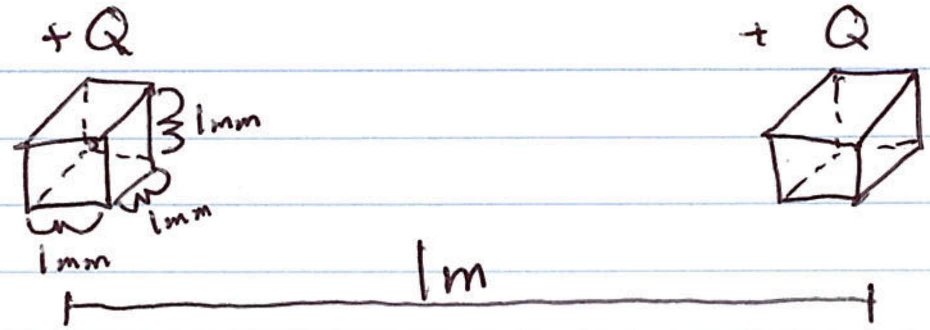
1.34)



Volume of 1 cube is: $V = s^3 = (.001\,m)^3$

~~Weight~~ Mass of 1 cube is: $M = VD = (.001m)^3(19300\ kg/m^3)$

Number of moles in 1 cube is: $\#\,mol = \frac{M}{m.w.} = \frac{VD}{m.w.} = \frac{s^3 D}{m.w.}$

$$\#\,mol = \frac{(.001\,m)^3(19300\ kg/m^3)}{(.197\ kg/mol)}$$

Number of electrons removed from 1 cube is:

$\#\,electrons = (\#\,mol)\cdot N_A = \frac{s^3 D N_A}{m.w.}$

Using conservation of charge & assuming gold is charge neutral, i.e., $Q=0$; then 1 cube has a charge:

$$Q = |e| \cdot \#\text{ of electrons}$$
$$= \frac{|e|\, s^3 D N_A}{m.w.}$$

Finally, force from Coulomb's Law: $F = \frac{kQQ}{r^2}$

$$F_c = \frac{k e^2 s^6 D^2 N_A^2}{(m.w.)^2 r^2}$$

Number of aircraft carriers that could be suspended:

$$\#_{ac} = \frac{F_c}{mg} = \frac{k e^2 s^6 D^2 N_A^2}{m g r^2 (m.w.)^2}$$

Note: Make sure you use same units, e.g., S.I. [m, kg, s].

Ans:

1.35)

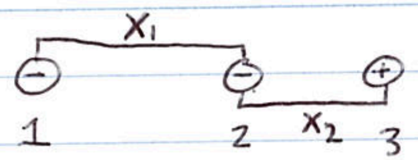
Balance $F_c = mg$

$$F_c = \frac{ke^2}{r^2} = m_e g \quad \Big| \text{ Solve for } r$$

$$\Rightarrow \quad r = \sqrt{\frac{ke^2}{m_e g}}$$

Ans:

1.40)



Potential from a point charge

$$U_{12} = \frac{kq_1q_2}{r_{12}}$$

Set total potential to zero from all three interactions (12, 13, 23)

$$U_{12} + U_{13} + U_{23} = 0$$

$$U_{12} = \frac{k(-e)(-e)}{x_1} \quad , \quad U_{13} = \frac{k(-e)(e)}{x_1 + x_2}$$

$$U_{23} = \frac{k(-e)(e)}{x_2}$$

$$ke^2\left(\frac{1}{x_1} - \frac{1}{x_1+x_2} - \frac{1}{x_2}\right) = 0$$

Solve for x_1 in terms of x_2

$$\frac{1}{x_1} - \frac{1}{x_2} = \frac{1}{x_1+x_2}$$

$$\Rightarrow \frac{x_2 - x_1}{x_1x_2} = \frac{1}{x_1+x_2}$$

$$\Rightarrow \frac{x_1x_2}{x_2-x_1} = x_1 + x_2 \qquad \Rightarrow \quad x_1x_2 = -x_1^2 + x_2^2$$

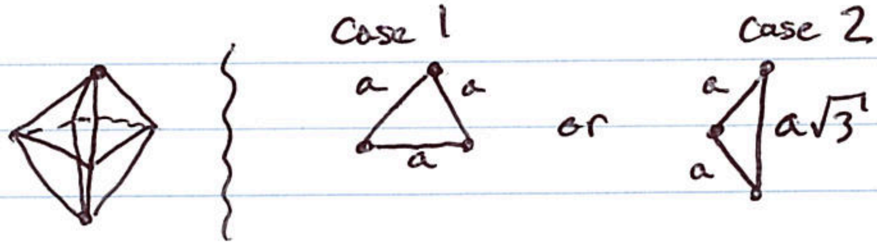
$$\Rightarrow x_1^2 + x_1x_2 - x_2^2 = 0$$

[Quadratic Equation $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$]

$$x_1 = x_2\,\frac{-1 \pm \sqrt{5}}{2}$$

Golden ratio: $\dfrac{1+\sqrt{5}}{2}$

1.41)

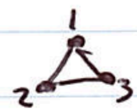


Case 1: i) Bring on in from ∞ — free!

ii) Bring second in from ∞

$$U_{12} = -\int_{\infty}^{a} \frac{kq^2}{r^2}\,dr = kq^2/a$$

iii) Bring in third charge from ∞

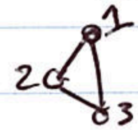


$$U_{13} = -\int_{\infty}^{a} \frac{kq^2}{r^2}\,dr = kq^2/a$$

$$U_{23} = -\int_{\infty}^{a} \frac{kq^2}{r^2}\,dr = kq^2/a$$

Total $U = U_{12} + U_{13} + U_{23} = \dfrac{3kq^2}{a}$

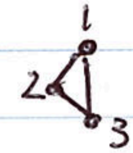
Case 2: i) Bring 1 in



— "free of charge"

ii) Bring 2 in

$$U_{12} = -\int_{\infty}^{a} \frac{kq^2}{r^2}\,dr = \frac{kq^2}{a}$$

iii) Bring charge 3 in from ∞ 

$$U_{13} = -\int_{\infty}^{\sqrt{3}a} \frac{kq^2}{r^2}\,dr = kq^2/\sqrt{3}a$$

$$U_{23} = -\int_{\infty}^{a} \frac{kq^2}{r^2}\,dr = kq^2/a$$

Total Potential Energy:

$$U_{TOT} = U_{12} + U_{23} + U_{13}$$
$$= kq^2\left(\frac{1}{a} + \frac{1}{\sqrt{3}a} + \frac{1}{a}\right)$$
$$= \frac{kq^2\,(2\sqrt{3}+1)}{\sqrt{3}\,a}$$