Phys 105A Solutions #3

(2.53) (1) $$m\frac{d\vec{v}}{dt} = \vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$$

Let $\vec{E} = E\hat{z}$, $\vec{B} = B\hat{z}$

Only the 1st term contributes to the z component of the force:

$$\boxed{m\frac{dv_z}{dt} = qE}$$

The 1st term does not contribute to the x & y components, so these are the same as in section (2.5):

$$\boxed{m\frac{dv_x}{dt} = qBv_y, \quad m\frac{dv_y}{dt} = -qBv_x}$$

The solution to the 1st eq. is

$$v_z = \frac{qE}{m}t + v_{z0}$$

$$\Rightarrow \boxed{z(t) = \frac{qE}{2m}t^2 + v_{z0}t + z_0}$$

The solution for $x(t)$ and $y(t)$ is the same as section (2.7)

$$\boxed{x + iy = (x_0 + iy_0)e^{-i\omega t}} \quad \text{with } \omega = \frac{qB}{m}$$

The particle moves in a helix about the z axis, with increasing distance between cycles due to the acceleration in the z direction.

(2.55) (2) $m\dot{\vec{v}} = q(\vec{E} + \vec{v}\times\vec{B})$ with $\vec{E} = E\hat{y}$, $\vec{B} = B\hat{z}$

(a) The z component of the force is zero.
So $m\dot{v}_z = 0 \Rightarrow v_z = \text{const}$
Initially, $z=0$, $v_z = 0$, so $z=0$ for all t.
The motion stays in a plane.

Other components: $\boxed{m\dot{v}_x = qBv_y, \quad m\dot{v}_y = qE - qBv_x}$

(b) The initial motion in x direction is undeflected if the y component of the force is zero:

$$qE - qBv_x = 0 \Rightarrow v_x = \boxed{E/B = v_{dr}}$$

This is the drift speed at which $\dot{v}_y = 0$, so if $v_y = 0$ initially, it stays zero & hence $\dot{v}_x = 0$.

(c) Let $u_x = v_x - v_{dr}$, $u_y = v_y$. Then

$$m\dot{u}_x = qBu_y, \quad m\dot{u}_y = -qBu_x$$

These are just the eqs solved in sec. (2.5).
Since the initial velocity is in $\hat{x}$ direction:

$$u_x = A\cos\omega t, \quad u_y = -A\sin\omega t$$

$$\Rightarrow \boxed{v_x = v_{dr} + A\cos\omega t, \quad v_y = -A\sin\omega t}$$

with initial cond:

$$v_{xo} = v_{dr} + A \Rightarrow \boxed{A = v_{xo} - v_{dr}}$$

(3.12) (3) (a) If mass of fuel is $.6\,m_0$, the final mass is $m = .4\,m_0$. From eq. (3.8)

$$v = v_0 + v_{ex} \ln (m_0/m)$$

Starting at rest, $v_0 = 0$, the final speed is

$$v = v_{ex} \ln(1/.4)$$

$$\boxed{v = .916\, v_{ex}}$$

(b) Speed after 1^{st} stage:

$$v_1 = v_{ex} \ln(1/.7) = .357\, v_{ex}$$

Mass at beginning of 2^{nd} stage: $m_1 = .6\,m_0$

" " end " " $m = .3\,m_0$

So

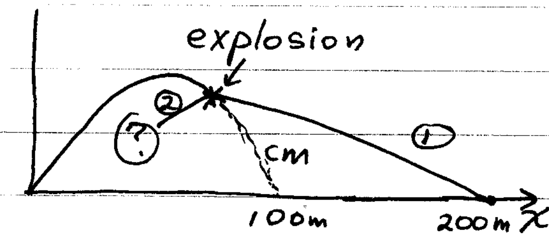
$$v = v_1 + v_{ex} \ln\left(\frac{.6}{.3}\right)$$

$$= (.357 + .693)\, v_{ex}$$

$\boxed{v = 1.05\, v_{ex}}$ which is greater than part (a).

(3.19) (4) (a) From eq. (3.12), the center of mass is only affected by external forces. Even after the explosion, it continues to follow a parabola.

(b)



Since the pieces land at the same time, the explosion added momentum $p_x^{(1)}$ to first piece and $p_x^{(2)}$ to second piece. Conservation of momentum implies $p_x^{(2)} = -p_x^{(1)}$. There are no forces in x-direction

The extra distance traveled by the first piece is

$$\frac{p_x^{(1)}}{m_1} t_0 = 100\,m \qquad t_0 = \text{time from explosion to landing}$$

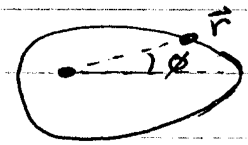
So the extra distance traveled by second piece is

$$\frac{p_x^{(2)}}{m_2} t_0 = -100\,m \qquad \text{since } m_2 = m_1, \; p_x^{(2)} = -p_x^{(1)}$$

Since center of mass was at $x = 100\,m$, the second piece lands back at the starting point!

(c) This would not be true if they landed at different times, since then t_0 would be replaced by t_1 or $t_2 \neq t_1$.

3.27) (5)(a)



The angular momentum is

$$\vec{l} = \vec{r} \times \vec{p} = m \vec{r} \times \vec{v}$$

In polar coordinates:

$$\vec{v} = \dot{r}\hat{r} + r\dot{\phi}\hat{\phi}$$

$\vec{r} = r\hat{r}$ and $\hat{r} \times \hat{r} = 0$, $\hat{r} \times \hat{\phi} = \hat{z}$, so

$$\vec{l} = m r^2 \dot{\phi} \hat{z}$$

Since $\dot{\phi} = \omega$, the magnitude of the angular momentum is $\boxed{l = m r^2 \omega}$

(b)

In a short time dt, the angle ϕ changes by $d\phi$

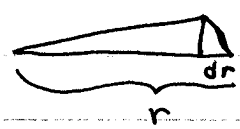
The area increases by $dA = \frac{1}{2}(\text{base}) \times (\text{height})$

$$= \frac{1}{2}(r)(r\,d\phi)$$

~~Notes~~

So $\boxed{\dfrac{dA}{dt} = \dfrac{1}{2} r^2 \dfrac{d\phi}{dt} = \dfrac{1}{2} r^2 \omega}$

Note:



The height is really $(r - dr)\,d\phi$. But in the limit of very small time intervals, the correction term goes to zero.

Finally $\dfrac{dA}{dt} = \dfrac{l}{2m}$

Since l is conserved, planets sweeps out equal areas in equal times.