Volume of 1 cube is \( V = s^3 = (0.001\text{ m})^3 \)

Mass weight of 1 cube is \( M = \rho V = (0.001\text{ m})^3(19300\text{ kg/m}^3) \)

Number of moles in 1 cube is \( \# \text{ mol} = \frac{M}{m.\text{w.}} = \frac{\rho V}{m.\text{w.}} \cdot \frac{s^3 \rho}{m.\text{w.}} \)

\[ \# \text{ mol} = \frac{(0.001\text{ m})^3(19300\text{ kg/m}^3)}{0.197\text{ kg/mol}} \]

Number of electrons removed from 1 cube is \( \# \text{ electrons} = (# \text{ mol}) \cdot N_A = \frac{s^3 D \cdot N_A}{m.\text{w.}} \)

Using conservation of charge & assuming gold is charge neutral, i.e., \( Q = 0 \), then 1 cube has a charge \( Q \).

\[ Q = |e| \cdot \# \text{ of electrons} = \frac{|e| s^3 D N_A}{m.\text{w.}} \]

Finally, force from Coulomb's Law is \( F = \frac{kQQ}{r^2} \)
\[ F_c = \frac{Re^2 s^6 D^2 N_A}{(m \cdot w)^2 r^2} \]

Number of aircraft carriers that could be suspended:
\[ \#_{ac} = \frac{F_c}{mg} = \frac{Re^2 s^6 D^2 N_A}{mg r^2 (m \cdot w)^2} \]

Note: Make sure you use same units, e.g., SI: [m, kg, s].

\[ \text{Ans}^o \]
1.35

\begin{align*}
\text{Proton} & \quad \text{Balance} \quad F_c = mg \\
\uparrow F_c & \\
\text{electron} & \\
\downarrow mg & \\

F_c &= \frac{k e^2}{r^2} = m_e g \quad \mid \text{Solve for } r \\

\Rightarrow r &= \sqrt{\frac{k e^2}{m_e g}} \\

\text{Ans:} & \\
\end{align*}
Potential from a point charge

\[ U_{12} = \frac{k q_1 q_2}{r_{12}} \]

Set total potential to zero from all three interactions (12, 13, 23)

\[ U_{12} + U_{13} + U_{23} = 0 \]

\[ U_{12} = \frac{k (-e)(-e)}{x_1}, \quad U_{13} = \frac{k (-e)(e)}{x_1 + x_2} \]

\[ U_{23} = \frac{k (-e)(e)}{x_2} \]

\[ k e^2 \left( \frac{1}{x_1} - \frac{1}{x_1 + x_2} - \frac{1}{x_2} \right) = 0 \]

Solve for \( x_1 \) in terms of \( x_2 \)

\[ \frac{1}{x_1} - \frac{1}{x_2} = \frac{1}{x_1 + x_2} \]

\[ \Rightarrow \frac{x_2 - x_1}{x_1 x_2} = \frac{1}{x_1 + x_2} \]

\[ \Rightarrow \frac{x_1 x_2}{x_2 - x_1} = x_1 + x_2 \quad \Rightarrow \quad x_1 x_2 = -x_1^2 + x_2^2 \]

\[ \Rightarrow x_1^2 + x_1 x_2 - x_2^2 = 0 \quad \text{[Quadratic Equation]} \]

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ x_1 = x_2 \frac{-1 \pm \sqrt{5}}{2} \quad \{ \text{Golden ratio} \quad \frac{1 + \sqrt{5}}{2} \} \]
Case 1:

(i) Bring on in from \( \infty \) - free!

\[ U_{i} = - \int_{0}^{a} \frac{kq^2}{r^2} \, dr = \frac{kq^2}{a} \]

(ii) Bring second in from \( \infty \)

(iii) Bring in third charge from \( \infty \)

\[ U_{13} = - \int_{0}^{a} \frac{kq^2}{r^2} \, dr = \frac{kq^2}{a} \]
\[ U_{23} = - \int_{0}^{a} \frac{kq^2}{r^2} \, dr = \frac{kq^2}{a} \]

Total \( U = U_{12} + U_{13} + U_{23} = \frac{3kq^2}{a} \)

Case 2:

(i) Bring 1 in - free of charge

(ii) Bring 2 in

\[ U_{12} = - \int_{0}^{a} \frac{kq^2}{r^2} \, dr = \frac{kq^2}{a} \]
iii) Bring charge 3 in from \(2\) to \(a\)

\[
U_{13} = -\int_{2}^{a} \frac{kq^{2}}{r^{2}} \, dr = \frac{kq^{2}}{\sqrt{3}a}
\]

\[
U_{23} = -\int_{a}^{2} \frac{kq^{2}}{r^{2}} \, dr = \frac{kq^{2}}{a}
\]

**Total Potential Energy**

\[
U_{\text{Tot}} = U_{12} + U_{23} + U_{13}
\]

\[
= kq^{2} \left( \frac{1}{a} + \frac{1}{\sqrt{3}a} + \frac{1}{a} \right)
\]

\[
= \frac{kq^{2} \left( 2\sqrt{3} + 1 \right)}{\sqrt{3}a}
\]