Angular Momentum

10.42. **IDENTIFY and SET UP:** $\vec{L}$ is conserved if there is no net external torque.
Use conservation of angular momentum to find ω at the new radius and use $K = \frac{1}{2}I\omega^2$ to find the change in kinetic energy, which is equal to the work done on the block.

EXECUTE: **(a)** Yes, angular momentum is conserved. The moment arm for the tension in the cord is zero so this force exerts no torque and there is no net torque on the block.

(b) $L_1 = L_2$ so $I_1\omega_1 = I_2\omega_2$. Block treated as a point mass, so $I = mr^2$, where *r* is the distance of the block from the hole.

$$mr_1^2\omega_1 = mr_2^2\omega_2$$

$$\omega_2 = \left(\frac{r_1}{r_2}\right)^2 \omega_1 = \left(\frac{0.300\text{ m}}{0.150\text{ m}}\right)^2 (1.75\text{ rad/s}) = 7.00\text{ rad/s}$$

(c) $K_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}mr_1^2\omega_1^2 = \frac{1}{2}mv_1^2$

$$v_1 = r_1\omega_1 = (0.300\text{ m})(1.75\text{ rad/s}) = 0.525\text{ m/s}$$

$$K_1 = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}(0.0250\text{ kg})(0.525\text{ m/s})^2 = 0.00345\text{ J}$$

$$K_2 = \tfrac{1}{2}mv_2^2$$

$$v_2 = r_2\omega_2 = (0.150\text{ m})(7.00\text{ rad/s}) = 1.05\text{ m/s}$$

$$K_2 = \tfrac{1}{2}mv_2^2 = \tfrac{1}{2}(0.0250\text{ kg})(1.05\text{ m/s})^2 = 0.01378\text{ J}$$

$$\Delta K = K_2 - K_1 = 0.01378\text{ J} - 0.00345\text{ J} = 0.0103\text{ J}$$

(d) $W_{\text{tot}} = \Delta K$

But $W_{\text{tot}} = W$, the work done by the tension in the cord, so $W = 0.0103$ J.

EVALUATE: Smaller *r* means smaller *I*. $L = I\omega$ is constant so ω increases and *K* increases. The work done by the tension is positive since it is directed inward and the block moves inward, toward the hole.

10.52. **IDENTIFY:** The angular momentum of Sedna is conserved as it moves in its orbit.
SET UP: The angular momentum of Sedna is $L = mvl$.

EXECUTE: **(a)** $L = mvl$ so $v_1l_1 = v_2l_2$. When $v_1 = 4.64$ km/s, $l_1 = 76$ AU.

$$v_2 = v_1\left(\frac{l_1}{l_2}\right) = (4.64\text{ km/s})\left(\frac{76\text{ AU}}{942\text{ AU}}\right) = 0.374\text{ km/s.}$$

(b) Since vl is constant the maximum speed is at the minimum distance and the minimum speed is at the maximum distance.

(c) $$\frac{K_1}{K_2} = \frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \left(\frac{v_1}{v_2}\right)^2 = \left(\frac{l_2}{l_1}\right)^2 = \left(\frac{942\text{ AU}}{76\text{ AU}}\right)^2 = 154.$$

EVALUATE: Since the units of *l* cancel in the ratios there is no need to convert from AU to m. The gravity force of the sun does work on Sedna as it moves toward or away from the sun and this changes the kinetic energy during the orbit. But this force exerts no torque, so the angular momentum of Sedna is constant.

Equilibrium

11.19. **IDENTIFY:** Apply the first and second conditions of equilibrium to the rod.
SET UP: The force diagram for the rod is given in Figure 11.19.

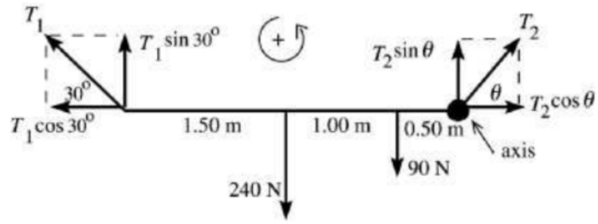


Figure 11.19

EXECUTE: $\sum \tau_z = 0$, axis at right end of rod, counterclockwise torque is positive

$(240 \text{ N})(1.50 \text{ m}) + (90 \text{ N})(0.50 \text{ m}) - (T_1 \sin 30.0°)(3.00 \text{ m}) = 0$

$T_1 = \dfrac{360 \text{ N}\cdot\text{m} + 45 \text{ N}\cdot\text{m}}{1.50 \text{ m}} = 270 \text{ N}$

$\sum F_x = ma_x$

$T_2 \cos\theta - T_1 \cos 30° = 0$ and $T_2 \cos\theta = 234 \text{ N}$

$\sum F_y = ma_y$

$T_1 \sin 30° + T_2 \sin\theta - 240 \text{ N} - 90 \text{ N} = 0$

$T_2 \sin\theta = 330 \text{ N} - (270 \text{ N})\sin 30° = 195 \text{ N}$

Then $\dfrac{T_2 \sin\theta}{T_2 \cos\theta} = \dfrac{195 \text{ N}}{234 \text{ N}}$ gives $\tan\theta = 0.8333$ and $\theta = 40°$

And $T_2 = \dfrac{195 \text{ N}}{\sin 40°} = 303 \text{ N}.$

EVALUATE: The monkey is closer to the right rope than to the left one, so the tension is larger in the right rope. The horizontal components of the tensions must be equal in magnitude and opposite in direction. Since $T_2 > T_1$, the rope on the right must be at a greater angle above the horizontal to have the same horizontal component as the tension in the other rope.

11.66. **IDENTIFY:** Apply $\sum \vec{F} = 0$ to each object, including the point where D, C and B are joined. Apply $\sum \tau_z = 0$ to the rod.

SET UP: To find T_C and T_D, use a coordinate system with axes parallel to the cords.

EXECUTE: A and B are straightforward, the tensions being the weights suspended:
$T_A = (0.0360 \text{ kg})(9.80 \text{ m/s}^2) = 0.353 \text{ N}$ and $T_B = (0.0240 \text{ kg} + 0.0360 \text{ kg})(9.80 \text{ m/s}^2) = 0.588 \text{ N}$. Applying $\sum F_x = 0$ and $\sum F_y = 0$ to the point where the cords are joined, $T_C = T_B \cos 36.9° = 0.470 \text{ N}$ and $T_D = T_B \cos 53.1° = 0.353 \text{ N}$. To find T_E, take torques about the point where string F is attached.

$T_E(1.00 \text{ m}) = T_D \sin 36.9°(0.800 \text{ m}) + T_C \sin 53.1°(0.200 \text{ m}) + (0.120 \text{ kg})(9.80 \text{ m/s}^2)(0.500 \text{ m})$ and $T_E = 0.833 \text{ N}$.

T_F may be found similarly, or from the fact that $T_E + T_F$ must be the total weight of the ornament. $(0.180\text{kg})(9.80\text{m/s}^2) = 1.76 \text{ N}$, from which $T_F = 0.931 \text{ N}$.

EVALUATE: The vertical line through the spheres is closer to F than to E, so we expect $T_F > T_E$, and this is indeed the case.

11.62. **IDENTIFY:** Apply the first and second conditions of equilibrium to the drawbridge.

SET UP: The free-body diagram for the drawbridge is given in Figure 11.62. H_v and H_h are the components of the force the hinge exerts on the bridge. In part (c), apply $\sum \tau_z = I\alpha$ to the rotating bridge and in part (d) apply energy conservation to the bridge.

EXECUTE: **(a)** $\sum \tau_z = 0$ with the axis at the hinge gives $-w(7.0\text{ m})(\cos 37°) + T(3.5\text{ m})(\sin 37°) = 0$ and

$$T = 2w\frac{\cos 37°}{\sin 37°} = 2\frac{(45{,}000\text{ N})}{\tan 37°} = 1.19\times 10^5\text{ N.}$$

(b) $\sum F_x = 0$ gives $H_h = T = 1.19\times 10^5$ N. $\sum F_y = 0$ gives $H_v = w = 4.50\times 10^4$ N.

$H = \sqrt{H_h^2 + H_v^2} = 1.27\times 10^5$ N. $\tan\theta = \dfrac{H_v}{H_h}$ and $\theta = 20.7°$. The hinge force has magnitude 1.27×10^5 N and is directed at 20.7° above the horizontal.

(c) We can treat the bridge as a uniform bar rotating around one end, so $I = 1/3\, mL^2$. $\sum \tau_z = I\alpha_z$ gives $mg(L/2)\cos 37° = 1/3\, mL^2\alpha$. Solving for α gives $\alpha = \dfrac{3g\cos 37°}{2L} = \dfrac{3(9.80\text{ m/s}^2)\cos 37°}{2(14.0\text{ m})} = 0.839\text{ rad/s}^2$.

(d) Energy conservation gives $U_1 = K_2$, giving $mgh = 1/2\, I\omega^2 = (1/2)(1/3\, mL^2)\omega^2$. Trigonometry gives $h = L/2\,\sin 37°$. Canceling m, the energy conservation equation gives $g(L/2)\sin 37° = (1/6)L^2\omega^2$. Solving for ω gives $\omega = \sqrt{\dfrac{3g\sin 37°}{L}} = \sqrt{\dfrac{3(9.80\text{ m/s}^2)\sin 37°}{14.0\text{ m}}} = 1.12\text{ rad/s.}$

EVALUATE: The hinge force is not directed along the bridge. If it were, it would have zero torque for an axis at the center of gravity of the bridge and for that axis the tension in the cable would produce a single, unbalanced torque.

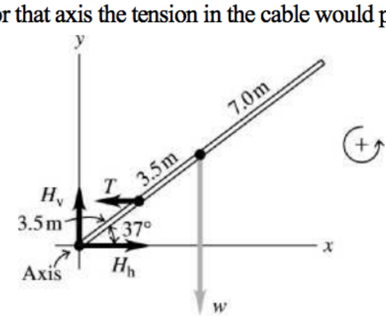
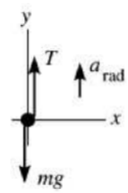


Figure 11.62

Young's Modulus

11.89. **IDENTIFY:** Apply Newton's second law to the mass to find the tension in the wire. Then apply Eq. (11.10) to the wire to find the elongation this tensile force produces.

(a) SET UP: Calculate the tension in the wire as the mass passes through the lowest point. The free-body diagram for the mass is given in Figure 11.89a.



The mass moves in an arc of a circle with radius $R = 0.50$ m. It has acceleration $\vec{a}_{rad}$ directed in toward the center of the circle, so at this point $\vec{a}_{rad}$ is upward.

Figure 11.89 a

EXECUTE: $\sum F_y = ma_y$

$T - mg = mR\omega^2$ so that $T = m(g + R\omega^2)$.

But ω must be in rad/s:

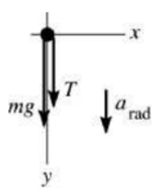
$\omega = (120 \text{ rev/min})(2\pi \text{ rad/1 rev})(1 \text{ min/60 s}) = 12.57$ rad/s.

Then $T = (12.0 \text{ kg})(9.80 \text{ m/s}^2 + (0.50 \text{ m})(12.57 \text{ rad/s})^2) = 1066$ N.

Now calculate the elongation Δl of the wire that this tensile force produces:

$$Y = \frac{F_\perp l_0}{A \Delta l} \text{ so } \Delta l = \frac{F_\perp l_0}{YA} = \frac{(1066 \text{ N})(0.50 \text{ m})}{(7.0 \times 10^{10} \text{ Pa})(0.014 \times 10^{-4} \text{ m}^2)} = 0.54 \text{ cm.}$$

(b) SET UP: The acceleration $\vec{a}_{rad}$ is directed in toward the center of the circular path, and at this point in the motion this direction is downward. The free-body diagram is given in Figure 11.89b.



EXECUTE:

$\sum F_y = ma_y$

$mg + T = mR\omega^2$

$T = m(R\omega^2 - g)$

Figure 11.89 b

$T = (12.0 \text{ kg})((0.50 \text{ m})(12.57 \text{ rad/s})^2 - 9.80 \text{ m/s}^2) = 830$ N

$$\Delta l = \frac{F_\perp l_0}{YA} = \frac{(830 \text{ N})(0.50 \text{ m})}{(7.0 \times 10^{10} \text{ Pa})(0.014 \times 10^{-4} \text{ m}^2)} = 0.42 \text{ cm.}$$

EVALUATE: At the lowest point T and w are in opposite directions and at the highest point they are in the same direction, so T is greater at the lowest point and the elongation is greatest there. The elongation is at most 1% of the length.

Fluid Mechanics

12.9. **IDENTIFY:** The gauge pressure $p - p_0$ at depth h is $p - p_0 = \rho g h$.

SET UP: Freshwater has density $1.00\times10^3\ \text{kg/m}^3$ and seawater has density $1.03\times10^3\ \text{kg/m}^3$.

EXECUTE: **(a)** $p - p_0 = (1.00\times10^3\ \text{kg/m}^3)(3.71\ \text{m/s}^2)(500\ \text{m}) = 1.86\times10^6\ \text{Pa}$.

(b) $h = \dfrac{p - p_0}{\rho g} = \dfrac{1.86\times10^6\ \text{Pa}}{(1.03\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 184\ \text{m}$

EVALUATE: The pressure at a given depth is greater on earth because a cylinder of water of that height weighs more on earth than on Mars.

12.37. **IDENTIFY** and **SET UP:** Apply Eq. (12.10). In part (a) the target variable is V. In part (b) solve for A and then from that get the radius of the pipe.

EXECUTE: **(a)** $vA = 1.20\ \text{m}^3/\text{s}$

$$v = \frac{1.20\ \text{m}^3/\text{s}}{A} = \frac{1.20\ \text{m}^3/\text{s}}{\pi r^2} = \frac{1.20\ \text{m}^3/\text{s}}{\pi(0.150\ \text{m})^2} = 17.0\ \text{m/s}$$

(b) $vA = 1.20\ \text{m}^3/\text{s}$

$v\pi r^2 = 1.20\ \text{m}^3/\text{s}$

$$r = \sqrt{\frac{1.20\ \text{m}^3/\text{s}}{v\pi}} = \sqrt{\frac{1.20\ \text{m}^3/\text{s}}{(3.80\ \text{m/s})\pi}} = 0.317\ \text{m}$$

EVALUATE: The speed is greater where the area and radius are smaller.