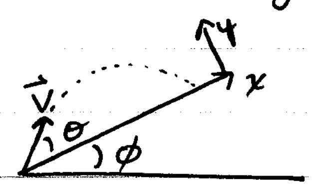
Phys. 105A Solution # 1

(1.19) 1) Since $\dot{\vec{r}} = \vec{v}$ and $\dot{\vec{v}} = \vec{a}$, we have

$$\frac{d}{dt}\left[\vec{a}\cdot(\vec{v}\times\vec{r})\right] = \dot{\vec{a}}\cdot(\vec{v}\times\vec{r}) + \vec{a}\cdot(\vec{a}\times\vec{r}) + \vec{a}\cdot(\vec{v}\times\vec{v})$$

But $\vec{v}\times\vec{v} = 0$, and $\vec{a}\times\vec{r}$ is orthogonal to $\vec{a}$

So only the first term remains.

(1.39) 2) 

Relative to the axes on the ~~inclide~~ inclined plane, the gravitational force has components

$$F_x = -mg\sin\phi$$
$$F_y = -mg\cos\phi$$

Notice that these are independent of the initial velocity of the ball & hence independent of θ.

The initial velocity has components

$$V_x = V_0\cos\theta \quad , \quad V_y = V_0\sin\theta$$

Solving $\vec{F} = m\vec{a}$ yields

$$x = v_x t + \tfrac{1}{2}a_x t^2 = v_0(\cos\theta)t - \tfrac{1}{2}g(\sin\phi)t^2$$

$$y = v_y t + \frac{1}{2} a_y t^2 = v_0 t \sin\theta - \frac{1}{2} g t^2 \cos\phi$$

The ball lands when $y=0 \Rightarrow t_0 = \dfrac{2 v_0 \sin\theta}{g \cos\phi}$

This corresponds to a distance

$$R = x(t_0) = \frac{2 v_0^2 \sin\theta \cos\theta}{g \cos\phi} - \frac{2 v_0^2 \sin^2\theta \sin\phi}{g \cos^2\phi}$$

$$= \frac{2 v_0^2 \sin\theta}{g \cos^2\phi} \left(\cos\theta \cos\phi - \sin\theta \sin\phi \right)$$

$$\Rightarrow \boxed{R = \frac{2 v_0^2 \sin\theta \cos(\theta+\phi)}{g \cos^2\phi}}$$

We now want to maximize this with respect to θ:

$$0 = \frac{d}{d\theta}\left[\sin\theta \cos(\theta+\phi)\right] = \cos\theta \cos(\theta+\phi) - \sin\theta \sin(\theta+\phi)$$

$$= \cos(2\theta+\phi)$$

$$\text{So } 2\theta + \phi = \frac{\pi}{2} \Rightarrow \theta = \frac{1}{2}\left(\frac{\pi}{2} - \phi\right)$$

Now $\sin\theta \cos(\theta+\phi) = \sin\frac{1}{2}\left(\frac{\pi}{2}-\phi\right) \cos\frac{1}{2}\left(\frac{\pi}{2}+\phi\right)$

$$= \sqrt{\frac{1-\cos\left(\frac{\pi}{2}-\phi\right)}{2}} \sqrt{\frac{1+\cos\left(\frac{\pi}{2}+\phi\right)}{2}}$$

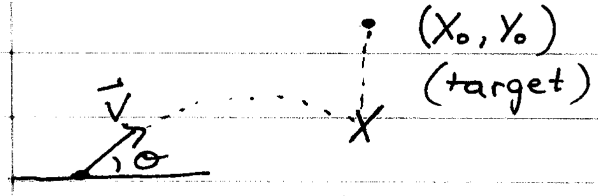
$$= \frac{1}{2}\left(1 - \sin\phi\right)$$

Since $\cos^2\phi = 1 - \sin^2\phi = (1-\sin\phi)(1+\sin\phi)$

We can write $R = \dfrac{(2 v_0^2)\frac{1}{2}(1-\sin\phi)}{g\cos^2\phi}$

as $\boxed{R = \dfrac{v_0^2}{g(1+\sin\phi)}}$

3)



In order for the projectile to hit the target, their y coordinates must be equal at the instant that the projectile reaches X_0. Thus

$v_{0x} t = X_0$ (no force in x direction)

$v_{0y} t - \frac{1}{2} g t^2 = Y_0 - \frac{1}{2} g t^2$ same ~~force~~ acceleration acts on both objects in y direction

$\Rightarrow$

$v_{0y} t = Y_0$

Thus $\boxed{\tan\theta = \dfrac{v_{0y}}{v_{0x}} = \dfrac{Y_0}{X_0}}$

You should aim where the target is at $t=0$!