Mass Continuity & Pressure

12.36. **IDENTIFY:** $v_1A_1 = v_2A_2$. The volume flow rate is vA.

SET UP: 1.00 h = 3600 s.

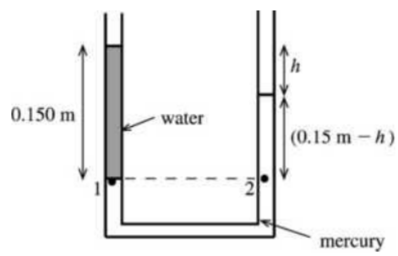
EXECUTE: **(a)** $v_2 = v_1\left(\frac{A_1}{A_2}\right) = (3.50\text{ m/s})\left(\frac{0.070\text{ m}^2}{0.105\text{ m}^2}\right) = 2.33\text{ m/s}$

(b) $v_2 = v_1\left(\frac{A_1}{A_2}\right) = (3.50\text{ m/s})\left(\frac{0.070\text{ m}^2}{0.047\text{ m}^2}\right) = 5.21\text{ m/s}$

(c) $V = v_1A_1t = (3.50\text{ m/s})(0.070\text{ m}^2)(3600\text{ s}) = 882\text{ m}^3$.

EVALUATE: The equation of continuity says the volume flow rate is the same at all points in the pipe.

12.59. **(a) IDENTIFY and SET UP:**



Apply $p = p_0 + \rho gh$ to the water in the left-hand arm of the tube. See Figure 12.59.

Figure 12.59

EXECUTE: $p_0 = p_a$, so the gauge pressure at the interface (point 1) is

$p - p_a = \rho gh = (1000\text{ kg/m}^3)(9.80\text{ m/s}^2)(0.150\text{ m}) = 1470\text{ Pa}$.

(b) IDENTIFY and SET UP: The pressure at point 1 equals the pressure at point 2. Apply Eq. (12.6) to the right-hand arm of the tube and solve for h.

EXECUTE: $p_1 = p_a + \rho_w g(0.150\text{ m})$ and $p_2 = p_a + \rho_{Hg} g(0.150\text{ m} - h)$

$p_1 = p_2$ implies $\rho_w g(0.150\text{ m}) = \rho_{Hg} g(0.150\text{ m} - h)$

$$0.150\text{ m} - h = \frac{\rho_w(0.150\text{ m})}{\rho_{Hg}} = \frac{(1000\text{ kg/m}^3)(0.150\text{ m})}{13.6\times10^3\text{ kg/m}^3} = 0.011\text{ m}$$

$h = 0.150\text{ m} - 0.011\text{ m} = 0.139\text{ m} = 13.9\text{ cm}$

EVALUATE: The height of mercury above the bottom level of the water is 1.1 cm. This height of mercury produces the same gauge pressure as a height of 15.0 cm of water.

Buoyancy

12.74. **IDENTIFY:** $B = \rho V_A g$. Apply Newton's second law to the beaker, liquid and block as a combined object and also to the block as a single object.

SET UP: Take $+y$ upward. Let F_D and F_E be the forces corresponding to the scale reading.

EXECUTE: Forces on the combined object: $F_D + F_E - (w_A + w_B + w_C) = 0$. $w_A = F_D + F_E - w_B - w_C$. D and E read mass rather than weight, so write the equation as $m_A = m_D + m_E - m_B - m_C$. $m_D = F_D/g$ is the reading in kg of scale D; a similar statement applies to m_E.

$m_A = 3.50\text{ kg} + 7.50\text{ kg} - 1.00\text{ kg} - 1.80\text{ kg} = 8.20\text{ kg}$.

Forces on A: $B + F_D - w_A = 0$. $\rho V_A g + F_D - m_A g = 0$. $\rho V_A + m_D = m_A$.

$$\rho = \frac{m_A - m_D}{V_A} = \frac{8.20\text{ kg} - 3.50\text{ kg}}{3.80\times10^{-3}\text{ m}^3} = 1.24\times10^3\text{ kg/m}^3$$

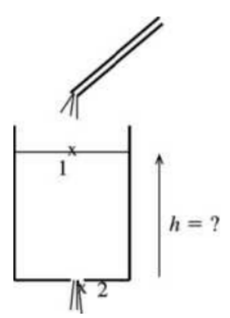
(b) D reads the mass of A: 8.20 kg. E reads the total mass of B and C: 2.80 kg.

EVALUATE: The sum of the readings of the two scales remains the same.

Bernoulli's Principle

12.90. **IDENTIFY:** Use Bernoulli's equation to find the velocity with which the water flows out the hole.

SET UP: The water level in the vessel will rise until the volume flow rate into the vessel, $2.40\times10^{-4}\text{ m}^3/\text{s}$, equals the volume flow rate out the hole in the bottom.



Let points 1 and 2 be chosen as in Figure 12.90.

Figure 12.90

EXECUTE: Bernoulli's equation: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

Volume flow rate out of hole equals volume flow rate from tube gives that $v_2 A_2 = 2.40\times10^{-4}\text{ m}^3/\text{s}$ and

$$v_2 = \frac{2.40\times10^{-4}\text{ m}^3/\text{s}}{1.50\times10^{-4}\text{ m}^2} = 1.60\text{ m/s}$$

$A_1 \gg A_2$ and $v_1 A_1 = v_2 A_2$ says that $\frac{1}{2}\rho v_1^2 \ll \frac{1}{2}\rho v_2^2$; neglect the $\frac{1}{2}\rho v_1^2$ term.

Measure y from the bottom of the bucket, so $y_2 = 0$ and $y_1 = h$.

$p_1 = p_2 = p_a$ (air pressure)

Then $p_a + \rho g h = p_a + \frac{1}{2}\rho v_2^2$ and $h = v_2^2/2g = (1.60\text{ m/s})^2/2(9.80\text{ m/s}^2) = 0.131\text{ m} = 13.1\text{ cm}$

EVALUATE: The greater the flow rate into the bucket, the larger v_2 will be at equilibrium and the higher the water will rise in the bucket.

Gravity

13.16. **IDENTIFY:** The gravity of Io limits the height to which volcanic material will rise. The acceleration due to gravity at the surface of Io depends on its mass and radius.

SET UP: The radius of Io is $R = 1.815\times10^6$ m. Use coordinates where $+y$ is upward. At the maximum height, $v_{0y} = 0$, $a_y = -g_{\text{Io}}$, which is assumed to be constant. Therefore the constant-acceleration kinematics formulas apply. The acceleration due to gravity at Io's surface is given by $g_{\text{Io}} = Gm/R^2$.

SOLVE: At the surface of Io, $g_{\text{Io}} = \dfrac{Gm}{R^2} = \dfrac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(8.94\times10^{22}\ \text{kg})}{(1.815\times10^6\ \text{m})^2} = 1.81\ \text{m/s}^2$. For constant acceleration (assumed), the equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ applies, so

$v_{0y} = \sqrt{-2a_y(y-y_0)} = \sqrt{-2(-1.81\ \text{m/s}^2)(5.00\times10^5\ \text{m})} = 1.345\times10^3$ m/s. Now solve for $y - y_0$ when $v_{0y} = 1.345\times10^3$ m/s and $a_y = -9.80\ \text{m/s}^2$. The equation $v_y^2 = v_{0y}^2 + 2a_y(y-y_0)$ gives

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{-(1.345\times10^3\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 92\ \text{km}.$$

EVALUATE: Even though the mass of Io is around 100 times smaller than that of the earth, the acceleration due to gravity at its surface is only about 1/6 of that of the earth because Io's radius is much smaller than earth's radius.

13.31. **IDENTIFY:** Section 13.6 states that for a point mass outside a uniform sphere the gravitational force is the same as if all the mass of the sphere were concentrated at its center. It also states that for a point mass a distance *r* from the center of a uniform sphere, where *r* is less than the radius of the sphere, the gravitational force on the point mass is the same as though we removed all the mass at points farther than *r* from the center and concentrated all the remaining mass at the center.

SET UP: The density of the sphere is $\rho = \dfrac{M}{\frac{4}{3}\pi R^3}$, where *M* is the mass of the sphere and *R* is its radius.

The mass inside a volume of radius $r < R$ is $M_r = \rho V_r = \left(\dfrac{M}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) = M\left(\dfrac{r}{R}\right)^3$. $r = 5.01$ m is outside the sphere and $r = 2.50$ m is inside the sphere.

EXECUTE: **(a)** (i) $F_g = \dfrac{GMm}{r^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(1000.0\ \text{kg})(2.00\ \text{kg})}{(5.01\ \text{m})^2} = 5.31\times10^{-9}$ N.

(ii) $F_g = \dfrac{GM'm}{r^2}$. $M' = M\left(\dfrac{r}{R}\right)^3 = (1000.0\ \text{kg})\left(\dfrac{2.50\ \text{m}}{5.00\ \text{m}}\right)^3 = 125$ kg.

$$F_g = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(125\ \text{kg})(2.00\ \text{kg})}{(2.50\ \text{m})^2} = 2.67\times10^{-9}\ \text{N}.$$

(b) $F_g = \dfrac{GM(r/R)^3 m}{r^2} = \left(\dfrac{GMm}{R^3}\right)r$ for $r < R$ and $F_g = \dfrac{GMm}{r^2}$ for $r > R$. The graph of F_g versus *r* is sketched in Figure 13.31.

EVALUATE: At points outside the sphere the force on a point mass is the same as for a shell of the same mass and radius. For $r < R$ the force is different in the two cases of uniform sphere versus hollow shell.

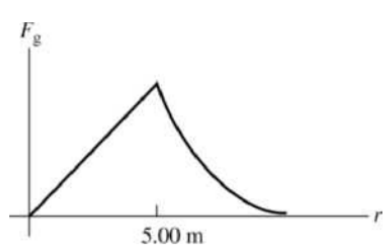


Figure 13.31