I. When does this work?

II. Examples

We can continue to invert the Taylor's series expansion around

\[ w = f(z) \quad \text{and already} \quad f'(z_0) = \frac{1}{g'(w_0)} \]

Because we have a series inverse.

Note: Something interesting may happen if \( f(z_0) = 0 \) (e.g., \( a_0 \) is singular)

or \( f(z_0) = \infty \) (e.g., \( a_0 \) is singular)

So long as \( z_0 \) is regular \& \( f'(z_0) \neq 0 \)

\( f \) preserves the angles at which curves intersect at \( z_0 \).

Note: we already saw some aspects of this.

For \( w = f(z) \), recall that the lines of

Constant \( u \& v \) are orthogonal in the \( z \)-plane.

General Case: Consider two curves \( C_1, C_2 \) that intersect at \( z_0 \).

Consider \( C_2 \) near \( z_0 \).

Then \( w = f(z) + f'(z_0) (z - z_0) + o(\|z - z_0\|^2) \)

\[ \lim_{w \to w_0} f'(z_0) (z - z_0) + o(\|z - z_0\|^2) \]

Now near \( z_0 \) each curve \( C_1, C_2 \) may be approximated

by the straight (smooth) line through \( z_0 \). Any such line has

an equation \( z - 2z_1 = r e^{i\theta} \) \( \theta \) fixed

where \( \theta \) varies over \( (0, \theta) \)

\[ z_1 \cdot \frac{z - z_1}{z - 2z_1} = 1 \quad \Rightarrow \quad z = 2z_1 \quad + \quad \frac{r e^{i\theta}}{1 + \frac{2r e^{i\theta}}{z_1}} \]

\[ z_1 \cdot \frac{2z - 2z_1}{z - 2z_1} = 1 \quad \Rightarrow \quad z = 2z_1 \quad + \quad \frac{2r e^{i\theta}}{1 - \frac{2r e^{i\theta}}{z_1}} \]
So \( f(z) \): \( w = w_0 = (x_0, y_0, z_0) e^{i (0, 2\pi)} \)
\( f(z) \): \( w = w_0 = (x_0, y_0, z_0) e^{i (\theta, 2\pi)} \)

1. Local conformal maps: curves are
2. scale
3. same angle x
4. relative angle
5. \( \theta \), \( \phi \)

Any map that preserves local angles in this way is called Conformal.

I.e. \( f \) is a conformal map so long as \( f(0) \), \( f'(0) \), \( f''(0) \), \( f'''(0) \).

Note: \( \theta \) you want to produce new curves on change the angle at which curves intersect, either if a \( f'' \) must be singular.

**Quick Example**

Find the electric field outside a conductor bar at an angle \( \alpha \):