Color from Geometry: Some Gauge Theory Applications of AdS/CFT

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Through the **gauge/gravity** (or **gauge/string**, or **holographic**) **correspondence**, string theory provides a useful tool to study **some** aspects of **certain strongly-coupled non-Abelian gauge theories**
Why do we care?

- Results are useful to develop some intuition on (at least some) strongly-coupled non-Abelian theories.
- Overlap with other field(s) gives good opportunity to learn some interesting real-world physics!
- Simple extraction of plausible gauge theory physics from string counterparts provides additional support for gauge/gravity correspondence.
Why should *you* care?

- Equivalence of gravitational and non-gravitational theories is stunning!! New theoretical paradigm
- Novel perspective on some difficult gravity problems
- Tools familiar to a relativist suddenly find new applications in high energy / nuclear (and condensed matter) physics
Disclaimers/Clarifications

- We are NOT claiming to have solved QCD!! The gauge theories under present control, while interesting, are at best toy models of real-world QCD.

- This string theory application is orthogonal to the search for a unified theory. We’re NOT looking for the Standard Model here. Then again, this IS string theory, arguably being useful.
Some Useful Reviews

- Gubser, Karch, arXiv:0901.0935
- Mateos, arXiv:0709.1523
- Aharony, Gubser, Maldacena, Ooguri, Oz, hep-th/9905111
- Myers, Vázquez, arXiv:0804.2423
- Hubeny, Rangamani, arXiv:1006.3675
- Son, Starinets, arXiv:0704.0240
- Erdmenger, Evans, Kirsch, Threlfall, arXiv:0711.4467
- Peeters, Zamaklar, arXiv:0708.1502
- Edelstein, Portugues, hep-th/0602021
- Klebanov, hep-ph/0509087
- Aharony, hep-th/0212193
Some Useful Reviews

Condensed Matter / Atomic Physics Applications

- Horowitz, arXiv:1002.1722
- McGreevy, arXiv:0909.0518
- Sachdev, arXiv:1002.2947
- Schäfer, Teaney, arXiv:0904.3107
Overall Plan

- **L1**: Motivation, String Theory & AdS/CFT Basics
- **L2**: AdS/CFT Dictionary, Entropy, Viscosity, Mesons, Screening, Energy Loss
- **L3**: Brownian, Damping, Unruh Generalizations of AdS/CFT (Gauge/Gravity)

**Plan for Lecture 1**

- Motivation: QCD and QGP
- String Theory Basics
- AdS/CFT `derivation`
Quarks

$q^{(F)}_C(x) \quad C = 1, 2, 3 \quad F = 1, \ldots, 6$

+ 3 colors (kinds of strong charge) $SU(3)_c$

Gluons

$A^\mu_{CC'}(x) \equiv A^\mu_I(x)t^I_{CC'} \quad C, C' = 1, 2, 3 \quad I = 1, \ldots, 8$

$S_{QCD} = \int d^4x \left[ -\frac{1}{2g_{YM}^2} \text{Tr}(F_{\mu\nu}F^{\mu\nu}) + \bar{q}^{(F)}(i\gamma^\mu D_\mu + m^{(F)})q^{(F)} \right]$

$D_\mu \equiv \partial_\mu - iA_\mu \quad F_{\mu\nu} \equiv i[D_\mu, D_\nu]$

(Note that $(A_\mu)_\text{here} = g_{YM}(A_\mu)_\text{usual}$)
Quarks

\[ q_C^{(F)}(x) \quad C = 1, 2, 3 \quad F = 1, \ldots, 6 \]

+ 

Gluons

\[ A_{CC'}^\mu(x) \equiv A_I^{\mu}(x)t_{CC'}^I \quad C, C' = 1, 2, 3 \quad I = 1, \ldots, 8 \]

\[ S_{QCD} = \int d^4x \left[ -\frac{1}{2g_{YM}^2} \text{Tr}(F_{\mu\nu}F^{\mu\nu}) + \bar{q}^{(F)}(i\gamma^\mu D_\mu + m^{(F)})q^{(F)} \right] \]

CLASSICALLY: invariant under local symmetry \( SU(3)_c \), and, if \( m^{(F)} = 0 \), rescalings and global internal symmetry

\[ SU(N_f)_V \times SU(N_f)_A \times U(1)_V \times U(1)_A \]
QCD

Quarks

\[ q^{(F)}_C(x) \quad C = 1, 2, 3 \quad F = 1, \ldots, 6 \]

Gluones

\[ A^{\mu}_{CC'}(x) \equiv A^{\mu}_I(x) t^I_{CC'} \quad C, C' = 1, 2, 3 \quad I = 1, \ldots, 8 \]

\[
S_{QCD} = \int d^4 x \left[ -\frac{1}{2 g_{YM}^2} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) + \bar{q}^{(F)}(i \gamma^\mu D_\mu + m^{(F)}) q^{(F)} \right]
\]

QUANTUMLY: invariant under local symmetry \( SU(3)_c \), and, even if \( m^{(F)} = 0 \), internal symmetry is only

\( SU(N_f)_V \times U(1)_V \) (chiral and rescaling symm. broken)
QCD: Asymptotic Freedom

The vacuum in a QFT is a polarizable medium...
QCD: Asymptotic Freedom

Virtual quarks screen, virtual gluons antiscreen

\[ \alpha_{YM}(Q) \equiv \frac{g_{YM}^2(Q)}{4\pi} \approx \frac{6\pi}{(11N_c - 2N_f) \log(Q/\Lambda_{QCD})} \]

[Gross, Wilczek; Politzer]
Asymptotic freedom

High energies: weak coupling

From: Bethke, hep-ex/0407021
QCD: Confinement

At low energies, we do NOT observe directly quarks and gluons, but hadrons (mesons, baryons, glueballs, etc.)

Only particles that are NEUTRAL under $SU(3)_c$

\[ V_{qq}(L) \approx \sigma L \]
QCD: Confinement

These ‘flux tubes’ are visible in numerical calculations on discretized spacetime—lattice QCD


Suggests connection between QCD and ‘fat’ strings (phenomenological model: ‘QCD string’ can reproduce “Regge behavior” \( J = \alpha' m^2 + \alpha(0) \))
QCD: Deconfinement

Note that strong coupling is necessary (although not sufficient!) to have confinement.

As we heat up a gas of hadrons, the coupling decreases...

We therefore expect phase transition to

Quark-Gluon Plasma (QGP)

at a certain deconfinement temperature

\[ T_c \approx \Lambda_{\text{QCD}} \approx 200 \text{ MeV} \approx 2 \times 10^{12} \text{K} \]
QCD: Deconfinement

Lattice calculations confirm this, with $T_c \simeq 170 - 190$ MeV

Energy and entropy density $\sim 0.8$ of ideal gas

From: F. Karsch, hep-lat/0106019
QGP at RHIC (Brookhaven)

Au+Au (400 nucleons)
100 GeV/nucleon

www.bnl.gov/RHIC/images/movies/Au-Au_200GeV.mpeg
QGP at RHIC (Brookhaven)

Au+Au (400 nucleons)  
100 GeV/nucleon  \rightarrow \textbf{QGP}  \rightarrow  5000 \text{ hadrons, etc.}  
\sim 10 \text{ GeV/hadron}

Size \sim 10^{-14} \text{ m}  
Duration \sim 10^{-22} \text{ s}  

www.bnl.gov/RHIC/images/movies/Au-Au_200GeV.mpeg
QGP at RHIC (Brookhaven)

- Temperature not far from $\Lambda_{QCD} \approx 200 \text{ MeV}$:
  $$T \sim (1-2)T_c \sim 170-300 \text{ MeV}$$

- Low viscosity/entropy [Teaney; Romatschke, Romatschke] :
  $$\frac{\eta}{s} \sim (0.1-0.2) \frac{\hbar}{k_B} \ll \left( \frac{\eta}{s} \right)_{\text{pQCD}} \sim \frac{1}{g_{YM}^4 \log \left( \frac{1}{g_{YM}^2} \right)} \frac{\hbar}{k_B}$$

  [Arnold, Moore, Yaffe]

- Short thermalization time, etc.
QGP at RHIC (Brookhaven)

⇒ **Strongly-Coupled Quark-Gluon Plasma (sQGP)**

\[ g_{QCD}^2 \sim 3 - 10 \quad \alpha_{QCD} \equiv \frac{g_{QCD}^2}{4\pi} \sim 0.3 - 1 \]

**Perturbative** expansion unreliable

(Euclidean) **Lattice** calculations useful to determine static properties, but NOT dynamical ones

Can construct effective **phenomenological models**...

Or can try to do first-principles calculations in a **different** (but hopefully similar) **theory**
Cousins of QCD

- 'QCD' w/o quarks ($N_f = 0$) = $SU(3)_c$ Yang-Mills
  Preserves asymptotic freedom & confinement
- $SU(N_c)$ Yang-Mills (w/ or w/o quarks) with $N_c > 3$
  Preserves asymptotic freedom & confinement
  Take $N_c \rightarrow \infty$ with $\lambda \equiv g_{YM}^2 N_c$ fixed ['t Hooft]
  Try $SU(3)_c \approx SU(\infty)_c + O(1/N_c^2)$

Planar diagrams only; organized by 2D topologies {P. Kraus}

$\lambda \ll 1$ Perturbative expansion is valid

$\lambda \gg 1$ Surfaces filled: strings with coupling $1/N_c$ ?!

Lattice: $N_c \rightarrow \infty$ is decent approximation to $N_c = 3$ !

[Teper et al.]
A (Distant) Cousin of QCD...

- \( SU(N_c) \) Yang-Mills (w/o quarks):
  \[ A^\mu_{CC'}(x) \quad C, C' = 1, \ldots, N_c \]
  + 6 real massless scalars:
  \[ \Phi^I_{CC'}(x) \quad I = 1, \ldots, 6 \]
  + 4 massless Weyl fermions:
  \[ \lambda^A_{CC'}(x) \quad A = 1, \ldots, 4 \]
  + carefully synchronized 3-pt and 4-pt interactions

= \( SU(N_c) \) Super-Yang-Mills with \( \mathcal{N} = 4 \) supersymmetry

\[
\mathcal{L} = \text{Tr} \left\{ -\frac{1}{2g_{YM}^2} F_{\mu\nu} F^{\mu\nu} - i\tilde{\lambda}^A \bar{\sigma}^\mu D_\mu \lambda^A - D_\mu \Phi^I D^\mu \Phi^I ight\}
\]

\[
+ g_{YM} C^A_{IB} \lambda^I_A \left[ \Phi^I_B, \lambda^I_B \right] + g_{YM} \bar{C}^A_{IB} \bar{\lambda}^I_A \left[ \Phi^I_B, \bar{\lambda}^I_B \right] + \frac{g_{YM}^2}{2} \left[ \Phi^I, \Phi^I \right] \left[ \Phi^I, \Phi^I \right]
\]
A (Distant) Cousin of QCD...

- $SU(N_c)$ Yang-Mills (w/o quarks):
  - $6$ real massless scalars: $\Phi^I_{CC'}(x)$, $I = 1, \ldots, 6$
  - $4$ massless Weyl fermions: $\lambda^A_{CC'}(x)$, $A = 1, \ldots, 4$
  - carefully synchronized $3$-pt and $4$-pt interactions

= $SU(N_c)$ Super-Yang-Mills with $\mathcal{N} = 4$ supersymmetry

Theory invariant under **rescalings** even at the quantum level... $g_{YM}$ does NOT run with energy!! [Sohnius, West]

Spacetime symmetry: $SO(4, 2) \supset Poincaré(3, 1) \supset SO(3, 1)$

Conformal group $\uparrow$ (+ fermionic part)
(dilatations + special conformal transf. + Poincaré)

Internal (R-)symmetry: $SU(4) \cong SO(6)$
A (Distant) Cousin of QCD...

- $SU(N_c)$ Yang-Mills (w/o quarks):
  - $A_{CC'}^\mu(x)$, $C, C' = 1, \ldots, N_c$
  - $\Phi^{I}_{CC'}(x)$, $I = 1, \ldots, 6$
  - $\lambda^A_{CC'}(x)$, $A = 1, \ldots, 4$
  - + 6 real massless scalars
  - + 4 massless Weyl fermions
  - + carefully synchronized 3-pt and 4-pt interactions

= $SU(N_c)$ Super-Yang-Mills with $\mathcal{N} = 4$ supersymmetry

Is this theory at least qualitatively similar to QCD??
**QCD vs. \( \mathcal{N} = 4 \) SYM**

- **\( T = 0 \):**
  - Asymptotically free: \( d g_{YM}^{2} / dE < 0 \)
  - Confined in IR
  - Only massive particles
  - Linear Potential
  - Non-Supersymmetric

- **\( T > T_{c} \):**
  - Approx. conformal: \( \varepsilon \sim T^{4} \)
  - Deconfined
  - Non-abelian plasma of gluons and matter in \textbf{fundamental} rep.
  - Screened Potential
  - No Supersymmetry

- **Conformal:**
  - \( d g_{YM}^{2} / dE = 0 \)
  - Deconfined
  - No mass scale
  - Coulomb Potential
  - Supersymmetric

- **\( T \approx T_{c} \):**
  - Temp. is only scale: \( \varepsilon \propto T^{4} \)
  - Deconfined
  - Non-abelian plasma of gluons and matter in \textbf{adjoint} rep.
  - Screened Potential
  - Supersymmetry broken
The AdS/CFT correspondence states that

\[ SU(N_c) \ \mathcal{N} = 4 \text{ SYM} \quad \overset{\text{[Maldacena]}}{=} \quad \text{A particular String Theory} \]

on a certain curved spacetime

On RHS, easy to compute in the strong-coupling region

\[ N_c \gg 1 \quad g_{YM}^2 N_c \gg 1 \]

(Remember: for sQGP we’re ultimately interested
in QCD at \( N_c = 3 \quad g_{YM}^2 N_c \sim 10 - 30 \))

N.B.: We can similarly study other (closer) cousins of QCD, but even this crudest setup gets us some mileage

Now, where does the correspondence come from?
**String Theory Basics**

**Perturbative** definition about some (not necessarily flat) spacetime background \{C. Johnson\}

- $m = \sqrt{8} / l_s$
- $m = \sqrt{4} / l_s$
- $m = 0$

For models of unification, $10^{35} \text{GeV} < m_s < 10^{14-10} \text{GeV}$

For application to QCD, $m_s \sim 1 \text{GeV}$
String Theory Basics

Field content of $\mathcal{N} = 1, 2$ SUGRA in $D = 9 + 1$

- $\varphi$
- $h_{\mu\nu}$
- $B_{\mu\nu}$
- $A_\mu$
- $C_{\mu_1...\mu_{p+1}}$

+ Fermions

Dilaton
Graviton
Gauge Bosons
String Theory Basics

Single basic interaction

Gravitational
Yang-Mills

... 

\[ g_s = e^\varphi \]

\[ G_N \sim g_s^2 l_s^8, \quad g_{YM}^2 \sim g_s l_s^{p-3} \]
String Theory Basics

Can compute scattering amplitudes:

UV finite!
String Theory Basics

Results summarized in effective action, e.g.,

\[
S_{\text{IIB}} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \left[ e^{-2\varphi} \left( R + 4(\partial_{\mu} \varphi)^2 - \frac{1}{12} \left( \partial_{[\mu} B_{\nu\lambda]} \right)^2 \right) \right.
\]

\[
- \sum_{p=-1,1,3} \frac{1}{(p + 2)!} \left( \partial_{[\mu_1} C_{\mu_2\ldots\mu_{p+2}]} \right)^2 + \cdots + \text{fermions}
\]

+ higher derivative terms suppressed by \( l_s^n \)

It is important to bear in mind that string theory is NOT really a theory of strings...
String Theory Basics

Recall that a ‘particle’ physicist is really a “field physicist”:

Particles = small excitations of a quantum field
String Theory Basics

Recall that a `particle’ physicist is really a “field physicist”:

Soliton = large (finite energy) excitation of a quantum field
In particular, when confronted with Gravity Spacetime, the particle/field physicist would start studying it as follows:
Within **string theory**, spacetime is only part of a much more complex structure (~ a “string field”)

whose small excitations are described by **strings**: 

\[ \varphi, h_{\mu\nu}, A_\mu, B_{\mu\nu}, C_{\mu_1...\mu_{p+1}} \]

+ fermions + etc.

size \( l_s \), coupling \( g_s \)
Within **string theory**, spacetime is only part of a much more complex structure (~ a “string field”)

and whose large, solitonic, excitations include various **branes**:

- 0-brane
- 1-brane
- 2-brane
- 3-brane

with masses $m \propto 1/g_s^2$ or $m \propto 1/g_s$ → **D-branes**
Within string theory, spacetime is only part of a much more complex structure (~ a “string field”)

and whose large, solitonic, excitations include various branes:

(Interestingly, each of these objects can be reinterpreted as the fundamental string of a different string theory! E.g., D1-brane of Type I = string of Heterotic SO(32))
Within **string theory**, spacetime is only part of a much more complex structure (~ a “string field”)

and whose large, solitonic, excitations include various **branes**:

(And, upon closer inspection, all of them have inner structure. Strings and branes are NOT fundamental. )
D-branes

D-branes are dynamical objects whose excitations are described by open strings.
Multiple D-branes

\[ E \ll 1/l_s \implies \mathcal{N} = 2^{(7-p)/2} \]

with \( U(N) \) gauge group and \( D = p + 1 \)

\[ g_{YM}^2 = (2\pi)^{p-2} g_s l_s^{p-3} \]

[Witten]

\( \Phi^i_{IJ} \), \( A_{\mu}^{IJ} \) + fermions + etc.

\( x^1, \ldots, x^p \)

N Dp-branes
In SYM: $\langle (\Phi^9)_{11} \rangle \neq 0 \implies (A_\mu)_{1J}$ acquires mass

$\langle (\Phi^i)_{IJ} \rangle , (A_\mu)_{IJ} + $ fermions + etc. $x^1, \ldots, x^p$
In SYM: \( \langle (\Phi^9)_{11} \rangle \neq 0 \Rightarrow (A_\mu)_{1J} \) acquires mass

In string theory: corresponding string acquires minimal \textit{length}

\[
\left( \Phi^i \right)_{IJ}, \left( A_\mu \right)_{IJ} + \text{fermions} + \text{etc.}
\]

\[
\mu = 0, \ldots, p \quad i = p + 1, \ldots, 9
\]

\[
d = \langle (\Phi^9)_{11} \rangle \neq 0
\]

\[
x^1, \ldots, x^p
\]
Multiple D-branes

\[ M_{Dp} = \frac{N V^p}{(2\pi)^p g_s l_s^{p+1}}, \quad Q_{Dp} = N \]

Do these branes deform spacetime?
Black Branes

Supergravity e.o.m. have solitonic solutions

Explicitly, for extremal (mass=charge) case:

\[
g_{\mu\nu}(r)dx^\mu dx^\nu = H^{-1/2} \left( -dx_0^2 + \cdots + dx_p^2 \right) + H^{1/2} \left( dr^2 + r^2 d\Omega_{8-p}^2 \right),
\]

\[
\exp(\varphi(r)) = g_s H^{(3-p)/4}, \quad C_{01\cdots p}(r) = 1 - H^{-1},
\]

\[
H(r) \equiv 1 + \left( 2\sqrt{\pi} \right)^{5-p} \Gamma\left( \frac{7-p}{2} \right) \frac{g_s NL_s^{7-p}}{r^{7-p}}.
\]
Black Branes

Extended (p-dim) versions of charged black holes:

\[ M = \frac{NV_p}{(2\pi)^p g_s l_s^{p+1}} \]

\[ Q = N \]

Omitted ‘longitudinal’ directions \( x^1, \ldots, x^p \)

Event horizon \( r = 0 \)

\[ S^{8-p} \times \mathbb{R}^p \]

[Horowitz, Strominger]
Black Branes vs. D-branes

Black p-brane
D-branes/Open strings?

N Dp-branes + Flat Spacetime
Curved Spacetime?

[Polchinski]
Black Branes vs. D-branes

Black p-brane

D-branes/Open strings?

N Dp-branes + Flat Spacetime
Curved Spacetime?

[Polchinski]
Overall Plan

- **L1:** Motivation, String Theory Basics
- **L2:** AdS/CFT `Derivation’ & Dictionary, Entropy, Viscosity, Screening, Energy Loss
- **L3:** Mesons, Brownian, Damping, Unruh, Generalizations of AdS/CFT (Gauge/Gravity)

**Plan for Lecture 2**

- `Derivation’ of AdS/CFT
- AdS/CFT Dictionary
- A Few Applications
Let’s take $p = 3$. At low energies, 

$$E \ll \frac{1}{(g_s N)^{1/4}} l_s \ll \frac{1}{l_s} ,$$

2 important things happen...
‘Derivation’ of AdS/CFT

Each system splits into 2 decoupled components...
‘Derivation’ of AdS/CFT

Free supergravity in flat 9+1 dim

and these components simplify drastically...
‘Derivation’ of AdS/CFT

\[ g_{\mu\nu}(r)dx^\mu dx^\nu = H^{-1/2} \left(-dx_0^2 + \cdots + dx_3^2\right) + H^{1/2} \left(dr^2 + r^2 d\Omega_5^2\right), \]

\[ e^{\phi(r)} = g_s, \quad C_{0123}(r) = 1 - H^{-1}, \]

\[ H(r) \equiv - \frac{4\pi g_s Nl_s^4}{r^4} \equiv \frac{L^4}{r^4} \]
‘Derivation’ of AdS/CFT

\[ g_{\mu\nu}(r)dx^\mu dx^\nu = \left[ \frac{r^2}{L^2}(-dt^2 + dx^2) + \frac{L^2}{r^2}dr^2 \right] + L^2d\Omega_5^2, \]

5-dim anti-de Sitter (Poincaré patch)
‘Derivation’ of AdS/CFT

Type IIB ST on $\text{AdS}_5 \times \text{S}^5 \equiv \text{SU}(N) \text{ SYM in } 3+1 \text{ dim} \phantom{!!}

[Maldacena; Gubser, Klebanov, Polyakov; Witten]
Anti-de Sitter Space

Maximally symmetric space with negative curvature, defined as (universal covering of) hyperboloid in $\mathbb{R}^{4,2}$:

$$-X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 - X_5^2 = -L^2$$

Global coords: $0 \leq \rho$, $-\infty < \tau < \infty$, $0 \leq \Omega_A \leq 1$

$X_0 = L \cosh \rho \cos \tau$

$X_A = L \sinh \rho \Omega_A$

$X_5 = L \cosh \rho \sin \tau$

$\Rightarrow \quad ds^2 = L^2 \left( -\cosh^2 \rho \, d\tau^2 + d\rho^2 + \sinh^2 \rho \, d\Omega_A^2 \right)$
Penrose diagram:

Anti-de Sitter Space

\[ \rho = \infty \]

\[ \tau = 0 \]

\[ \tau = \pi \]

\[ \tau = -\pi \]

AdS is NOT globally hyperbolic. Boundary conditions are important.

Spatial (and null) infinity is timelike surface.
Anti-de Sitter Space

Maximally symmetric space with negative curvature, defined as (universal covering of) hyperboloid in $\mathbb{R}^{4,2}$:

$$-X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 - X_5^2 = -L^2$$

Poincaré (or horospheric) coords: $0 \leq r, \quad -\infty < t, \bar{x} < \infty$

$$X_0 = \frac{L^2}{2r} \left( 1 + \frac{r^2}{L^4} \left( L^2 + \bar{x}^2 - t^2 \right) \right), \quad \bar{X} = \frac{r}{L} \bar{x}$$

$$X_4 = \frac{L^2}{2r} \left( 1 - \frac{r^2}{L^4} \left( L^2 - \bar{x}^2 + t^2 \right) \right), \quad X_5 = \frac{r}{L} t$$

$$\Rightarrow \quad ds^2 = \frac{r^2}{L^2} \left( -dt^2 + d\bar{x}^2 \right) + \frac{L^2}{r^2} dr^2$$
Penrose diagram:

Anti-de Sitter Space

Constant r observers share *acceleration horizon* at r=0 (with T=0)

Poincaré coords. cover only a portion of AdS

\[ r = 0 \]
\[ r = \infty \]
\[ t = \infty \]
\[ t = -\infty \]
Anti-de Sitter Space

From now on, will depict Poincaré patch like this:

\[ ds^2 = \left(\frac{r}{L}\right)^2 (-dt^2 + d\vec{x}^2) + \left(\frac{L}{r}\right)^2 dr^2 \]
Anti-de Sitter Space

Or, equivalently:

\[ ds^2 = \left( \frac{L}{z} \right)^2 (-dt^2 + d\vec{x}^2 + dz^2) \]
String Theory on AdS

Stringy fluctuations of this background can be small

\[ ds^2 = \left( \frac{L}{z} \right)^2 (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2) \]
String Theory on AdS

Stringy fluctuations of this background can be small or large

\[ ds^2 = \left( \frac{L}{z} \right)^2 \left( g_{\mu\nu}(x, z) dx^\mu dx^\nu + dz^2 \right) \]
AdS/CFT Correspondence

$SU(N_c) \ \mathcal{N}=4 \ \text{SYM} \ \overset{\text{[Maldacena]}}{=} \ \text{IIB String Theory on}\ \text{asymptotically} \ \text{AdS}_5 \times S^5$

Geometry on RHS is dynamical:

Pure $\text{AdS}_5$ spacetime corresponds to SYM vacuum

Excitations on top of $\text{AdS}_5$ correspond to other SYM states

e.g., black hole in $\text{AdS}_5$ corresponds to finite temperature

$$ds_{\text{SchwAdS}}^2 = \left( \frac{L}{z} \right)^2 \left[ -\left( 1 - \frac{z^4}{z_h^4} \right) dt^2 + d\vec{x}^2 \right] + \frac{dz^2}{\left( 1 - \frac{z^4}{z_h^4} \right)}$$
**AdS/CFT Dictionary**

\[ SU(N_c) \mathcal{N} = 4 \text{ SYM} \quad \cong \quad \text{IIB String Theory on (planar)} \quad \text{[Maldacena]} \]

\[ \text{SchwAdS}_5(L, z_h) \times S^5(L) \]

Gluon (+ adjoint scalar & fermion) plasma

Can deduce this either by repeating AdS/CFT derivation starting w/ *near*-extremal black 3-brane, OR by using AdS/CFT dictionary to reconstruct geometry dual to thermal plasma

Black hole (brane) in AdS
AdS/CFT Dictionary

\[ SU(N_c) \mathcal{N} = 4 \quad \text{SYM} \equiv \quad \text{IIB String Theory on } a\text{AdS}_5 \times S^5 \]

\[ D = 3+1: \quad (t, \bar{x}) \]

\[ D = 9+1: \quad (t, \bar{x}, r; \theta_1, \ldots, \theta_5) \]
AdS/CFT Dictionary

$SU(N_c) \mathcal{N} = 4$ SYM $\equiv$ IIB String Theory on $a\text{AdS}_5 \times S^5$

Internal space $= \theta_1, \ldots, \theta_5$
AdS/CFT Dictionary

$SU(N_c) \mathcal{N} = 4 \; \text{SYM} \equiv \text{IIB String Theory on } a\text{AdS}_5 \times S^5$

Conformal group $SO(4, 2) = SO(4, 2)$ Isometries $\text{AdS}_5$

in particular, dilatation

$$(t, \vec{x}) \rightarrow (st, s\vec{x}) \quad \leftrightarrow \quad (t, \vec{x}, z) \rightarrow (st, s\vec{x}, sz)$$

So $z$ scales like a length,

$$r = \frac{L^2}{z} \quad \text{scales like an energy...}$$
AdS/CFT Dictionary

$SU(N_c) \ N = 4 \quad \text{SYM} \equiv \text{IIB String Theory on } a\text{AdS}_5 \times S^5$

Energy scale $E = r / L^2$  

[Susskind, Witten; Polchinski, Peet]

Diagram:
- AdS$_5$($L$)
- UV region
- IR region
- Energy scale $r$
AdS/CFT Dictionary

$SU(N_c) \mathcal{N} = 4$ SYM $\equiv$ IIB String Theory on $a\text{AdS}_5 \times S^5$

\[
\frac{g_{YM}^2}{g_s} = 4\pi g_s
\]

\[
N_c = \text{Units of } F_{(5)} \text{ flux through } S^5
\]

't Hooft coupling $\lambda \equiv g_{YM}^2 N_c = L^4 / l_s^4$

NOTE: RHS is under calculational control only if spacetime is weakly curved and strings are weakly coupled

\[
\Rightarrow g_{YM}^2 N_c \gg 1, \quad N_c \gg 1
\]

i.e., when LHS is strongly coupled

Easiest: $N_c \rightarrow \infty$, $g_{YM}^2 N_c \rightarrow \infty$

With more effort: $1 / g_{YM}^2 N_c$ corrections
AdS/CFT Dictionary

\( SU(N_c) \ N = 4 \ \text{SYM} \equiv \text{IIB String Theory on } a\text{AdS}_5 \times S^5 \)

Conformal group \( SO(4, 2) = SO(4, 2) \ \text{Isometries } \text{AdS}_5 \)

Internal symmetry \( SO(6) = SO(6) \ \text{Isometries } S^5 \)

Gauge group \( SU(N_c) \leftrightarrow \text{Nothing !} \)

Gauge-invt. operator \( O(x) \leftrightarrow \phi(x, r) \ \text{Field } (S^5 \text{ harmonic}) \)

[Gubser,Klebanov,Polyakov;Witten]
Black Branes vs. D-branes

\[ \phi(x, r) \]

Black \( p \)-brane

\[ \phi(x, r) \]

\[ \{ \text{Polchinski} \} \]

\[ O(x) \]

\[ N \text{ Dp-branes} + \text{Flat Spacetime} \]
AdS/CFT Dictionary

$SU(N_c) \ \mathcal{N}=4 \ \text{SYM} = \ \text{IIB String Theory on aAdS}_5 \times S^5$

Conformal group $SO(4,2) = SO(4,2)$ Isometries AdS$_5$

Internal symmetry $SO(6) = SO(6)$ Isometries $S^5$

Gauge group $SU(N_c) \leftrightarrow$ Nothing!

Gauge-invt. operator $O(x) \leftrightarrow \phi(x,r)$ Field ($S^5$ harmonic)

\[ \text{e.g., } \text{Tr}[F^2(x) + \ldots] \leftrightarrow \phi(x,r) \text{ dilaton (s wave)} \]

\[ T_{\mu \nu}(x) \leftrightarrow h_{\mu \nu}(x,r) \text{ graviton} \]

Find perfect match for all supergravity modes [Witten]
AdS/CFT Dictionary

Black p-brane

\(\phi(x, r)\)

[Polchinski]

N Dp-branes + Flat Spacetime

\(\phi(x, r)\)
AdS/CFT Dictionary

\( SU(N_c) \, \mathcal{N} = 4 \, \text{SYM} \equiv \text{IIB String Theory on aAdS}_5 \times S^5 \)

Gauge-inv. operator \( O(x) \leftrightarrow \phi(x,z) \) Field (\( S^5 \) harmonic)

\[
\int D(\text{SYM}) \exp \left[ iS_{\text{SYM}} + i \int d^4x \, O(x) J(x) \right] = \int D(\text{ST}) \exp \left[ iS_{\text{ST}} \right]
\]

[Gubser, Klebanov, Polyakov; Witten] \( \phi(x,z=0) = z^{4-\Delta} J(x) \)

In \( N_c \to \infty \), \( g_{YM}^2 N_c \to \infty \) limit, RHS simplifies to

\[
\int D(\text{ST}) \exp \left[ -S_{\text{ST}} \right] = \exp \left[ -S_{\text{SUGRA}}^{\text{on-shell}} \right]
\]

\( 1/g_{YM}^2 N_c \) corrections: higher derivative terms

\( 5 \times 5a \text{AdS}_5 \times S^5 \)

\( c \times \text{SU}_N \)
AdS/CFT Dictionary

\[ SU(N_c) \mathcal{N} = 4 \text{ SYM} \equiv \text{IIB String Theory on } a\text{AdS}_5 \times S^5 \]

Gauge-invt. operator \( O(x) \leftrightarrow \phi(x, z) \) Field (\( S^5 \) harmonic)

\[
\int D(\text{SYM}) \exp \left[ iS_{\text{SYM}} + i \int d^4 x O(x) J(x) \right] = \exp \left[ iS_{\text{on-shell}}^{\text{SUGRA}} \right]
\]

[Gubser,Klebanov,Polvakov;Witten] \( \phi(x,z=0) = z^{4-\Delta} J(x) \)

with \( \Delta = 2 + \sqrt{4 + m^2 L^2} \) the \textbf{conformal dimension} of the dual operator: \( O(x) \rightarrow s^\Delta O(sx) \) under dilatations

\[
\phi(x, z) = z^{4-\Delta} J(x) + z^{\Delta} \left< O(x) \right> + \ldots
\]

External source: determines THEORY

[Balasubramanian,Kraus,Lawrence]

Expectation value: determines STATE
AdS/CFT Dictionary

\[ SU(N_c) \mathcal{N} = 4 \text{ SYM} \equiv \text{IIB String Theory on (planar) } \text{SchwAdS}_5(L, z_h) \times S^3(L) \]

Gluon (+ adjoint scalar & fermion) plasma \equiv \text{Black hole (brane) in AdS}
AdS/CFT Dictionary

$SU(N_c) \ N = 4 \ \text{SYM} \equiv \text{IIB ST on SchwAdS}_5 \times S^5$

\[ T = \frac{r_h}{\pi L^2} = \frac{1}{\pi z_h} = T_H \]  

[Witten]
Entropy of SYM Plasma

\[ S_{\text{plasma}} = S_{\text{BH}} = \frac{A_H}{4G_N} \]
Entropy of SYM Plasma

\[ g_{YM}^2 N \ll 1 \Rightarrow \]
\[ S_{\text{plasma}} = \frac{2\pi^2}{3} N^2 T^3 V \]

\[ g_{YM}^2 N \gg 1 \Rightarrow \]
\[ S_{\text{BH}} = \frac{2\pi^2}{3} N^2 T^3 V \left( \frac{3}{4} \right) \]

[Gubser, Klebanov, Peet]
Entropy of SYM Plasma

\[ S_{\text{plasma}} = \frac{2\pi^2}{3} N^2 T^3 V \left( 1 - \frac{3g_{YM}^2 N}{4\pi^2} + \cdots \right) \]

\[ S_{\text{BH}} = \frac{2\pi^2}{3} N^2 T^3 V \left( \frac{3}{4} + \frac{45\zeta(3)}{64\sqrt{2} \left( g_{YM}^2 N \right)^{3/2}} + \cdots \right) \]

[Glubser, Klebanov, Peet; Fotopoulos, Taylor]

[Glubser, Klebanov, Peet; Gubser, Klebanov, Tseytlin]
Entropy of SYM Plasma

\[ S_{\text{plasma}} = \frac{2\pi^2}{3} N^2 T^3 V \left( 1 - \frac{3g_{YM}^2 N}{4\pi^2} + \cdots \right) \]

\[ S_{\text{BH}} = \frac{2\pi^2}{3} N^2 T^3 V \left( \frac{3}{4} + \frac{45\zeta(3)}{64\sqrt{2} \left( g_{YM}^2 N \right)^{3/2}} + \cdots \right) \]

\[ \frac{S(g_{YM}^2 N_c = \infty)}{S(g_{YM}^2 N_c = 0)} = 0.75 \quad \text{close to lattice QCD @} \quad T \sim (1 - 4)T_c \]
Lattice QCD

Deconfinement transition with $T_c \approx 170 - 190 \text{ MeV}$

Energy and entropy densities $\sim 0.80 - 0.85$ of ideal gas

From: F. Karsch, hep-lat/0106019
Shear Viscosity of SYM Plasma

\[
\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int d^4x e^{i\omega t} \left\langle \left[ T_{xy}(x), T_{xy}(0) \right] \right\rangle = \lim_{\omega \to 0} \frac{1}{16\pi G_N} \sigma_{h_{\mu\nu}}(\omega)
\]

[Kubo]

[Callan; Gubser, Klebanov, Polyakov; Witten]
Shear Viscosity of SYM Plasma

\[
\eta \sim \frac{1}{s (g_{YM}^2 N_c)^2 \log(1/ g_{YM}^2 N_c)} \gg 1
\]

[Arnold, Moore, Yaffe]

Universal: same for all gauge theories with gravity dual!!

\[
\eta = \frac{1}{s} + \frac{135 \zeta(3)}{64 \sqrt{2} \pi (g_{YM}^2 N_c)^{3/2}} + \ldots < 1
\]

[Policastro, Son, Starinets; Buchel, Liu, Starinets]
Shear Viscosity of SYM Plasma

\[
\eta \sim \frac{1}{s (g_{YM}^2 N_c)^2 \log(1/g_{YM}^2 N_c)} \gg 1 \quad \Rightarrow \quad \text{[Arnold, Moore, Yaffe]}
\]

\[
g_{YM}^2 N \ll 1 \quad \Rightarrow \quad \eta = \frac{1}{4\pi} + \frac{135\zeta(3)}{64\sqrt{2}\pi (g_{YM}^2 N_c)^{3/2}} + \ldots < 1
\]

\[
\text{[Policastro, Son, Starinets; Buchel, Liu, Starinets]}
\]

... and, close to \(~0.1\text{-}0.2\) at RHIC!
Other Transport Coefficients

Besides the shear viscosity, there exist AdS/CFT calculations of bulk viscosity, charge diffusion constant, electrical conductivity, thermal conductivity, etc. of SYM and other plasmas.

(Can also obtain these from quasinormal modes of BH, or via fluid/gravity correspondence {V. Hubeny})

[Policastro, Son, Starinets; Buchel, Liu, Starinets; Buchel; Parnachev, Starinets; Kovtun, Starinets; Benincasa, Buchel, Starinets; Janik, Peschanski; Mas; Son, Starinets; Saremi; Buchel; Maeda, Natsuume, Okamura; Benincasa, Buchel; Benincasa, Buchel, Naryshkin; Janik; Mateos, Myers, Thomson; Liao, Shuryak; Bak, Janik; etc.]
Overall Plan

- **L1:** Motivation, String Theory Basics
- **L2:** AdS/CFT ‘Derivation’ & Dictionary, Entropy, Viscosity
- **L3:** Energy Loss, Brownian, Unruh, Damping,
- “L4”: Mesons, Screening, Limiting Velocity, Generalizations of AdS/CFT (Gauge/Gravity)

Plan for Lecture 3

- Adding Quarks
- Energy Loss
- Brownian Motion, Unruh, Damping
Adding Quarks to SYM

Recall: before taking low energy (Maldacena) limit, had

\[
\begin{align*}
\left( A_\mu \right)_{IJ}, \left( \Phi^i \right)_{IJ} + \text{fermions} + \text{etc.} \quad & I, J = 1, \ldots, N_c \\
E \ll 1/l_s \\
\Rightarrow \quad & \mathcal{N} = 4 \quad U(N) \quad \text{Super-Yang-Mills in 3+1 dim}
\end{align*}
\]

[Witten]
Adding Quarks to SYM

Now introduce additional branes,

\[ N_c \text{ D3-branes} \]

\[ \text{fermions} + \text{etc.} \]

\[ I = 1, \ldots, N_c \]

\[ x^1, x^2, x^3 \]

\[ N_f \text{ D7-branes} \]

\[ x^4, x^5, x^6, x^7 \]

\( E \ll 1/l_s \)

\[ \mathcal{N} = 4 \ U(N) \] Super-Yang-Mills in 3+1 dim

\[ + N_f \] sets of 4 scalars + 2 spinors (\( \mathcal{N} = 2 \) hyperm.)

in \textbf{fundamental} rep of gauge group: “quarks”

[Karch,Katz]
Adding Quarks to SYM

In low-energy limit, dual description replaces D3s with AdS.

If $N_f \ll N_c$, can disregard backreaction of D7s on AdS geometry (= `quenched’ approx. of lattice QCD).

[Karch, Katz]

$\text{AdS}_5 \times S^5$

$S^3 \times S^5 \{x^1, x^2, x^3\}$
Adding Quarks to SYM

\[ \text{SU}(N_c) \, \mathcal{N} = 4 \, \text{SYM} \]
\[ + N_f \ll N_c \text{ flavors of } (\mathcal{N} = 2) \text{ fundamental rep} \]

\[ = \text{IIB ST on AdS}_5 \times S^5 \]
\[ + N_f \text{ D7-branes} \text{ (wrapped on } S^3 \subset S^5 \text{)} \]

[Karch, Katz]
Adding Quarks to SYM

\[ SU(N_c) \, \mathcal{N} = 4 \, \text{SYM} + N_f \ll N_c \, \text{flavors of (} \mathcal{N} = 2 \text{) matter in fundamental rep} \]

\[ = \text{IIB ST on } \text{AdS}_5 \times S^5 + N_f \, \text{D7-branes (wrapped on } S^3 \subset S^5 ) \]

\[ m = \frac{r_m}{2\pi l_s^2} = \sqrt{g_{YM}^2 N_c} \]

\[ = \frac{r_m}{2\pi z_m} \]
Adding Quarks to SYM

Finite-mass Quark \[=\] String w/endpoint at \( r_m < \infty \)
Adding Quarks to SYM

Infinitely heavy Quark

\[ \langle \text{Tr} F^2 \rangle, \langle T_{\mu\nu} \rangle, \ldots \]

\[ = \]

String w/endpoint at \( r \rightarrow \infty \)

E.g.,

\[ \langle \text{Tr} \left[ F^2(\vec{x}, t) + \ldots \right] \rangle_q = \frac{\sqrt{g_{YM}^2 N_c}}{32\pi^2 \left| \vec{x} \right|^4} \]

[Danielsson, Kruczenski, Keski-Vakkuri]

Coulombic profile (as expected by conformal invariance)
Quark and Antiquark superposed

\[ \text{Quark and Antiquark} \quad \equiv \quad \text{2 Strings w/opposite orientations} \]
Quark-Antiquark Potential

\[
\left\langle \text{Tr} \left[ F^2 (\vec{x}, t) + \ldots \right] \right\rangle_{q\bar{q}} \bigg|_{|\vec{x}| \gg L} = \frac{15 \Gamma \left( \frac{1}{4} \right)^4 \sqrt{g_{YM}^2 N_c}}{8 (2\pi)^5} \frac{L^3}{|\vec{x}|^7} \quad \text{(cf.} \frac{L^2}{|\vec{x}|^6})
\]

[Callan,AG] [Klebanov, Maldacena, Thorn]
Quark-Antiquark Potential

\[
V_{q\bar{q}}^{T=0}(L) = -\frac{4\pi^2 \sqrt{g_{YM}^2 N}}{\Gamma\left(\frac{1}{4}\right)^4 L} [\text{Rey, Yee; Maldacena}]
\]

Quark-Antiquark \quad \equiv \quad 1 \text{ String w/BOTH endpoints at } r \rightarrow \infty
Quark-Antiquark Potential

Quark-Antiquark $\equiv$ 1 String w/BOTH endpoints at $r \to \infty$

Endpoints $\leftrightarrow$ Quarks, String $\leftrightarrow$ Gluonic field

I.e., ‘QCD string’ really lives in 5 (+5) dimensions!!
Quark-Antiquark Potential

Quark-Antiquark $\equiv$ 1 String w/BOTH endpoints at $r \rightarrow \infty$

$V_{qq}^{T=0} (L) \propto L$ [Witten; Sonnenschein et al.; etc.]

In **confining** theories, the string is prevented from penetrating arbitrarily far into the bulk, and we then reproduce expected linear potential
Adding Quarks to SYM

Finite-mass Quark \[ \equiv \] String w/endpoint at \( r_m < \infty \)
Adding Quarks to SYM

Finite mass quark is automatically “dressed” or “composite”:

width of ‘gluon cloud’ (i.e., Compton wavelength)

\[ \sqrt{g_{YM}^2 N_c / 2\pi m} = z_m \]

\[ \langle \text{Tr} \left[ F^2(\vec{x},t) + \ldots \right] \rangle_q = \frac{\sqrt{g_{YM}^2 N_c}}{32\pi^2 |\vec{x}|^4} \left( 1 + \frac{5}{2} \left( \frac{2\pi m |\vec{x}|}{\sqrt{g_{YM}^2 N_c}} \right)^2 \right)^{5/2} \]

[Hovdebo, Kruczenski, Mateos, Myers, Winters]
Adding Quarks to SYM

Notice we are coupling 2nd-quantized gluonic (+etc.) fields to 1st-quantized quark:

\[ \int Dx(\tau) DA_\mu(x') \exp \left(i S[A(x'), x(\tau)]\right) \]

Integral over \( A_\mu(x') \) is done EXACTLY (with AdS) but that over \( x^\mu(\tau) \) is treated in saddle point approx.

(later today: some semiclassical corrections, \( \sim \lambda^{-1/2} \))
Energy Loss: Heavy Quark

RHIC finds significant energy loss for partons crossing the QGP ("jet quenching"):

How much energy is lost by quark? Where does this energy go?
Energy Loss: Heavy Quark

Static quark in vacuum $\equiv$ Vertical string on pure AdS
Energy Loss: Heavy Quark

Quark with constant $\nu$  $\equiv$  Vertical string at constant $\nu$

No energy loss

The situation changes if:
Quark accelerates
or/and
Quark is placed inside a medium

consider this first
Energy Loss: Heavy Quark

Heavy quark in thermal SYM plasma \( (m \gg T) \)  

\[ \implies \]  

String extending on Schw-AdS from \( z = z_m \) to \( z = z_h \gg z_m \)
String e.o.m. follows from std. Nambu-Goto action

\[ S_{NG} = -\frac{1}{2\pi l_s^2} \int d^2\sigma \sqrt{-\gamma} = -\frac{1}{2\pi l_s^2} \int d^2\sigma \sqrt{-\text{det}\left(G_{mn}\partial_a X^m \partial_b X^n\right)} \]

\[ = -\frac{1}{2\pi l_s^2} \int dt dz \sqrt{\left(\dot{X}(z,t) \cdot X'(z,t)\right)^2 - \dot{X}(z,t)^2 X'(z,t)^2} \]

\[ \dot{X}^m \equiv \partial_t X^m, \quad X'^m \equiv \partial_z X^m, \quad \ddot{X} \cdot X' \equiv G_{mn} \dot{X}^m X'^n, \quad \text{etc.} \]

on the Schw-AdS geometry

\[ ds^2_{\text{SchwAdS}} = \left(\frac{L}{z}\right)^2 \left[ \left(-h \, dt^2 + d\bar{x}^2\right) + \frac{dz^2}{h} \right], \quad h(z) \equiv 1 - \frac{z^4}{z_h^4} \]

E.o.m. state that conjugate momentum densities are conserved currents (b/c of translation invariance),

\[ \Pi_i^a \equiv \frac{\partial L_{NG}}{\partial \left(\partial_a X^i\right)}, \quad \partial_t \Pi_i^t + \partial_z \Pi_i^z = 0 \quad \Rightarrow \quad \Pi_i^z = \text{const.} \]

for stationary configurations
Energy Loss: Heavy Quark

Stationary solution has

and takes the form

\[ X(z, t) = v \left[ t - \frac{z_h}{4} \ln \left( \frac{z_h + z}{z_h - z} \right) + \frac{z_h}{2} \tan^{-1} \left( \frac{z}{z_h} \right) \right] \]

[Herzog, Karch, Kovtun, Kozcaz, Yaffe; Gubser]

(Related work: [Casalderrey-Solana, Teaney; Liu, Rajagopal, Wiedemann])
Energy Loss: Heavy Quark

Rates at which momentum and energy flow along string (supplied by external force) are given by

\[
\Pi_x^z = -\frac{\pi \sqrt{\lambda} T^2}{2} \frac{v}{\sqrt{1-v^2}} \equiv \frac{dp_x}{dt}, \quad \Pi_t^z = -\frac{\pi \sqrt{\lambda} T^2}{2} \frac{v^2}{\sqrt{1-v^2}} \equiv \frac{dE}{dt}
\]

Drag force on quark (rate of momentum loss)  
Rate of energy loss for quark
Energy Loss: Heavy Quark

\[ p_x(t) = p_x(0) \exp\left(-\frac{t}{t_r}\right) \]

\[ t_r = \frac{2m}{\pi \sqrt{g_{YM}^2 NT^2}} \]

E.g.,
\[ t_r \text{ (charm)} \approx 0.6 - 2.1 \text{ fm/c} \]

cf. pQCD
\[ t_r \text{ (charm)} \approx 4 - 12 \text{ fm/c} \]

[van Hees, Rapp]

[van Hees, Rapp]
Energy Loss: Meson and Baryon

Meson in plasma feels NO drag force, as a result of its being color-neutral
[Peeters, Sonnenschein, Zamaklar; Liu, Rajagopal, Wiedemann; Chernicoff, García, AG]

Baryon (=D5-brane) similarly feels no drag
[Witten; Brandhuber et al.; Callan, AG, Savvidy]

Drag does appear in $1/N_c^2$ corrections
[Dusling, Erdmenger, Kaminski, Rust, Teaney, Young]
Energy Loss: Heavy Quark

Can generalize calculation to **accelerated** quark,

[Chernicoff,AG]

and to somewhat more realistic case where quark
is created within the plasma...

[Herzog,Karch,Kovtun,Kozcaz,Yaffe; Chernicoff,AG]
Energy Loss: Pair Creation

**Singlet** back-to-back quark-antiquark pair [Herzog, Karch, Kovtun, Kozcaz, Yaffe]

Initial velocity fixed at $v_{\text{max}}$; ‘start feeling the plasma’ according to stationary formula when q-qbar separation reaches (v-dependent) screening length [Chernicoff, AG; related work: Hatta, Iancu, Mueller]

U-shaped string with initially coincident endpoints
Energy Loss: Spatial Distribution

Can determine the spatial profile of dissipated energy from

\[ \langle T_{\mu \nu} (x) \rangle_{q,v} \leftrightarrow h_{\mu \nu} (x, r = \infty) \]

[Friess,Gubser,Michalogiorgakis; Friess,Gubser,Michalogiorgakis,Pufu; Yarom; Gubser,Pufu; Gubser,Pufu,Yarom; Chesler,Yaffe; Noronha,Torrieri,Gyulassy; Betz,Gyulassy,Noronha,Torrieri; etc.]
Energy Loss: Spatial Distribution

Energy density in wake generated by the quark
[Guβer, Pufu, Yarom; Chesler, Yaffe]

\[ \nu = 0.25 < \nu_s = 1/\sqrt{3} \]

From: Chesler, Yaffe, arXiv:0706.0368
Energy Loss: Spatial Distribution

Energy density in wake generated by the quark

\[ \nu = 0.75 > \nu_s \]

\[ \theta_M = \sin^{-1} \left( \frac{\nu_s}{\nu} \right) \approx 50^\circ \]

Gives credence to sonic shock wave proposal

From: Chesler, Yaffe, arXiv:0706.0368
Energy Loss: Spatial Distribution

Energy density in wake generated by the quark
[Gubser,Pufu,Yarom; Chesler,Yaffe]

More generally, behavior far from quark is in accord with linearized hydrodynamics (see {V. Hubeny} for nonlinear hydro)

From: Chesler,Yaffe, arXiv:0706.0368
Energy Loss: Spatial Distribution

From: Chesler, Yaffe, arXiv:0712.0050

There is a “neck” region close to quark where hydro is not applicable, but AdS/CFT technology is, and gives interesting result...
Energy Loss: Spatial Distribution

Recall energy loss ("jet quenching") setup at RHIC/LHC:
Energy Loss: Spatial Distribution

Recall energy loss ("jet quenching") setup at RHIC/LHC:

From: Torrieri, Betz, Noronha, Gyulassy, arXiv:0901.0230
AdS/CFT hydro region alone (Mach cone + diffusion wake) does not emulate this, but "neck" region does...
Energy Loss Distribution

Result from perturbative QCD vs. AdS/CFT:

From: Betz, Gyulassy, Noronha, Torrieri arXiv:0807.4526

(But, see 3-jet proposal by [Ayala, Jalilian-Marian, Magnin, Ortiz, Paic, Tejeda-Yeomans])
Brownian Motion

Static heavy quark in thermal $\mathcal{N} = 4$ SYM plasma

Would expect quark to undergo Brownian motion...

Static string from $z = z_m$ to $z = z_h$ on Schw-AdS geometry

How does this come about in string description?
String e.o.m. follows from std. Nambu-Goto action

\[ S_{\text{NG}} = -\frac{1}{2\pi l_s^2} \int dt dz \sqrt{-\gamma} = -\frac{1}{2\pi l_s^2} \int dt dz \sqrt{-\det(G_{mn} \partial_a X^m \partial_b X^n)} \]

on the Schw-AdS geometry

\[ ds^2_{\text{SchwAdS}} = \left(\frac{L}{z}\right)^2 \left[ (\gamma dt^2 + d\bar{x}^2) + \frac{dz^2}{h} \right], \quad h(z) \equiv 1 - \frac{z^4}{z_h^4} \]

Small fluctuations of embedding field around static configuration feel induced metric,

\[ \ddot{X}(z, t) = 0 + \delta \ddot{X}(z, t) \]

\[ S_{\text{NG}} = -\frac{1}{2\pi l_s^2} \int dt dz \sqrt{-\gamma} \left[ 1 + \gamma^{ab} G_{ij} \partial_a \delta X^i \partial_b \delta X^j + O(\delta X^2) \right] \]

i.e., they are free massless fields living on 2 dim black hole geometry
Expanding into modes and quantizing,

\[ X(t, z) = \sum_{\omega > 0} \left[ a_\omega u_\omega (t, z) + a_\omega^\dagger u_\omega^* (t, z) \right] \]

know that Hawking radiation emerging from the black hole will excite the modes of the string.

Semiclassically, these modes are thermally populated,

\[ \langle a_\omega^\dagger a_{\omega'} \rangle = \frac{\delta_{\omega\omega'}}{e^{\beta\omega} - 1} \]

from which one can determine correlators of the fluctuating position of the string endpoint (=quark)

\[ \langle x(t)x(0) \rangle \]

(Conversely, if we could compute this correlator in gauge theory, we could infer precise character of Hawking radiation!)
Brownian Motion

The position of the endpoint is found to obey a generalized Langevin equation

\[ m\ddot{x} + \int dt' \eta(t,t') \dot{x}(t') = \xi(t), \quad \langle \xi(t) \xi(t') \rangle = \kappa(t,t') \]

[de Boer, Hubeny, Rangamani, Shigemori; Son, Teaney]
Brownian Motion

The position of the endpoint is found to obey a generalized Langevin equation

\[ m\dddot{x} + \int dt' \eta(t,t') \dot{x}(t') = \xi(t), \quad \langle \xi(t)\xi(t') \rangle = \kappa(t,t') \]

[de Boer, Hubeny, Rangamani, Shigemori; Son, Teaney]
So, in the AdS/CFT context, Hawking = Brown!!
Can generalize to case of quark moving at constant velocity (=trailing stationary string), where horizon on induced metric on the worldsheet no longer coincides with horizon of bulk metric

\[ z_v = (1 - v^2)^{1/4} z_h \]

[Gubser; Casalderrey, Teaney; Giecold, Iancu, Mueller; Casalderrey, Teaney]
Fluctuations in Vacuum

At zero temperature, string embedding is known analytically for arbitrary string trajectory [Mikhailov]
For an accelerating (and therefore radiating) quark, an event horizon develops on string worldsheet [Chernicoff,AG]
At **zero** temperature, string embedding is known analytically for *arbitrary* string trajectory [Mikhailov]

For an accelerating (and therefore radiating) quark, an event horizon develops on string worldsheet [Chernicoff,AG].

Get quantum fluctuations about average quark trajectory induced by Hawking radiation in string theory description and by emitted radiation in gauge theory description.
Fluctuations in Vacuum

Can work this out explicitly for quark with uniform proper acceleration $A$

$z_h = \sqrt{A^{-2} + z_m^2}$

[Cáceres, Chernicoff, AG, Pedraza]

The calculation makes direct contact with previous story of Brownian motion in a thermal medium, as expected from Unruh effect...

(Related work: [Xiao; Hirayama, Kao, Kawamoto, Lin])
Changing to Rindler coordinates where quark is static, CFT metric takes the form

\[ ds_{\text{CFT}}^2 = e^{2Ax'} \left( -dt'^2 + dx'^2 \right) + d\vec{x}'_\perp^2 \]

Both in CFT and AdS, find acceleration horizon at

\[ x' = -\infty \quad \forall \, z', \vec{x}'_\perp \]
Unruh en AdS/CFT

Can remove horizon from CFT by via \textbf{Weyl transformation}

\[
\begin{align*}
  ds^2_{\text{CFT}} &= e^{2Ax'} \left( -dt'^2 + dx'^2 \right) + d\vec{x}'_\perp \\
  &\quad \rightarrow \quad -dt''^2 + dx''^2 + e^{-2Ax''} d\vec{x}_\perp^2
\end{align*}
\]

This corresponds to a change of radial foliation in the bulk

\{Kraus\}...

[Imbimbo, Schwimmer, Theisen, Yankielowicz]
Unruh in AdS/CFT

In this new presentation of the bulk, the string is vertical, and the bulk horizon lies at a fixed radius

\[ z_h'' = A^{-1} = \frac{2\pi}{T_U} \]

indicating that thermal medium is still present

Setup coincides \textbf{precisely} with that of (3-dim) Brownian motion calculation of [de Boer, Hubeny, Rangamani, Shigemori]!
Unruh in AdS/CFT

In this new presentation of the bulk, the string is vertical, and the bulk horizon lies at a fixed radius

$$z''_h = A^{-1} = 2\pi / T_U$$

indicating that thermal medium is still present

From the CFT perspective, medium now arises not from entanglement with region behind a horizon, but from direct application of a temperature (analogous to familiar Schw-AdS case)
Going back to the classical description of the quark/string, note that the question of energy loss in a strongly-coupled non-Abelian gauge theory is interesting already in vacuum (where AdS/CFT gives us full analytic control).

Expect accelerating quark to radiate, and experience damping force due to emitted radiation.
Radiation Damping

In classical E&M, Abraham-Lorentz equation (NR electron)

\[ m \left( \frac{d^2 \vec{x}}{dt^2} - t_e \frac{d^3 \vec{x}}{dt^3} \right) = \vec{F} \]

\[ t_e \equiv \frac{2e^2}{3mc^3} \sim \text{classical electron radius} \]

damping term

and (Abraham-)Lorentz-Dirac equation

\[ m \left( \frac{d^2 x^\mu}{d\tau^2} - t_e \left[ \frac{d^3 x^\mu}{d\tau^3} - \frac{1}{c^2} \frac{d^2 x_v}{d\tau^2} \frac{d^2 x^\nu}{d\tau^2} \frac{dx^\mu}{d\tau} \right] \right) = \mathcal{F}^\mu \]

Schott (near field) term  radiation reaction term
Radiation Damping

For **accelerating** quark, dual string trails behind endpoint, and acts as an energy sink.

i.e., quark has a ‘tail’, and it is this tail that is responsible for damping effect.

In fact, standard boundary condition for open string can be easily shown to take the form of an equation of motion for the quark...
Generalized Lorentz-Dirac Eqn

E.o.m. for dressed quark:

\[
\frac{d}{d\tau} \left( m \frac{dx^\mu}{d\tau} - \frac{\sqrt{\lambda}}{2\pi m} \mathcal{F}^\mu \right) = \frac{\mathcal{F}^\mu - \frac{\sqrt{\lambda}}{2\pi m^2} \mathcal{F}^2 \frac{dx^\mu}{d\tau}}{1 - \frac{\lambda}{4\pi^2 m^4} \mathcal{F}^2}
\]

[Chernicoff, García, AG]

Recall \( z_m = \frac{\sqrt{\lambda}}{2\pi m} \) plays the role of Compton wavelength
Generalized Lorentz-Dirac Eqn

E.o.m. for dressed quark:

\[
\frac{d}{d\tau} \left( m \frac{dx^\mu}{d\tau} - \frac{\sqrt{\lambda}}{2\pi m} \mathcal{F}^\mu \right) = \mathcal{F}^\mu - \frac{\sqrt{\lambda}}{2\pi m^2} \mathcal{F}^2 \frac{dx^\mu}{d\tau}
\]

When \( \frac{\sqrt{\lambda}}{2\pi m^2 \sqrt{\mathcal{F}^2}} \ll 1 \), can ignore denominators...

[Chernicoff, García, AG ]
Generalized Lorentz-Dirac Eqn

To zeroth order in \( \frac{\sqrt{\lambda}}{2\pi m^2 \sqrt{\mathcal{F}^2}} \ll 1 \), recover

\[ m \frac{d^2 x^\mu}{d\tau^2} \approx \mathcal{F}^\mu \]

To first order,

\[ \frac{d}{d\tau} \left( m \frac{dx^\mu}{d\tau} - \frac{\sqrt{\lambda}}{2\pi m} \mathcal{F}^\mu \right) \approx \mathcal{F}^\mu - \frac{\sqrt{\lambda}}{2\pi m^2} \mathcal{F}^2 \frac{dx^\mu}{d\tau} \]

which is equivalent to

\[ m \left( \frac{d^2 x^\mu}{d\tau^2} - \frac{\sqrt{\lambda}}{2\pi m} \frac{d^3 x^\mu}{d\tau^3} \right) \approx \mathcal{F}^\mu - \frac{\sqrt{\lambda}}{2\pi} \left( \frac{d^2 x^\nu}{d\tau^2} \frac{d^2 x_\nu}{d\tau^2} \right) \frac{dx^\mu}{d\tau} \]

Exact same structure as Lorentz-Dirac (with \( t_e \to z_m \))!
To second order in $\frac{\sqrt{\lambda}}{2\pi m^2} \sqrt{\mathcal{F}^2} \ll 1$, similarly obtain

$$m \frac{d^2 x^\mu}{d\tau^2} - \frac{\sqrt{\lambda}}{2\pi} \left( \frac{d^3 x^\mu}{d\tau^3} - \frac{d^2 x_\nu}{d\tau^2} \frac{d^2 x^\nu}{d\tau^2} \frac{dx^\mu}{d\tau} \right)$$

$$+ \frac{\lambda}{4\pi^2 m} \left( \frac{d^4 x^\mu}{d\tau^4} - (1+2) \frac{d^2 x_\nu}{d\tau^2} \frac{d^3 x^\nu}{d\tau^2} \frac{dx^\mu}{d\tau} \right)$$

$$- \frac{\lambda}{4\pi^2 m} \left( \frac{1}{2} + 1 \right) \frac{d^2 x_\nu}{d\tau^2} \frac{d^2 x^\nu}{d\tau^2} \frac{d^2 x^\mu}{d\tau^2} \approx \mathcal{F}^\mu$$
To second order in \( \frac{\sqrt{\lambda}}{2\pi m^2} \sqrt{\mathcal{F}^2} \ll 1 \), similarly obtain

\[
\begin{align*}
  m \frac{d^2 x^\mu}{d\tau^2} & - \frac{\sqrt{\lambda}}{2\pi} \left( \frac{d^3 x^\mu}{d\tau^3} - \frac{d^2 x_\nu}{d\tau^2} \frac{d^2 x^\nu}{d\tau^2} \frac{dx^\mu}{d\tau} \right) \\
  + \frac{\lambda}{4\pi^2 m} \left( \frac{d^4 x^\mu}{d\tau^4} - (1+2) \frac{d^2 x_\nu}{d\tau^2} \frac{d^3 x^\nu}{d\tau^2} \frac{dx^\mu}{d\tau} \right) \\
  - \frac{\lambda}{4\pi^2 m} \left( \frac{1}{2} + 1 \right) \frac{d^2 x_\nu}{d\tau^2} \frac{d^2 x^\nu}{d\tau^2} \frac{d^2 x^\mu}{d\tau^2} & \approx \mathcal{F}^\mu
\end{align*}
\]

radiation reaction terms
Generalized Lorentz-Dirac Eqn

To second order in \( \frac{\sqrt{\lambda}}{2\pi m^2} \sqrt{\mathcal{F}^2} \ll 1 \), similarly obtain

\[
\frac{m}{d\tau^2} \frac{d^2 x^\mu}{d\tau^2} - \frac{\sqrt{\lambda}}{2\pi} \left( \frac{d^3 x^\mu}{d\tau^3} - \frac{d^2 x_\nu}{d\tau^2} \frac{d^2 x^\nu}{d\tau^2} \frac{dx^\mu}{d\tau} \right) + \frac{\lambda}{4\pi^2 m} \left( \frac{d^4 x^\mu}{d\tau^4} - (1+2) \frac{d^2 x_\nu}{d\tau^2} \frac{d^3 x^\nu}{d\tau^3} \frac{dx^\mu}{d\tau} \right) - \frac{\lambda}{4\pi^2 m} \left( \frac{1}{2} + 1 \right) \frac{d^2 x_\nu}{d\tau^2} \frac{d^2 x^\nu}{d\tau^2} \frac{d^2 x^\mu}{d\tau^2} \approx \mathcal{F}^\mu
\]

near field terms
Generalized Lorentz-Dirac Eqn

Full e.o.m. for dressed quark

\[
\frac{d}{d\tau} \left( m \frac{dx^\mu}{d\tau} - \frac{\sqrt{\lambda}}{2\pi m} \mathcal{F}_\mu \right) = \frac{\mathcal{F}_\mu - \sqrt{\lambda} \mathcal{F}^2}{2\pi m^2} \frac{dx^\mu}{d\tau}
\]

is non-linear generalization of Lorentz-Dirac eqn

(And, unlike Lorentz-Dirac, has no self-accelerating solutions)
Wilson Loops

The quark=string entry leads to a recipe for computing Wilson loops \textit{(non-local gauge-invariant operators)}:

\[
\left\langle \text{Tr} \left[ P \exp \left( i \oint_C dx^\mu A_\mu (x) + \ldots \right) \right] \right\rangle_{\text{SYM}} = \int DX \exp \left[ i S_{\text{string}} [X] \right] \bigg|_{X(r = \infty) = C}
\]

[Rey,Yee; Maldacena; Drukker,Gross,Ooguri]
Wilson Loops

The quark=string entry leads to a recipe for computing Wilson loops (non-local gauge-invariant operators):

\[
\left\langle \text{Tr} \left[ P \exp \left( i \oint_C dx^\mu A_\mu (x) + \ldots \right) \right] \right\rangle_{\text{SYM}} = \int DX \exp \left[ i S_{\text{string}} [X] \right]

X (r = \infty) = C

= \exp \left[ i S_{\text{on-shell}}^{\text{string}} [X] \right]

\text{in } N_c \to \infty , \ g_{YM}^2 N_c \to \infty \ \text{limit}

(The trace in LHS above is taken in the fundamental rep of the gauge group. For traces in other reps, RHS involves strings bound to D3- or D5-branes)

[Drukker,Fiol; Hartnoll,Prem Kumar; Yamaguchi; Gomis,Passerini]
Quark-Antiquark Potential

Quark-Antiquark $\equiv$ 1 String w/BOTH endpoints at $r \to \infty$

$$V^{T=0}_{q\bar{q}}(L) = \frac{-4\pi^2 \sqrt{g_{YM}^2 N}}{\Gamma(\frac{1}{4})^4 L}$$

[Rey, Yee; Maldacena]
Screening from AdS/CFT

\[ V_{qq}^{T}(L) \left[ \sqrt{g_{YM}^{2}NT/4} \right] \]

\[ L \left[ \frac{1}{2\pi T} \right] \]

[Rey, Theisen, Yee; Brandhuber, Itzhaki, Sonnenschein, Yankielowicz]
[Bak, Karch, Yaffe]
Screening from AdS/CFT

Meson moving through SYM plasma

\[ V^{T}_{qq} (L, v) \]

U-shaped string moving on AdS black hole geometry

[Chernicoff, García, AG; Liu, Rajagopal, Wiedemann]
Screening Length

\[ L_s(T, v) \left[ \frac{1}{2\pi T} \right] \]

\[ \sim (1 - v^2)^{1/3} \]

\[ l_* = l_{max} \rightarrow (1 - v^2)^{1/4} \]

Possibly relevant for J/psi suppression [Matsui,Satz]

\[ T_{diss} \propto (1 - v^2)^p \]

(Suppression enhanced for charmonium w/larger \( p_T \))

[Liu,Rajagopal,Wiedemann; Chernicoff,García,AG]

[Liu,Rajagopal,Wiedemann; Cáceres,Natsuume,Okamura]
Meson Spectrum

Can also determine **microscopic** meson spectrum, e.g.

\[
M_s = \frac{2\pi m_q}{\sqrt{g_{YM}^2 N}} \sqrt{(n+l+1)(n+l+2)}
\]

for scalar mesons

[Kruczenski,Mateos,Myers,Winters]

Notice that \( M_s \ll m_q \): **mesons** are lightest d.o.f.

(Can in fact recover quark as a soliton made of mesons!)
Meson Spectrum at Finite $T$

$r_m$ related to quark mass $m$

$r_h$ proportional to temperature $T$

For large enough $T/m$, the D7-branes end INSIDE BH
Meson Spectrum at Finite $T$

View omitting SYM directions but including $S^5$:

Discrete meson spectrum

$$M_{\text{mes}} \sim T_{\text{fun}}$$
(stable: survive deconfinement)

+ Massive quarks

Continuous spectrum

with NO quasi-particles!!

First order phase transition at

$$T_{\text{fun}} \sim \frac{m_g}{\sqrt{g_{\text{YM}}^2 N}}$$

[Mateos, Myers, Thomson]
Meson Spectrum at Finite $T$

Meson dispersion relations at increasing $T < T_{\text{fun}}$:

[Mateos, Myers, Thomson; Ejaz, Faulkner, Liu, Rajagopal, Wiedemann]

**Limiting velocity less than 1!!**
Meson Spectrum at Finite $T$

Origin: local speed of light at edge of D7-branes,

$$v_{\text{max}} = \sqrt{1 - \left(\frac{z_m}{z_h}\right)^4}$$

$$\approx 1 - \left(\frac{\sqrt{\lambda}T}{m_q}\right)^4$$

[Argyres, Edalati, Vázquez-Poritz]
Generalizations

What we’ve discussed so far is the best understood example of more general \textbf{gauge/gravity (or gauge/string) correspondence} which can involve backgrounds w/different asymptotics.

Various other examples are known, relating certain gauge theories (some of which are more `QCD-like’) to string theory on different curved spacetimes...

[Sakai-Sugimoto(-Witten); Klebanov-Strassler; Maldacena-Núñez; Polchinski-Strassler; Freedman-Gubser-Pilch-Warner; etc.]
Other SYM Geometries

For starters: since SYM is conformal/Weyl invariant (up to Weyl anomaly), can change from Minkowski to a different (nondynamical) 3+1 geometry via Weyl transformation

$$\eta_{\mu\nu} \rightarrow g_{\mu\nu}(x) = e^{2\omega(x)}\eta_{\mu\nu}$$

On string side, this corresponds to a bulk diffeo \{Kraus\}

$$z \rightarrow z'' = e^{\omega(x)}z, \quad x^\mu \rightarrow x''^\mu(x, z)$$

which modifies the radial foliation

[Witten; Imbimbo, Schwimmer, Theisen, Yankielowicz]

SYM on Minkowski arises from Poincaré foliation of AdS, but different foliations of the same geometry can lead, e.g., to SYM on the closed or open Einstein static universe

[Maldacena; Witten; Emparan; Birmingham; Emparan, Johnson, Myers; etc.]
Other SYM Geometries

SYM on 3+1 Minkowski
Other SYM Geometries

SYM on $S^3 \times \mathbb{R}$
Other SYM Geometries

SYM on $S^3 \times \mathbb{R}$ at finite $T$ has confining/deconfining phase transition, which turns out to be dual to Hawking-Page transition in AdS!
Deconfined phase = large \textit{spherical} Schw-AdS BH

[Witten]

Weyl anomaly of SYM (quantum effect due to UV divergences) is perfectly reproduced by AdS/CFT prescription for energy-momentum tensor (classical effect due to IR divergences)!! {Kraus}

[Witten; Henningson,Skenderis; etc.]
Other AdS/CFT Examples

We got to AdS$_5 \times S^5$ by considering D3-branes on 9+1 Minkowski, $M^{9,1}$. If we start instead with D3-branes on certain other 9+1 geometries, we can arrive at Type IIB String Theory on AdS$_5 \times X^5$
finding this to be dual to a 3+1 dim theories distinct from SYM. These are CFTs (b/c of AdS factor) with fewer or no supersymmetries (b/c no longer have $S^5$)

It’s also possible to engineer CFTs in other dimensions by using string theory objects other than D3-branes,
e.g., $d = 2 \quad d = 3 \quad d = 4 \quad d = 6$
from D1-D5 M2 D3 M5

[Klebanov, Witten; etc.]

[Maldacena; etc.]
Gauge/Gravity Correspondence

We also know various ways to obtain “nonAdS/nonCFT” correspondences (again w/various amounts of susy):

Can consider D\(p\)-branes (or other branes) on Minkowski, for other values of \(p\)

[Itzhaki,Maldacena,Sonnenschein,Yankielowicz; Boonstra,Skenderis,Townsend;etc.]

Can deform known AdS/CFT examples adding relevant terms to the Lagrangian (=switching on non-normalizable part of dual bulk fields). In these cases CFT starting point is UV fixed point

[Polchinski-Strassler; Girardello,Petrini,Porrati,Zaffaroni; Freedman-Gubser-Pilch-Warner; etc.]

[Sakai-Sugimoto(-Witten); Klebanov-Strassler; Maldacena-Núñez; etc.]
Gauge/Gravity Correspondence

We also know various ways to obtain "nonAdS/nonCFT" correspondences (again w/various amounts of susy):

Consider D$p$-branes (or other branes), for other values of $p$. For 3+1 theory, can start w/ $p>3$ and compactify

[Itzhaki,Maldacena,Sonnenschein,Yankielowicz; Maldacena-Núñez; Kruczenski,Mateos,Myers,Winters; Sakai-Sugimoto(-Witten); etc.]

Deform known AdS/CFT examples adding IR-relevant terms to the Lagrangian (=switching on non-normalizable part of dual bulk fields).

Original CFT is then UV fixed point

[Polchinski-Strassler; Girardello,Petrini,Porrati,Zaffaroni; Pilch-Warner; Freedman-Gubser-Pilch-Warner; Johnson,Peet,Polchinski; etc.]

Deform previous setups to yield different asymptotics

[Klebanov-Strassler; etc.]
Among these generalizations we find QCD-like gauge theories, with confinement and chiral symmetry breaking.

What comes closest to QCD is the **Sakai-Sugimoto(-Witten) model**, which uses D4-branes + D8-branes + anti-D8-branes to obtain a non-CFT & non-SUSY theory with quarks, confinement and non-Abelian chiral symmetry breaking.

By dialing a continuous dimensionless parameter in this model, we would precisely obtain QCD.

(Sakai-Sugimoto is also connected to the (phenomenologically useful) Nambu-Jona-Lasinio model)

[Antonyan, Harvey, Jensen, Kutasov]
Gauge/Gravity Correspondence

Finite temperature studies show that, in the regime of SS where 4-fermion interactions become important, the confinement and chiral-symm-breaking transitions no longer coincide [Aharony, Sonnenschein, Yankielowicz; Parnachev, Sahakyan]

This separation has been later verified with lattice calculation [Sinclair]

Unfortunately, moving towards QCD takes us outside the supergravity approximation, so we lose ability to compute (conversely, in supergravity regime where we can calculate, SS is truly 4+1 dim at scale of $\Lambda_{\text{QCD}}$)
This is generic: in all cases we get QCD (or YM) + junk, and the latter CANNOT be ignored within supergravity description.

Basic problem is that UV region in QCD is asymptotically free, which translates into strongly-curved, and therefore highly stringy, geometry.

Another way to state the problem: validity of supergravity requires large separation between masses of spin J=0,1,2 vs. J>2 modes. QCD is not like that.
Gauge/Gravity Correspondence

Nevertheless, we can use the known examples as toy models to develop intuition. This is interesting and useful in its own right.

Armed with that, to move toward QCD we can:

* Build **phenomenological** models that attempt to bridge the gap between our toy models and real-world QCD,

* Search for appropriately **universal** quantities, which allow us to extrapolate to QCD

There’s similar very interesting current work on possible applications to a variety of condensed matter and atomic physics systems
Gauge/Gravity Correspondence

The approach we’ve discussed so far is “top-down”: it starts with a known string/brane construction, and therefore gives us some control over what the theory on each side of the correspondence should be.

One can also try to follow a “bottom-up” approach, where one starts with a gauge theory of interest and tries to guess a phenomenological realization in terms of a gravity dual (“AdS/QCD”).

[Polchinski, Strassler; Erlich, Katz, Son; Da Rold, Pomarol; Karch, Katz, Son, Stephanov; etc.]

... or, starts with a given geometry (not necessarily string-related) and study the properties of the putative field theory dual (dS/CFT, Kerr/CFT, AdS/CMT,...)

[Strominger; Strominger; Son; Balasubramanian, McGreevy; Kachru, Liu, Mulligan; Gubser; Hartnoll, Herzog, Horowitz; etc.]
Conclusions

1) AdS/CFT is an efficient tool for calculations in certain strongly-coupled gauge theories. It is an established theoretical tool, and already makes suggestions for phenomenological models.

2) The sQGP produced at RHIC/LHC, and some strongly-coupled condensed matter / atomic physics systems, appear to be the most promising sites for eventually obtaining firm experimental predictions from AdS/CFT (& string theory)...But a lot remains to be done!

3) AdS/CFT is also been used in the opposite direction, where it gives some interesting suggestions on the problem of quantum gravity. Key idea: Holography

[‘t Hooft; Susskind] (See, e.g. [Horowitz,Polchinski; Horowitz])
[Beresteen; Lin,Lunin,Maldacena; Mathur; Bena,Warner; Kraus,Larsen; Berenstein; Yamaguchi; Balasubramanian,de Boer,Jejjala,Simon; Grant et al.; Balasubramanian,Czech,Larjo,Marolf,Simon; Mandal; etc.]