Chapter 10

10.6. **IDENTIFY:** Knowing the force on a bar and the point where it acts, we want to find the position vector for the point where the force acts and the torque the force exerts on the bar.

SET UP: The position vector is $\vec{r} = x\hat{i} + y\hat{j}$ and the torque is $\vec{\tau} = \vec{r} \times \vec{F}$.

EXECUTE: **(a)** Using $x = 3.00$ m and $y = 4.00$ m, we have $\vec{r} = (3.00)\hat{i} + (4.00)\hat{j}$.

(b) $\vec{\tau} = \vec{r} \times \vec{F} = [(3.00 \text{ m})\hat{i} + (4.00 \text{ m})\hat{j}] \times [(7.00 \text{ N})\hat{i} + (-3.00 \text{ N})\hat{j}]$.

$\vec{\tau} = (-9.00 \text{ N}\cdot\text{m})\hat{k} + (-28.0 \text{ N}\cdot\text{m})(-\hat{k}) = (-37.0 \text{ N}\cdot\text{m})\hat{k}$. The torque has magnitude $37.0 \text{ N}\cdot\text{m}$ and is in the $-z$-direction.

EVALUATE: Applying the right-hand rule for the vector product to $\vec{r} \times \vec{F}$ shows that the torque must be in the $-z$-direction because it is perpendicular to both $\vec{r}$ and $\vec{F}$, which are both in the x-y plane.

10.19. **IDENTIFY:** Since there is rolling without slipping, $v_{\text{cm}} = R\omega$. The kinetic energy is given by Eq. (10.8). The velocities of points on the rim of the hoop are as described in Figure 10.13 in Chapter 10.

SET UP: $\omega = 3.00$ rad/s and $R = 0.600$ m. For a hoop rotating about an axis at its center, $I = MR^2$.

EXECUTE: **(a)** $v_{\text{cm}} = R\omega = (0.600 \text{ m})(3.00 \text{ rad/s}) = 1.80$ m/s.

(b) $K = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}(MR^2)(v_{\text{cm}}/R^2) = Mv_{\text{cm}}^2 = (2.20 \text{ kg})(1.80 \text{ m/s})^2 = 7.13 \text{ J}$

(c) (i) $v = 2v_{\text{cm}} = 3.60$ m/s. $\vec{v}$ is to the right. (ii) $v = 0$

(iii) $v = \sqrt{v_{\text{cm}}^2 + v_{\tan}^2} = \sqrt{v_{\text{cm}}^2 + (R\omega)^2} = \sqrt{2}v_{\text{cm}} = 2.55$ m/s. $\vec{v}$ at this point is at 45° below the horizontal.

(d) To someone moving to the right at $v = v_{\text{cm}}$, the hoop appears to rotate about a stationary axis at its center. (i) $v = R\omega = 1.80$ m/s, to the right. (ii) $v = 1.80$ m/s, to the left. (iii) $v = 1.80$ m/s, downward.

EVALUATE: For the special case of a hoop, the total kinetic energy is equally divided between the motion of the center of mass and the rotation about the axis through the center of mass. In the rest frame of the ground, different points on the hoop have different speed.

Chapter 11

11.1. **IDENTIFY:** Use Eq. (11.3) to calculate x_{cm}. The center of gravity of the bar is at its center and it can be treated as a point mass at that point.

SET UP: Use coordinates with the origin at the left end of the bar and the $+x$ axis along the bar. $m_1 = 0.120$ kg, $m_2 = 0.055$ kg, $m_3 = 0.110$ kg.

EXECUTE: $x_{\text{cm}} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \dfrac{(0.120 \text{ kg})(0.250 \text{ m}) + 0 + (0.110 \text{ kg})(0.500 \text{ m})}{0.120 \text{ kg} + 0.055 \text{ kg} + 0.110 \text{ kg}} = 0.298$ m. The fulcrum should be placed 29.8 cm to the right of the left-hand end.

EVALUATE: The mass at the right-hand end is greater than the mass at the left-hand end. So the center of gravity is to the right of the center of the bar.

11.77. **IDENTIFY:** Apply the first and second conditions of equilibrium to the gate.
SET UP: The free-body diagram for the gate is given in Figure 11.77.

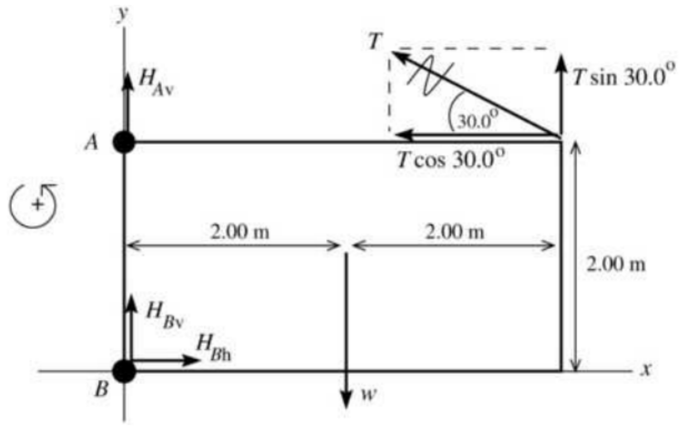


Figure 11.77

Use coordinates with the origin at B. Let $\vec{H}_A$ and $\vec{H}_B$ be the forces exerted by the hinges at A and B. The problem states that $\vec{H}_A$ has no horizontal component. Replace the tension $\vec{T}$ by its horizontal and vertical components.

EXECUTE: **(a)** $\sum \tau_B = 0$ gives $+(T\sin 30.0°)(4.00\text{ m}) + (T\cos 30.0°)(2.00\text{ m}) - w(2.00\text{ m}) = 0$

$T(2\sin 30.0° + \cos 30.0°) = w$

$$T = \frac{w}{2\sin 30.0° + \cos 30.0°} = \frac{500\text{ N}}{2\sin 30.0° + \cos 30.0°} = 268\text{ N}$$

(b) $\sum F_x = ma_x$ says $H_{Bh} - T\cos 30.0° = 0$

$H_{Bh} = T\cos 30.0° = (268\text{ N})\cos 30.0° = 232\text{ N}$

(c) $\sum F_y = ma_y$ says $H_{Av} + H_{Bv} + T\sin 30.0° - w = 0$

$H_{Av} + H_{Bv} = w - T\sin 30.0° = 500\text{ N} - (268\text{ N})\sin 30.0° = 366\text{ N}$

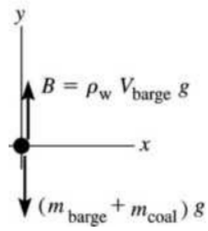
EVALUATE: T has a horizontal component to the left so H_{Bh} must be to the right, as these are the only two horizontal forces. Note that we cannot determine H_{Av} and H_{Bv} separately, only their sum.

IDENTIFY: Use Eq. (11.3) to locate the x-coordinate of the center of gravity of the block combinations.

Chpater 12

12.61. **IDENTIFY:** Apply Newton's second law to the barge plus its contents. Apply Archimedes's principle to express the buoyancy force B in terms of the volume of the barge.

SET UP: The free-body diagram for the barge plus coal is given in Figure 12.61.



EXECUTE: $\sum F_y = ma_y$

$B - (m_{barge} + m_{coal})g = 0$

$\rho_w V_{barge} g = (m_{barge} + m_{coal})g$

$m_{coal} = \rho_w V_{barge} - m_{barge}$

Figure 12.61

$V_{barge} = (22\text{ m})(12\text{ m})(40\text{ m}) = 1.056\times10^4\text{ m}^3$

The mass of the barge is $m_{barge} = \rho_s V_s$, where s refers to steel.

From Table 12.1, $\rho_s = 7800\text{ kg/m}^3$. The volume V_s is 0.040 m times the total area of the five pieces of steel that make up the barge

$$V_s = (0.040\text{ m})[2(22\text{ m})(12\text{ m}) + 2(40\text{ m})(12\text{ m}) + (22\text{ m})(40\text{ m})] = 94.7\text{ m}^3.$$

Therefore, $m_{barge} = \rho_s V_s = (7800\text{ kg/m}^3)(94.7\text{ m}^3) = 7.39\times10^5\text{ kg}$.

Then $m_{coal} = \rho_w V_{barge} - m_{barge} = (1000\text{ kg/m}^3)(1.056\times10^4\text{ m}^3) - 7.39\times10^5\text{ kg} = 9.8\times10^6\text{ kg}$.

The volume of this mass of coal is $V_{coal} = m_{coal}/\rho_{coal} = 9.8\times10^6\text{ kg}/1500\text{ kg/m}^3 = 6500\text{ m}^3$; this is less than V_{barge} so it will fit into the barge.

EVALUATE: The buoyancy force B must support both the weight of the coal and also the weight of the barge. The weight of the coal is about 13 times the weight of the barge. The buoyancy force increases when more of the barge is submerged, so when it holds the maximum mass of coal the barge is fully submerged.

Chapter 13

13.9. **IDENTIFY:** Use Eq. (13.2) to calculate the gravitational force each particle exerts on the third mass. The equilibrium is stable when for a displacement from equilibrium the net force is directed toward the equilibrium position and it is unstable when the net force is directed away from the equilibrium position.
SET UP: For the net force to be zero, the two forces on M must be in opposite directions. This is the case only when M is on the line connecting the two particles and between them. The free-body diagram for M is given in Figure 13.9. $m_1 = 3m$ and $m_2 = m$. If M is a distance x from m_1, it is a distance $1.00\text{ m} - x$ from m_2.

EXECUTE: **(a)** $F_x = F_{1x} + F_{2x} = -G\dfrac{3mM}{x^2} + G\dfrac{mM}{(1.00\text{ m} - x)^2} = 0.3\ (1.00\text{ m} - x)^2 = x^2.$

$1.00\text{ m} - x = \pm x/\sqrt{3}$. Since M is between the two particles, x must be less than 1.00 m and $x = \dfrac{1.00\text{ m}}{1 + 1/\sqrt{3}} = 0.634\text{ m}$. M must be placed at a point that is 0.634 m from the particle of mass $3m$ and 0.366 m from the particle of mass m.

(b) (i) If M is displaced slightly to the right in Figure 13.9, the attractive force from m is larger than the force from $3m$ and the net force is to the right. If M is displaced slightly to the left in Figure 13.9, the attractive force from $3m$ is larger than the force from m and the net force is to the left. In each case the net force is away from equilibrium and the equilibrium is unstable.
(ii) If M is displaced a very small distance along the y axis in Figure 13.9, the net force is directed opposite to the direction of the displacement and therefore the equilibrium is stable.
EVALUATE: The point where the net force on M is zero is closer to the smaller mass.