\frac{20 M_\odot}{M_b}\right)^3 \left(\frac{a}{1\mathrm{AU}}\right)^4 (1 - e^2)^{7/2} \text{ yrs.} \qquad (1)$$

- M_b is the binary mass, a the semi–major axis, and e the eccentricity
- If this was the only mechanism and orbits were fairly circular, there would never have been a two 30 $M_\odot$ BHB merger detection
- Luckily there is a strong dependence on the eccentricity of the BHB
- Highly eccentric orbits, $e = 0.999$, could merge in a ~40,000 years

† Okay, a bit leading because we never said how to get to an AU and if that mechanism breaks down

Kozai–Lidov Mechanism (KL)

- Consider a triple system, from dynamical stability easiest when inner binary has far orbiting third body
- Coherent perturbations over timescales much longer than an individual orbit are called secular effects
- Importantly, the Kozai–Lidov cycling can play an important role in the evolution of triples
- This effect has been invoked in many planetary and black hole studies, originally used for asteroid perturbed by Jupiter studies



KL: The quick and dirty overview of the path

- Consider constructing the Hamiltonian for a hierarchical triple system in terms of some expansion coefficient $\alpha \equiv \frac{a_1}{a_2}$, for hierarchal triples $\alpha << 1$
- The answer

$$H = \frac{k^2 m_1 m_2}{2a_1} + \frac{k^2 m_3 (m_1 + m_2)}{2a_2} + \frac{k^2}{a_2} \sum_j \alpha^j M_j \left(\frac{r_1}{a_1}\right)^j \left(\frac{a_2}{r_2}\right)^{j+1} P_j\left(\cos(\Phi)\right)$$

- Obviously not a trivial answer to arrive at, Legendre polynomials, some weird mass factor†, but first two terms are intuitive.
- After a coordinate transformations and doing some Gauss orbiting averaging on this Hamiltonian, the Kozai–Lidov oscillation is massaged out

† $M_j = m_1 m_2 m_3 \frac{m_1^{j-1} - (-m_2)^{j-1}}{(m_1+m_2)^j}$

Kozai–Lidov Mechanism (KL)

- General idea: exchange inclination for eccentricity while conserving angular momentum
- Kozai found for an initial inclination separation, $i_{tot,0}$, the maximum eccentricity of the binary reached†

$$e_{1,max} \approx \sqrt{1 - \frac{5}{3}\cos^2(i_{tot,0})} \qquad (2)$$

- Higher order corrections and general relativistic effects included via N–body simulations

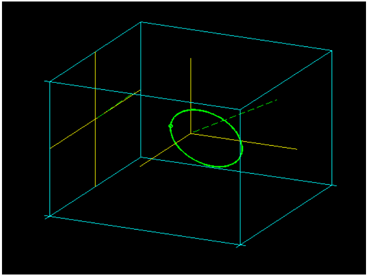


Figure: Credit CfA: Smadar Naoz

† For circular outer & initial inclination above the critical inclination, $i \gtrsim 39.2°$

Radius of Influence

- Three conditions need to be met for the BHB to undergo merging via KL oscillation
    i. Member of hierarchical triple system
    ii. Inclination above critical KL angle
    iii. Duration of high eccentricity long enough
- BHB are sufficiently influenced by SMBH within radius of influence, forming the triple†

$$R_{\rm infl} = \frac{GM_{\rm SMBH}}{\sigma^2} \approx \sqrt{\frac{M_{\rm SMBH}}{10^6\, M_\odot}}\ {\rm pc} \tag{3}$$

† $M_{\rm SMBH} \propto \sigma^{\alpha}$, where $\alpha = 4$.

TABLE 1

Timescale	Note
$T_{\rm KL} = 1.4 \times 10^{6}\,{\rm yr} \left(\frac{M_{\rm SMBH}}{10^{6}{\rm M}_{\odot}}\right)^{-1} \left(\frac{M_{\rm b}}{20{\rm M}_{\odot}}\right)^{1/2} \left(\frac{a_1}{1{\rm AU}}\right)^{-3/2} \left(\frac{a_2}{0.1{\rm pc}}\right)^{3} (1-e_2^2)^{3/2}$	1
$T_{\rm ER} = 4.4 \times 10^{7}\,{\rm yr} \left(\frac{M_{\rm SMBH}}{10^{6}{\rm M}_{\odot}}\right) \left(\frac{M_{\rm b}}{20{\rm M}_{\odot}}\right)^{-1} \left(\frac{a_2}{0.1{\rm pc}}\right)^{1/2}$	2
$T_{\rm RR} = 2.3 \times 10^{7}\,{\rm yr} \left(\frac{M_{\rm SMBH}}{10^{6}{\rm M}_{\odot}}\right)^{1/2} \left(\frac{M_{\rm b}}{20{\rm M}_{\odot}}\right)^{-1} \left(\frac{a_2}{0.1{\rm pc}}\right)^{3/2}$	3
$T_{\rm VRR} = 4.6 \times 10^{4}\,{\rm yr} \left(\frac{M_{\rm SMBH}}{10^{6}{\rm M}_{\odot}}\right)^{1/4} \left(\frac{M_{\rm b}}{20{\rm M}_{\odot}}\right)^{-1} \left(\frac{a_2}{0.1{\rm pc}}\right)$	4
$T_{\rm GR} = 6.0 \times 10^{4}\,{\rm yr} \left(\frac{M_{\rm b}}{20{\rm M}_{\odot}}\right)^{-3/2} \left(\frac{a_1}{1{\rm AU}}\right)^{5/2} (1-e_1^2)$	5

NOTE. —
1. Kozai-Lidov timescale (Innanen et al. 1997). The timescale over which Kozai-Lidov oscillations occur. Here $a_2 = 0.1R_{\rm infl}$ is the superorbit semi-major axis, e_2 is the superorbit eccentricity, and a_1 is the interior orbit semi-major axis.
2. Energy Relaxation timescale (Spitzer 1987). The timescale over which objects can significantly alter their orbital energy. This is the same process as dynamical friction. We use a Keplerian velocity dispersion since our region of interest is inside $R_{\rm infl}$. The M–σ relation and a number density of stars $n \propto r^{-\alpha}$ with $\alpha = 2$ sets the number of stars within the volume. We choose α primarily to speed up the simulations, though a value of $\alpha \approx 2$ is physically motivated (e.g., Genzel et al. 2003). The average stellar mass is set to 1 $M_{\odot}$.
3. Resonant Relaxation timescale (Rauch & Tremaine 1996). The timescale over which objects can significantly alter their angular momentum.
4. Vector Resonant Relaxation timescale (Hopman & Alexander 2006a). VRR is due to the averaged mass distribution over many orbits of an individual star, which exerts a torque that can alter the plane of the BHB orbit without affecting its energy or the magnitude of its angular momentum. Here once again the M–σ relation and stellar number density are used to set the number of stars in the volume.
5. General Relativistic Precession timescale. If this precession is too fast, it can suppress the KL cycles. Previous work (Blaes et al. 2002; Hollywood & Melia 1997) has found the precise condition at which this happens. In our simulations this condition is rarely met.

Change in i_{tot}

- Timescale for merging is dependent e, which KL evolution depends on i_{tot}
- The evolution of i_{tot} is therefore important, but still debated
- Author ran a modified 3 body simulation in attempts to elucidate matter†
- Points out orbits with $i_{tot} \approx 85°$ can evolve to $90°$, which rapidly merge

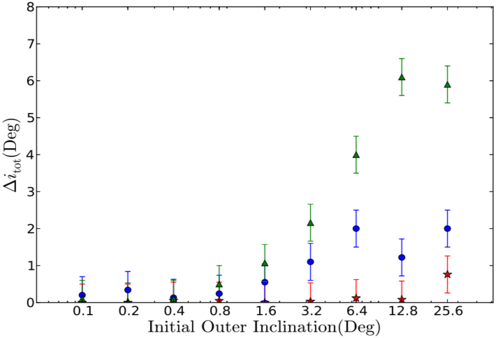


Fig. 2.— Change in the mutual inclination of an inner and outer binary in a hierarchical triple as a function of the initial inclination of the outermost object, which increases its inclination by a factor of $e \approx 2.718$ over one KL cycle. The initial mutual inclination, i_{tot}, is 55° for red stars, 70° for blue circles, and 85° for green triangles. Error bars represent the variability of the inclinations. All values of Δi_{tot} within 0.5° of 0° are consistent with no change. Although Δi_{tot} is much smaller than the change in the inclination of the outermost object, it is not negligible, especially for large initial i_{tot}.

† my feelings: unconvinced

Reasons for Numerics

- Goals: simulate BHB in star dense region near a SMBH, capturing physics
- Complications, timescale disparity between BHB internal orbit or close approaches and orbit around the SMBH and star field
- *NBODY6* used to simulate the large field of stars and black holes, limits SMBH size due to close encounters
- *HNBODY* used to simulate the BHB and SMBH, includes first order GR corrections to forces† and GR radiation as a drag force

† precessions

Procedure

1. Generate ICs for large scale N–body simulation
2. Simulate until BHB would be tidally disrupted
3. Generate ICs for small scale close approach N–body† simulation
4. Update center of mass of BHB to match large scale simulation
5. Simulate until BHB merges or end of large scale simulation‡

†3–body: BHB+SMBH

‡BHB was tidally disrupted

Intial Conditions *NBODY6*

- Number density of field stars $n \propto r^{-2}$
- d_{init} set by $t_{KL} = t_{relax}$, since further distance the BHB will not undergo appreciable KL oscillations
- $d_{max} \sim 4d_{init}$, deep embedding
- Eccentricities drawn from Maxwellian distribution
- All other orbital angles drawn from uniform distribution

Simulation	1k	2.5k	5k	10k
Mass of SMBH ($M_\odot$)	10^3	2.5×10^3	5×10^3	10^4
Number of Stars	307	680	2000	4400
$d_{\rm init}$ for BHB (AU)	500	700	1500	2500
$d_{\rm max}$ for Stars (AU)	2000	3000	6000	9000
Simulation Time (yrs)	4×10^4	6×10^4	1.12×10^5	3.5×10^5

Figure: Table of parameters used across simulations

Initial Conditions *HNBODY*

- BHB are circular 1 AU orbits
- Other orbital elements are similarly drawn from uniform distributions

Result 1, Figure 3

- Superorbit evolution over time
- Can see dynamical friction on BHB, but overplayed since field stars are uniformly $1 M_\odot$
- Eccentricity and inclination wander, but this is not rationalized by Kozai–Lidov oscillation[†]

[†] What is the third body perturber?

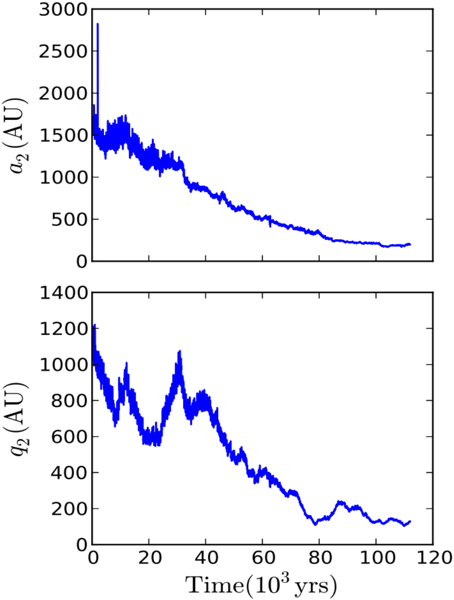


FIG. 3.— Example output of the COM orbit of the BHB from simulation 5k_3 around a $5 \times 10^3 M_\odot$ SMBH converted into Keplerian orbital elements. The superorbit semi-major axis (a_2), eccentricity (e_2), and pericenter (q_2) are plotted along with the inclination of the COM to a reference plane. The effects of the stellar potential on the orbit of the BHB are clearly visible. The eccentricity and inclination wander significantly over the simulation. The semi-major axis shrinks from 1500 AU to 200 AU due to mass segregation.

Result 2, Figures 4

- Shows a BHB merger at 8×10^4 yrs, when $e \to 0.999985$ and $t_{GW} = 100$ yrs
- Change in Δi_{tot} maps to i wandering of super orbit
- Clear indication of Kozai–Lidov oscillations in eccentricity and inclination

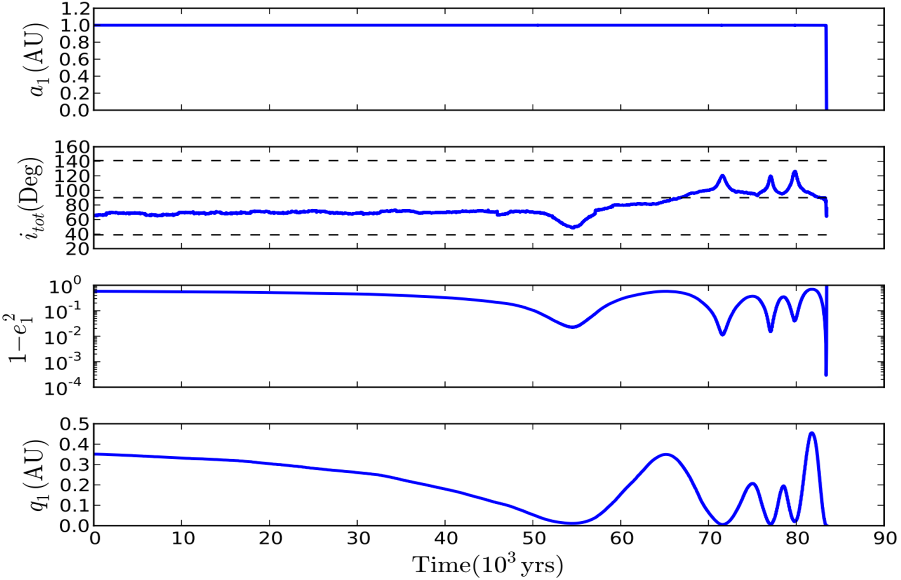


FIG. 4.— Example of the evolution of the internal orbital elements of a BHB from simulation 5k_3_2. The interior semi-major axis (a_1), eccentricity (e_1) and pericenter (q_1) are plotted along with the total inclination (i_{tot}). The eccentricity is plotted as $1-e_1^2$ to better show the different peaks as well as to relate to $T_{\rm GW}$ and $T_{\rm KL}$. The dashed lines in the plot of i_{tot} show the range of inclinations that lead to KL oscillations in the quadrupole limit, with the critical angle at 90°. KL oscillations are readily apparent in the eccentricity and total inclination. The BHB merges at the end of this simulation as GW emission becomes efficient when the eccentricity reaches a large enough value.

Result 3, Figures 5

- Much the same as the last results, but instead the BHB is ionized (~ 10 AU from SMBH)
- Shows a BHB ionized at 16×10^4 yrs, note KL inoperable until around 200 AU from SMBH
- All BHB reach either this or the previous fate, and simulations only terminate when either occurs.

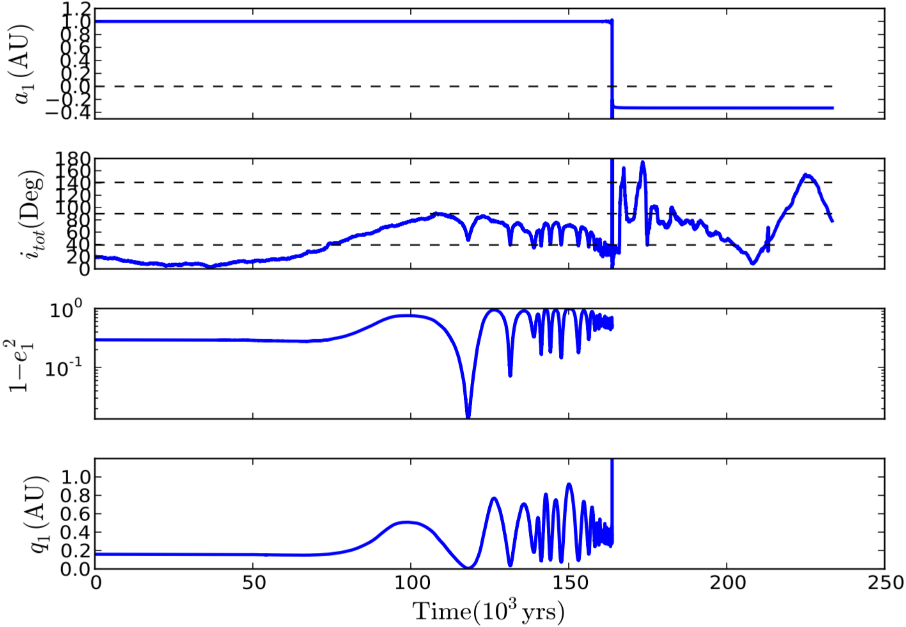


FIG. 5.— Example of the evolution of the internal elements of a BHB from simulation 10k_4_9. The dashed line in the plot of a_1 is the line at which a binary becomes ionized. This BHB is tidally separated before it can merge due to the KL mechanism. At around 1.6×10^5 yrs the semi-major axis becomes negative and the eccentricity larger than unity indicating ionization.

Result 4 & 5, Figure 6 & 7

- Greyed background lines are super orbit semi–major axis
- Markers demark where the merger occurred in the super orbit
- Notice that most occur close to the SMBH
- This is easier to see given the mass scaling of the radius of influence, in figure 7
- Timing of mergers appears to be focused around a certain time, not too early, not too late†
- Many explanations to alter the timing and refilling of BHB under the SMBH influence

†Pretty much when you fall into the right radius

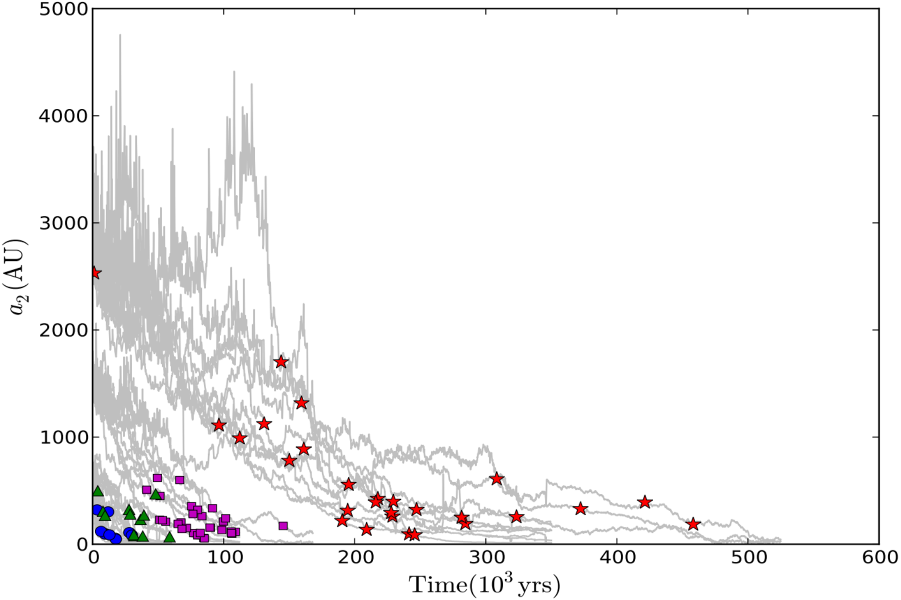


FIG. 6.— BHB mergers plotted as a function of the superorbit semi-major axis (a_2) and time at which they merged. Blue circles are mergers from 1k simulations, green triangles from 2.5k simulations, purple squares from 5k simulations, and red stars from 10k simulations. These are plotted on top of the tracks of the evolution of a_2 over time. Many of the mergers occur relatively close to the SMBH. There is in general an absence of mergers at early times.

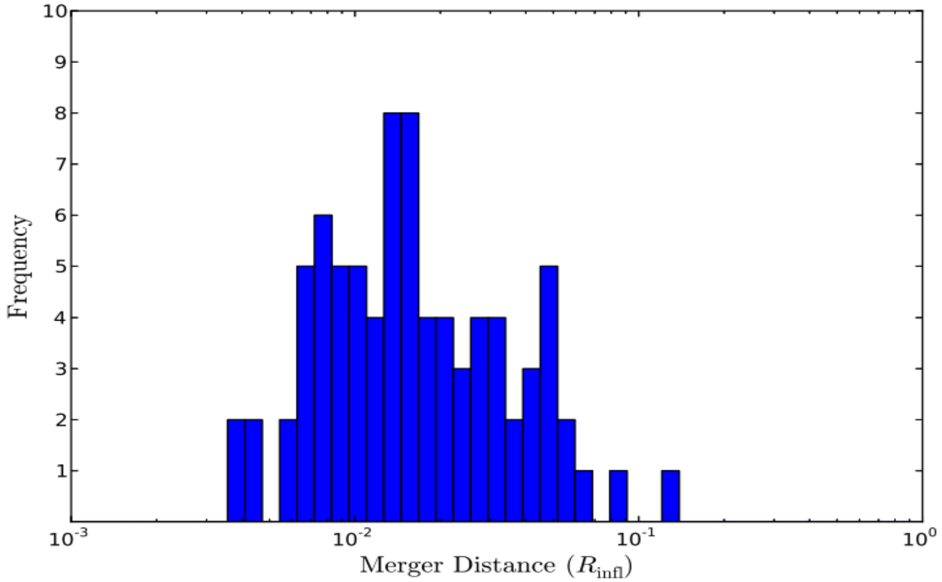


FIG. 7.— Histogram of the frequency of BHB merger at various distances from the SMBH relative to $R_{\rm infl}$. The most common merger distance is $\sim 0.01 R_{\rm infl}$.

Result 6, Figures 8

- Shows the cumulative merger fraction as a function of the simulations relaxation time
- Clearly 1k and 2.5k have lower merger fraction
- Author says due to time scales, t_{KL} increases with smaller SMBH mass while t_{ER} decreases
- Claims similarity of 5k and 10k captures the true result

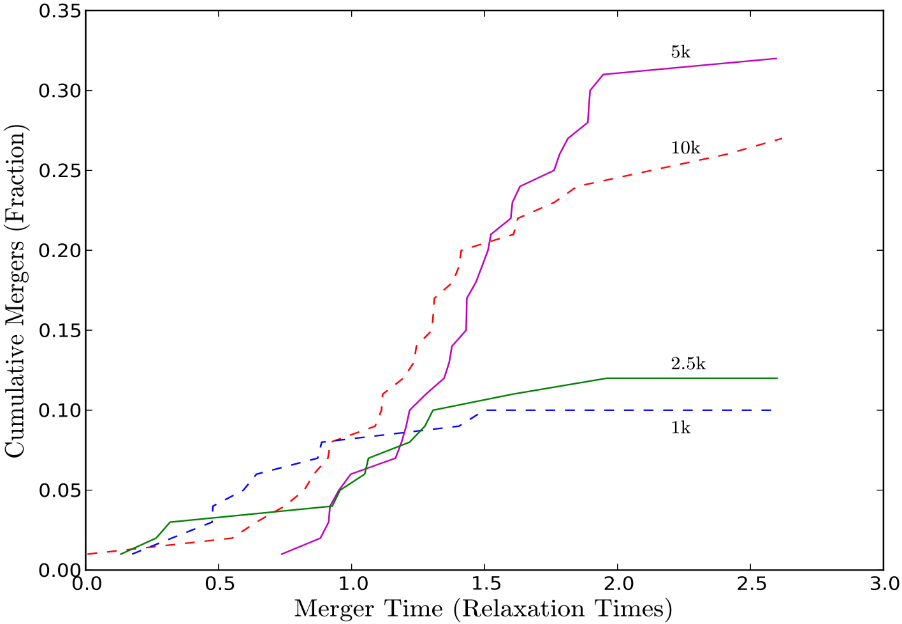


FIG. 8.— Fraction of simulations of each of the four classes that have merged over time. The time is scaled to the relaxation time for each simulation class (See Table 2). The simulation classes are marked on the plot. A significant fraction of the simulations of all classes merge over several relaxation times.

Detection Rates

- The author makes a rough order of magnitude estimation of what the detection rate would be from our own Sgr A*
- Mergers per year

$$\frac{\Delta N_{\rm merge}}{\Delta t} = \frac{f_{merge} N_{star} f_{BH} f_{bin}}{t_{relax}} \approx 2\ \mathrm{Myr}^{-1} \tag{4}$$

- Where f_{merge} is their model merger fraction, and $N_{star} f_{BH} f_{bin}$ is the total number of binary black holes, estimated from number of observable stars
- Assuming the Milky Way Equivalent Galaxy density is $\sim \frac{1}{100}$ Mpc^{-3}, then we have $N \sim 200$ Gpc^{-3} yr^{-1}
- This is probably an upper limit according to the author, but is also only one merge pathway

Eccentric Mergers

- GW signals from eccentric mergers[†] will have different signals than circular coalescences
- By checking eccentricity when BHB a 10 Hz frequency (aLIGO's sensitivity range) evaluate *HNBODY* when $a_1 = 10^{-6}$ AU
- Only found one simulation where this was satisfied, out of 400
- Thus unlikely but possible mechanism

[†] Define eccentric as $e \gtrsim .1$

Caveats

- \- Probably over estimated the change in mutual inclination since t_{KL} ¡ t_{VRR}
- $+$ SMBH are orders small than realistically, t_{KL} decreases while t_{VRR} and t_{ER} increase
- \- R_{ion} increases proportionally to SMBH mass while R_{info} only goes as root
- ? Small number of stars missing important stochastic processes
- \- Better IMFs for mass distribution, mutli–phase power law for number density, better distribution for BHB semi–major axis
- ? Replenishment of BHB to galatic nuclei