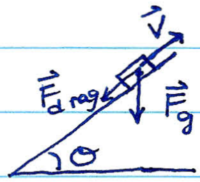
Phys 105A Solution #2

(1) 

The component of the gravitational force in the direction of the velocity (along the slope) is $-mg\sin\theta$. The drag force is $\vec{F}_{drag} = -b\vec{v}$. So

$$m\frac{dv}{dt} = -mg\sin\theta - bv \qquad (*)$$

$$\Rightarrow \underbrace{\int_{v_0}^{v} \frac{d\tilde{v}}{g\sin\theta + b\tilde{v}/m}}_{\frac{m}{b}\ln\left(g\sin\theta + \frac{b\tilde{v}}{m}\right)\Big|_{v_0}^{v}} = -\int_0^t d\tilde{t} = -t$$

Exponentiate: $$\frac{g\sin\theta + bv/m}{g\sin\theta + bv_0/m} = e^{-bt/m}$$

Solving for v: $$\boxed{v(t) = \frac{mg}{b}\sin\theta\left(e^{-bt/m} - 1\right) + v_0 e^{-bt/m}}$$

Note that we solved the equation of motion (*) using separation of variables. You can also do it by defining $u = v + \frac{mg}{b}\sin\theta$ and solving a simpler equation for u.

(2.8) (2) $$m\frac{dv}{dt} = F(v) = -c\,v^{3/2}$$

So $$\frac{dv}{v^{3/2}} = -\frac{c}{m}\,dt$$

Integrating both sides:

$$\frac{c}{m}t = -\int_{v_0}^{v}\frac{dv}{v^{3/2}} = \frac{2}{v^{1/2}}\Big|_{v_0}^{v} = \frac{2}{\sqrt{v}} - \frac{2}{\sqrt{v_0}}$$

Solving for $v(t)$: $$\frac{4}{v} = \left(\frac{ct}{m} + \frac{2}{\sqrt{v_0}}\right)^2$$

$$\Rightarrow \boxed{v(t) = \left(\frac{ct}{2m} + \frac{1}{\sqrt{v_0}}\right)^{-2}}$$

It takes an ∞ time for the mass to come to rest.

(2.11) 3. (a) $m\frac{dv_y}{dt} = -mg - bv$ Set $v_{ter} \equiv \frac{mg}{b}$

This was solved in class (and is very similar to the solution 2.29-2.31 in the text). The solution is:

$$\boxed{v_y(t) = (v_0 + v_{ter})\, e^{-bt/m} - v_{ter}}$$

We get the position by integrating & setting $y(0)=0$:

$$\boxed{y(t) = \frac{m}{b}(v_0 + v_{ter})\left(1 - e^{-bt/m}\right) - v_{ter}\, t}$$

(b) At the highest point, $v_y = 0$. This occurs at a time $\frac{v_0 + v_{ter}}{v_{ter}} = e^{bt/m}$

So

$$\boxed{t_{top} = \frac{m}{b}\ln\left(1 + \frac{v_0}{v_{ter}}\right)}$$

and

$y_{max} = y(t_{top})$. Using $v_y(t_{top}) = 0$, we have

$$\boxed{y_{max} = \frac{m}{b}\left[v_0 - v_{ter}\ln\left(1 + \frac{v_0}{v_{ter}}\right)\right]}$$

(c) When $v_0/v_{ter} \ll 1$, we can expand

$$\ln\left(1 + \frac{v_0}{v_{ter}}\right) \approx \frac{v_0}{v_{ter}} - \frac{1}{2}\left(\frac{v_0}{v_{ter}}\right)^2$$

So $y_{max} = \frac{m}{b}\left[v_0 - v_0 + \frac{1}{2}\frac{v_0^2}{v_{ter}}\right] = \frac{v_0^2}{2g}$

(2.31) 4. (a) For quadratic air resistance $v_{ter} = \sqrt{\frac{mg}{c}}$

where $c = \gamma D^2$, $\gamma = .25\, Ns^2/m^4$ (eq. 2.6).
Plugging in $m = .6$ kg, $g = 9.8$ m/sec^2, $D = .24$ m
we get

$$\boxed{v_{ter} = 20.2 \text{ m/s}}$$

(b) From eq. (2.58): $y = \frac{v_{ter}^2}{g} \ln\left[\cosh\left(\frac{gt}{v_{ter}}\right)\right]$

So $\cosh\left(\frac{gt}{v_{ter}}\right) = e^{gy/v_{ter}^2}$

$$t = \frac{v_{ter}}{g} \operatorname{arccosh}\left(e^{gy/v_{ter}^2}\right)$$

Setting $y = 30$m, and using the above value for v_{ter} yields

$$\boxed{t = 2.78 \text{ sec}}$$

From eq. (2.57): $v = v_{ter} \tanh\left(\frac{gt}{v_{ter}}\right)$

Plugging in the values for g, t, v_{ter} yields

$$\boxed{v = 17.7 \text{ m/sec}}$$

In vacuum, $t = \left(\frac{2y}{g}\right)^{1/2} = 2.47$ sec ~~in vac~~

and $v = gt = (2gy)^{1/2} = 24.2$ m/sec

(2.40) 5.

$$m\frac{dv}{dt} = -bv - cv^2$$

$$\frac{m\,dv}{bv+cv^2} = -dt \Rightarrow \int_{v_0}^{v} \frac{dv}{bv+cv^2} = -\frac{t}{m}$$

$$\Rightarrow \frac{1}{b}\ln\frac{v}{b+cv}\Big|_{v_0}^{v} = -\frac{t}{m}$$

$$\Rightarrow \ln\frac{v}{b+cv} - \ln\frac{v_0}{b+cv_0} = -\frac{bt}{m}$$

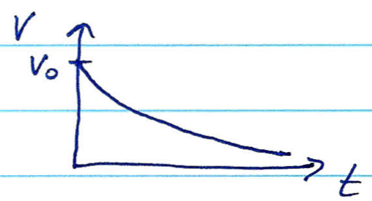
$$\Rightarrow \frac{v}{b+cv} = \frac{v_0}{b+cv_0}e^{-bt/m} \equiv a e^{-bt/m}$$

where we defined $\boxed{a = \frac{v_0}{b+cv_0}}$. Now solve for $v(t)$:

$$v = (b+cv)\,a e^{-bt/m}$$

$$v\left(1 - cae^{-bt/m}\right) = bae^{-bt/m}$$

$$\Rightarrow \boxed{v(t) = \frac{abe^{-bt/m}}{1-ace^{-bt/m}}}$$



$v(t)$ decays exponentially at late time. The linear term dominates since v is small.