# Kepler's Laws

**13.27.** **IDENTIFY:** In part (b) apply the results from part (a).

**SET UP:** For Pluto, $e = 0.248$ and $a = 5.92\times10^{12}$ m. For Neptune, $e = 0.010$ and $a = 4.50\times10^{12}$ m. The orbital period for Pluto is $T = 247.9$ y.

**EXECUTE:** **(a)** The result follows directly from Figure 13.18 in the textbook.

**(b)** The closest distance for Pluto is $(1-0.248)(5.92\times10^{12}\text{ m}) = 4.45\times10^{12}$ m. The greatest distance for Neptune is $(1+0.010)(4.50\times10^{12}\text{ m}) = 4.55\times10^{12}$ m.

**(c)** The time is the orbital period of Pluto, $T = 248$ y.

**EVALUATE:** Pluto's closest distance calculated in part (a) is $0.10\times10^{12}\text{ m} = 1.0\times10^{8}$ km, so Pluto is about 100 million km closer to the sun than Neptune, as is stated in the problem. The eccentricity of Neptune's orbit is small, so its distance from the sun is approximately constant.

# Gravitational Potential for non-spherical mass distribution

**13.32.** **IDENTIFY:** The gravitational potential energy of a pair of point masses is $U = -G\frac{m_1 m_2}{r}$. Divide the rod into infinitesimal pieces and integrate to find $U$.

**SET UP:** Divide the rod into differential masses $dm$ at position $l$, measured from the right end of the rod. $dm = dl(M/L)$.

**EXECUTE:** **(a)** $U = -\frac{Gm\,dm}{l+x} = -\frac{GmM}{L}\frac{dl}{l+x}$.

Integrating, $U - \frac{GmM}{L}\int_0^L \frac{dl}{l+x} = -\frac{GmM}{L}\ln\left(1+\frac{L}{x}\right)$. For $x \gg L$, the natural logarithm is $\sim(L/x)$, and $U \to -GmM/x$.

**(b)** The $x$-component of the gravitational force on the sphere is

$F_x = -\frac{\partial U}{\partial x} = \frac{GmM}{L}\frac{(-L/x^2)}{(1+(L/x))} = -\frac{GmM}{(x^2+Lx)}$, with the minus sign indicating an attractive force. As $x \gg L$, the denominator in the above expression approaches $x^2$, and $F_x \to -GmM/x^2$, as expected.

**EVALUATE:** When $x$ is much larger than $L$ the rod can be treated as a point mass, and our results for $U$ and $F_x$ do reduce to the correct expression when $x \gg L$.

# SHO 101

**14.4.** **IDENTIFY:** The period is the time for one cycle and the amplitude is the maximum displacement from equilibrium. Both these values can be read from the graph.

**SET UP:** The maximum $x$ is 10.0 cm. The time for one cycle is 16.0 s.

**EXECUTE:** **(a)** $T = 16.0$ s so $f = \frac{1}{T} = 0.0625$ Hz.

**(b)** $A = 10.0$ cm.

**(c)** $T = 16.0$ s

**(d)** $\omega = 2\pi f = 0.393$ rad/s

**EVALUATE:** After one cycle the motion repeats.

# Physical Pendulum

**14.56.** **IDENTIFY:** The ornament is a physical pendulum: $T = 2\pi\sqrt{I/mgd}$ (Eq.14.39). $T$ is the target variable.

**SET UP:** $I = 5MR^2/3$, the moment of inertia about an axis at the edge of the sphere. $d$ is the distance from the axis to the center of gravity, which is at the center of the sphere, so $d = R$.

**EXECUTE:** $T = 2\pi\sqrt{5/3}\sqrt{R/g} = 2\pi\sqrt{5/3}\sqrt{0.050\text{ m}/(9.80\text{m/s}^2)} = 0.58\text{ s}$.

**EVALUATE:** A simple pendulum of length $R = 0.050\text{ m}$ has period 0.45 s; the period of the physical pendulum is longer.