Phys. 105A  
Solution #1

(1.19) 1) Since \( \ddot{r} = \ddot{v} \) and \( \ddot{v} = \ddot{a} \), we have

\[
\frac{d}{dt} [\ddot{a} \cdot (\dddot{v} \times \ddot{r})] = \dddot{a} \cdot (\dddot{v} \times \ddot{r}) + \ddot{a} \cdot (\dddot{a} \times \ddot{r}) + \dddot{a} \cdot (\dddot{v} \times \dddot{v})
\]

But \( \dddot{v} \times \dddot{v} = 0 \), and \( \dddot{a} \times \ddot{r} \) is orthogonal to \( \dddot{a} \)

So only the first term remains.

(1.39) 2) Relative to the axes on the inclined plane, the gravitational force has components:

\[
\begin{align*}
F_x &= -mg \sin \phi \\
F_y &= -mg \cos \phi
\end{align*}
\]

Notice that these are independent of the initial velocity of the ball and hence independent of \( \theta \).

The initial velocity has components:

\[
V_x = V_0 \cos \theta, \quad V_y = V_0 \sin \theta
\]

Solving \( F = ma \) yields:

\[
x = v_x t + \frac{1}{2} a_x t^2 = v_0 (\cos \theta) t - \frac{1}{2} g (\sin \phi) t^2
\]
\[ y = v_y t + \frac{1}{2} a_y t^2 = v_0 t \sin \theta - \frac{1}{2} g t^2 \cos \phi \]

The ball lands when \( y = 0 \Rightarrow t = \frac{2 v_0 \sin \theta}{g \cos \phi} \)

This corresponds to a distance

\[ R = x(t) = \frac{2 v_0^2 \sin \theta \cos \phi}{g \cos \phi} - \frac{2 v_0^2 \sin^2 \theta \sin \phi}{g \cos^2 \phi} \]

\[ = \frac{2 v_0^2 \sin \theta}{g \cos^2 \phi} \left( \cos \theta \cos \phi - \sin \theta \sin \phi \right) \]

\[ R = \frac{2 v_0^2 \sin \theta \cos (\theta + \phi)}{g \cos^2 \phi} \]

We now want to maximize this with respect to \( \theta \):

\[ \frac{\partial}{\partial \theta} \left[ \sin \theta \cos (\theta + \phi) \right] = \cos \theta \cos (\theta + \phi) - \sin \theta \sin (\theta + \phi) \]

\[ = \cos (2\theta + \phi) \]

So \( 2\theta + \phi = \frac{\pi}{2} \Rightarrow \theta = \frac{1}{2} \left( \frac{\pi}{2} - \phi \right) \)

Now \( \sin \theta \cos (\theta + \phi) = \sin \frac{1}{2} (\frac{\pi}{2} - \phi) \cos \frac{1}{2} (\frac{\pi}{2} + \phi) \)

\[ = \sqrt{1 - \cos (\frac{\pi}{2} - \phi)} \sqrt{1 + \cos (\frac{\pi}{2} + \phi)} \]

\[ = \frac{1}{2} \left( 1 - \sin \phi \right) \]
Since \( \cos^2 \phi = 1 - \sin^2 \phi = (1 - \sin \phi)(1 + \sin \phi) \)

We can write \( R = \frac{(2V_0^2) \frac{1}{2} (1 - \sin \phi)}{g \cos^2 \phi} \)

as \[ R = \frac{V_0^2}{g(1 + \sin \phi)} \]

3) \( (x_0, y_0) \)

(target)

\( \vec{v}_0 \)

\( X \)

In order for the projectile to hit the target, their \( y \) coordinates must be equal at the instant that the projectile reaches \( x_0 \). Thus \( V_{0x} t = x_0 \) (no force in \( x \) direction)

\[ V_{0y} t - \frac{1}{2} g t^2 = y_0 - \frac{1}{2} g t^2 \] same force acts on both objects in \( y \) direction

\[ v_{0y} t = y_0 \]

Thus \[ \tan \Theta = \frac{v_{0y}}{v_{0x}} = \frac{y_0}{x_0} \]

You should aim where the target is at \( t = 0 \)!