Kepler's Laws

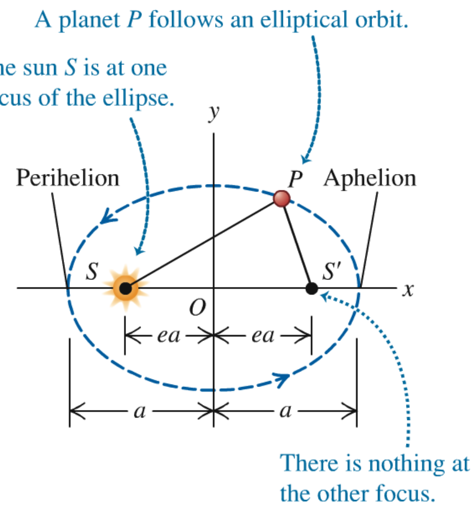
13.5 Kepler's Laws and the Motion of Planets

1. **Each planet moves in an elliptical orbit, with the sun at one focus of the ellipse.**
2. **A line from the sun to a given planet sweeps out equal areas in equal times.**
3. **The periods of the planets are proportional to the $\frac{3}{2}$ powers of the major axis lengths of their orbits.**

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} \qquad T = \frac{2\pi a^{3/2}}{\sqrt{Gm_S}}$$

13.27 • (a) Use Fig. 13.18 to show that the sun–planet distance at perihelion is $(1 - e)a$, the sun–planet distance at aphelion is $(1 + e)a$, and therefore the sum of these two distances is $2a$. (b) When the dwarf planet Pluto was at perihelion in 1989, it was almost 100 million km closer to the sun than Neptune. The semi-major axes of the orbits of Pluto and Neptune are 5.92×10^{12} m and 4.50×10^{12} m, respectively, and the eccentricities are 0.248 and 0.010. Find Pluto's closest distance and Neptune's farthest distance from the sun. (c) How many years after being at perihelion in 1989 will Pluto again be at perihelion?

13.18 Geometry of an ellipse. The sum of the distances SP and $S'P$ is the same for every point on the curve. The sizes of the sun (S) and planet (P) are exaggerated for clarity.

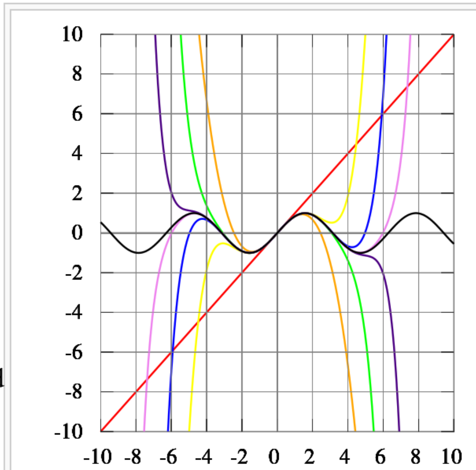


Math aside: Taylor Series

Taylor series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

where n! denotes the factorial of n and $f^{(n)}(a)$ denotes the nth derivative of f evaluated at the point a. The derivative of order zero of f is defined to be f itself and $(x - a)^0$ and 0! are both defined to be 1. When a = 0, the series is also called a Maclaurin series.

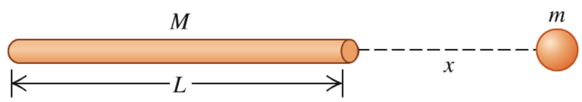


As the degree of the Taylor polynomial rises, it approaches the correct function. This image shows sin(*x*) and its Taylor approximations, polynomials of degree 1, 3, 5, 7, 9, 11 and 13.

Gravitational Potential for non-spherical mass distribution

13.32 • **CALC** A thin, uniform rod has length L and mass M. A small uniform sphere of mass m is placed a distance x from one end of the rod, along the axis of the rod (Fig. E13.32). (a) Calculate

Figure **E13.32**



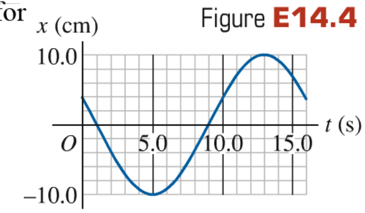
the gravitational potential energy of the rod–sphere system. Take the potential energy to be zero when the rod and sphere are infinitely far apart. Show that your answer reduces to the expected result when x is much larger than L. (*Hint:* Use the power series expansion for $\ln(1 + x)$ given in Appendix B.) (b) Use $F_x = -dU/dx$ to find the magnitude and direction of the gravitational force exerted on the sphere by the rod (see Section 7.4). Show that your answer reduces to the expected result when x is much larger than L.

SHO 101

14.4 • The displacement of an oscillating object as a function of time is shown in Fig. E14.4. What are (a) the frequency; (b) the amplitude; (c) the period; (d) the angular frequency of this motion?
(e) Write the equation for the displacement as a function of time.
Hint: $x = A\cos(\omega t + \phi)$ (displacement in SHM)

(f) Write out similar equations for veloicty and acceleration.

Figure **E14.4**



Physical Pendulum

14.56 •• **CP** A holiday ornament in the shape of a hollow sphere with mass $M = 0.015$ kg and radius $R = 0.050$ m is hung from a tree limb by a small loop of wire attached to the surface of the sphere. If the ornament is displaced a small distance and released, it swings back and forth as a physical pendulum with negligible friction. Calculate its period. (*Hint:* Use the parallel-axis theorem to find the moment of inertia of the sphere about the pivot at the tree limb.) $I_P = I_{cm} + Md^2$ (parallel-axis theorem)