

**Ideal Gas,  $pV=NkT$  or  $pV=nRT$** 

**18.26. IDENTIFY:**  $pV = nRT = NkT$

**SET UP:** At STP,  $T = 273$  K,  $p = 1.01 \times 10^5$  Pa.  $N = 6 \times 10^9$  molecules.

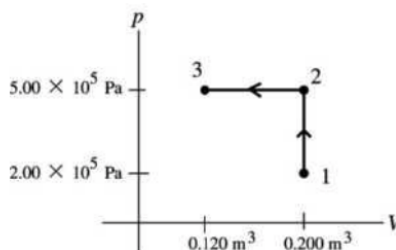
**EXECUTE:**  $V = \frac{NkT}{p} = \frac{(6 \times 10^9 \text{ molecules})(1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K})(273 \text{ K})}{1.01 \times 10^5 \text{ Pa}} = 2.24 \times 10^{-16} \text{ m}^3$ .

$L^3 = V$  so  $L = V^{1/3} = 6.1 \times 10^{-6} \text{ m}$ .

**EVALUATE:** This is a small cube.

## First Law of Thermodynamics, $\Delta U = Q - W$

- 19.6. (a) **IDENTIFY and SET UP:** The  $pV$ -diagram is sketched in Figure 19.6.



**Figure 19.6**

- (b) Calculate  $W$  for each process, using the expression for  $W$  that applies to the specific type of process.

**EXECUTE:**  $1 \rightarrow 2$ ,  $\Delta V = 0$ , so  $W = 0$

$2 \rightarrow 3$

$p$  is constant; so  $W = p \Delta V = (5.00 \times 10^5 \text{ Pa})(0.120 \text{ m}^3 - 0.200 \text{ m}^3) = -4.00 \times 10^4 \text{ J}$  ( $W$  is negative since the volume decreases in the process.)

$$W_{\text{tot}} = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} = -4.00 \times 10^4 \text{ J}$$

**EVALUATE:** The volume decreases so the total work done is negative.

- 19.11. **IDENTIFY:** Part  $ab$  is isochoric, but  $bc$  is not any of the familiar processes.

**SET UP:**  $pV = nRT$  determines the Kelvin temperature of the gas. The work done in the process is the area under the curve in the  $pV$  diagram.  $Q$  is positive since heat goes into the gas.  $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$ .

$$1 \text{ L} = 1 \times 10^{-3} \text{ m}^3. \quad \Delta U = Q - W.$$

**EXECUTE:** (a) The lowest  $T$  occurs when  $pV$  has its smallest value. This is at point  $a$ , and

$$T_a = \frac{p_a V_a}{nR} = \frac{(0.20 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})(2.0 \text{ L})(1.0 \times 10^{-3} \text{ m}^3/\text{L})}{(0.0175 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})} = 278 \text{ K}.$$

(b)  $a$  to  $b$ :  $\Delta V = 0$  so  $W = 0$ .

$b$  to  $c$ : The work done by the gas is positive since the volume increases. The magnitude of the work is the area under the curve so  $W = \frac{1}{2}(0.50 \text{ atm} + 0.30 \text{ atm})(6.0 \text{ L} - 2.0 \text{ L})$  and

$$W = (1.6 \text{ L} \cdot \text{atm})(1 \times 10^{-3} \text{ m}^3/\text{L})(1.013 \times 10^5 \text{ Pa/atm}) = 162 \text{ J}.$$

(c) For  $abc$ ,  $W = 162 \text{ J}$ .  $\Delta U = Q - W = 215 \text{ J} - 162 \text{ J} = 53 \text{ J}$ .

**EVALUATE:** 215 J of heat energy went into the gas. 53 J of energy stayed in the gas as increased internal energy and 162 J left the gas as work done by the gas on its surroundings.

- 19.29. **IDENTIFY:** For an adiabatic process of an ideal gas,  $p_1 V_1^\gamma = p_2 V_2^\gamma$ ,  $W = \frac{1}{\gamma - 1}(p_1 V_1 - p_2 V_2)$  and

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}.$$

**SET UP:** For a monatomic ideal gas  $\gamma = 5/3$ .

$$\text{EXECUTE: (a) } p_2 = p_1 \left( \frac{V_1}{V_2} \right)^\gamma = (1.50 \times 10^5 \text{ Pa}) \left( \frac{0.0800 \text{ m}^3}{0.0400 \text{ m}^3} \right)^{5/3} = 4.76 \times 10^5 \text{ Pa}.$$

(b) This result may be substituted into Eq. (19.26), or, substituting the above form for  $p_2$ ,

$$W = \frac{1}{\gamma - 1} p_1 V_1 \left( 1 - (V_1/V_2)^{\gamma-1} \right) = \frac{3}{2} (1.50 \times 10^5 \text{ Pa})(0.0800 \text{ m}^3) \left( 1 - \left( \frac{0.0800}{0.0400} \right)^{2/3} \right) = -1.06 \times 10^4 \text{ J}.$$

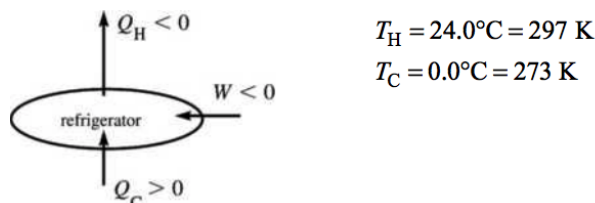
(c) From Eq. (19.22),  $(T_2/T_1) = (V_2/V_1)^{\gamma-1} = (0.0800/0.0400)^{2/3} = 1.59$ , and since the final temperature is higher than the initial temperature, the gas is heated.

**EVALUATE:** In an adiabatic compression  $W < 0$  since  $\Delta V < 0$ .  $Q = 0$  so  $\Delta U = -W$ .  $\Delta U > 0$  and the temperature increases.

## Second Law of Thermodynamics, $\Delta S \geq 0$

- 20.16. IDENTIFY and SET UP:** The device is a Carnot refrigerator. We can use Eqs. (20.2) and (20.13).

(a) The operation of the device is sketched in Figure 20.16.



**Figure 20.16**

The amount of heat taken out of the water to make the liquid  $\rightarrow$  solid phase change is

$Q = -mL_f = -(85.0 \text{ kg})(334 \times 10^3 \text{ J/kg}) = -2.84 \times 10^7 \text{ J}$ . This amount of heat must go into the working substance of the refrigerator, so  $Q_C = +2.84 \times 10^7 \text{ J}$ . For Carnot cycle  $|Q_C|/|Q_H| = T_C/T_H$ .

**EXECUTE:**  $|Q_H| = |Q_C|(T_H/T_C) = 2.84 \times 10^7 \text{ J}(297 \text{ K}/273 \text{ K}) = 3.09 \times 10^7 \text{ J}$

(b)  $W = Q_C + Q_H = +2.84 \times 10^7 \text{ J} - 3.09 \times 10^7 \text{ J} = -2.5 \times 10^6 \text{ J}$

**EVALUATE:**  $W$  is negative because this much energy must be supplied to the refrigerator rather than obtained from it. Note that in Eq. (20.13) we must use Kelvin temperatures.

- 20.41. IDENTIFY:**  $pV = nRT$ , so  $pV$  is constant when  $T$  is constant. Use the appropriate expression to calculate

$Q$  and  $W$  for each process in the cycle.  $e = \frac{W}{Q_H}$ .

**SET UP:** For an ideal diatomic gas,  $C_V = \frac{5}{2}R$  and  $C_p = \frac{7}{2}R$ .

**EXECUTE:** (a)  $p_a V_a = 2.0 \times 10^3 \text{ J}$ .  $p_b V_b = 2.0 \times 10^3 \text{ J}$ .  $pV = nRT$  so  $p_a V_a = p_b V_b$  says  $T_a = T_b$ .

(b) For an isothermal process,  $Q = W = nRT \ln(V_2/V_1)$ .  $ab$  is a compression, with  $V_b < V_a$ , so  $Q < 0$  and

heat is rejected.  $bc$  is at constant pressure, so  $Q = nC_p \Delta T = \frac{C_p}{R} p \Delta V$ .  $\Delta V$  is positive, so  $Q > 0$  and heat is

absorbed.  $ca$  is at constant volume, so  $Q = nC_V \Delta T = \frac{C_V}{R} V \Delta p$ .  $\Delta p$  is negative, so  $Q < 0$  and heat is

rejected.

(c)  $T_a = \frac{p_a V_a}{nR} = \frac{2.0 \times 10^3 \text{ J}}{(1.00)(8.314 \text{ J/mol} \cdot \text{K})} = 241 \text{ K}$ .  $T_b = \frac{p_b V_b}{nR} = T_a = 241 \text{ K}$ .

$T_c = \frac{p_c V_c}{nR} = \frac{4.0 \times 10^3 \text{ J}}{(1.00)(8.314 \text{ J/mol} \cdot \text{K})} = 481 \text{ K}$ .

(d)  $Q_{ab} = nRT \ln\left(\frac{V_b}{V_a}\right) = (1.00 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(241 \text{ K}) \ln\left(\frac{0.0050 \text{ m}^3}{0.010 \text{ m}^3}\right) = -1.39 \times 10^3 \text{ J}$ .

$$Q_{bc} = nC_p \Delta T = (1.00) \left(\frac{7}{2}\right) (8.314 \text{ J/mol} \cdot \text{K})(241 \text{ K}) = 7.01 \times 10^3 \text{ J}.$$

$$Q_{ca} = nC_V \Delta T = (1.00) \left(\frac{5}{2}\right) (8.314 \text{ J/mol} \cdot \text{K})(-241 \text{ K}) = -5.01 \times 10^3 \text{ J}. \quad Q_{\text{net}} = Q_{ab} + Q_{bc} + Q_{ca} = 610 \text{ J}.$$

$$W_{\text{net}} = Q_{\text{net}} = 610 \text{ J}.$$

$$(e) \quad e = \frac{W}{Q_H} = \frac{610 \text{ J}}{7.01 \times 10^3 \text{ J}} = 0.087 = 8.7\%$$

**EVALUATE:** We can calculate  $W$  for each process in the cycle.  $W_{ab} = Q_{ab} = -1.39 \times 10^3 \text{ J}$ .

$W_{bc} = p \Delta V = (4.0 \times 10^5 \text{ Pa})(0.0050 \text{ m}^3) = 2.00 \times 10^3 \text{ J}$ .  $W_{ca} = 0$ .  $W_{\text{net}} = W_{ab} + W_{bc} + W_{ca} = 610 \text{ J}$ , which