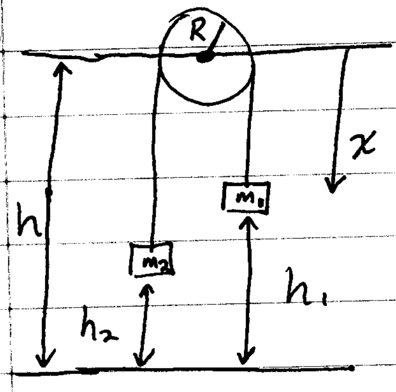
Phys 105A Solutions # 5

(4.35) (1)



Let m_1 be a height h_1 off ground
m_2 " " " h_2 " "
Let string have total length L
Let pulley have height h

(a) Potential energy of two masses:

$$U = m_1 g h_1 + m_2 g h_2 = m_1 g (h-x) + m_2 g [h - (L - x - \pi R)]$$

$$= (m_2 - m_1) g x + \text{const}$$

The potential energy of the pulley just adds a constant.

The velocity of m_1 is $\dot{x}$ and velocity of m_2 is $-\dot{x}$, so the total kinetic energy (including the pulley) is

$$T = \tfrac{1}{2} m_1 \dot{x}^2 + \tfrac{1}{2} m_2 \dot{x}^2 + \tfrac{1}{2} I \omega^2$$

But $\omega = \dot{x}/R$, so the total energy is (up to const.)

$$\boxed{E = \tfrac{1}{2}\left(m_1 + m_2 + \frac{I}{R^2}\right)\dot{x}^2 + (m_2 - m_1) g x}$$

(b) $\dfrac{dE}{dt} = 0 \Rightarrow \tfrac{1}{2}\left(m_1 + m_2 + \frac{I}{R^2}\right) 2\dot{x}\ddot{x} + (m_2 - m_1) g \dot{x} = 0$

$$\Rightarrow \boxed{\ddot{x} = \frac{(m_1 - m_2) g}{m_1 + m_2 + I/R^2}}$$

(4.35) (cont'd)

$m_1 \ddot{x} = m_1 g - F_{T1}$ (last term is tension in rope)

$m_2 \ddot{x} = F_{T2} - m_2 g$

$I \dot{\omega} = (F_{T1} - F_{T2}) R$ (net torque on pulley)

Using $\dot{\omega} = \ddot{x}/R$, $F_{T1} = m_1 g - m_1 \ddot{x}$, $F_{T2} = m_2 \ddot{x} + m_2 g$

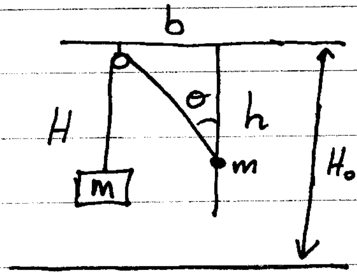
we get

$$\frac{I \ddot{x}}{R} = \left[(m_1 - m_2) g - (m_1 + m_2) \ddot{x}\right] R$$

$$\Rightarrow \ddot{x} = \frac{(m_1 - m_2) g}{m_1 + m_2 + I/R^2}$$

This is exactly the same eq. obtained above.

(4.36)(2)



(a) Assume the pulley is a height H_0 above the ground. Then the potential energy is

$$U = mg(H_0 - h) + Mg(H_0 - H)$$

But $H = l - \frac{b}{\sin\theta}$, $h = b\cot\theta = b\cos\theta/\sin\theta$

So $$U = -\frac{mgb\cos\theta}{\sin\theta} - Mg\left(l - \frac{b}{\sin\theta}\right) + \text{const}$$

$$\Rightarrow \boxed{U(\theta) = \frac{Mgb - mgb\cos\theta}{\sin\theta}} + \text{const}$$

(b) $$U'(\theta) = +mgb - \frac{(Mgb - mgb\cos\theta)\cos\theta}{\sin^2\theta}$$

$$= \frac{mgb - Mgb\cos\theta}{\sin^2\theta}$$

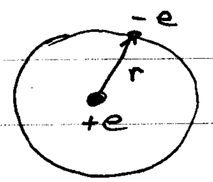
So $U'(\theta_0) = 0 \Rightarrow \boxed{\cos\theta_0 = m/M}$

This is an equilibrium position. At this angle:

$$U''(\theta_0) = \frac{(m + M\sin\theta_0)gb}{\sin^2\theta_0} > 0$$

So it is stable.

(4.53) (3) (a)



Coulomb force $\vec{F} = \frac{-Ke^2}{r^2}\hat{r}$

This causes centripetal acceleration

$$\frac{Ke^2}{r^2} = \frac{mv^2}{r}$$

So the kinetic energy is $T = \frac{1}{2}mv^2 = \frac{1}{2}\frac{Ke^2}{r}$

The potential energy between the proton and electron is $U = -\frac{Ke^2}{r}$. So $\boxed{T = -\frac{1}{2}U}$

(b) Let r_1 be distance from proton to first e^-

r_2 " " " " " second e^-

r_{12} " " between two electrons.

Then

$$E = T_1 + T_2 + U_1 + U_2 + U_{12}$$

where

$$U_1 = -\frac{Ke^2}{r_1}, \quad U_2 = -\frac{Ke^2}{r_2}, \quad U_{12} = +\frac{Ke^2}{r_{12}}$$

T_2 ~~is given~~ $= \frac{1}{2}mv_2^2$ and $T_1 = \frac{1}{2}mv_1^2$

(c) Long before the collision, particle 2 is far away so $U_2 = 0$, $U_{12} = 0$, T_2 is given and $T_1 = -\frac{1}{2}U_1$ from part (a). So

$$\boxed{E = T_2 + \frac{1}{2}U_1 = T_2 - \frac{Ke^2}{2r}}$$

4.53 (cont'd) Long after the collision: $\tilde{U}_1 = 0$, $\tilde{U}_{12} = 0$

Since the second e^- is in a circular orbit of radius r': $\tilde{U}_2 = -\frac{ke^2}{r'}$

and by part (a): $\tilde{T}_2 = -\frac{1}{2}\tilde{U}_2$

So

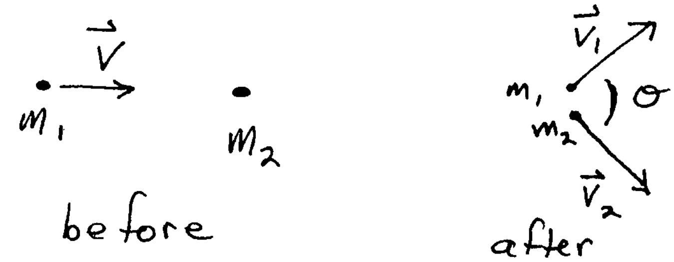
$$E = \tilde{T}_1 + \frac{1}{2}\tilde{U}_2 = \tilde{T}_1 - \frac{ke^2}{2r'}$$

Since energy is conserved: $\tilde{T}_1 - \frac{ke^2}{2r'} = T_2 - \frac{ke^2}{2r}$

So at late times:

$$\boxed{\tilde{T}_1 = T_2 + \frac{ke^2}{2}\left(\frac{1}{r'} - \frac{1}{r}\right)}$$

(4)



We will work in the lab frame

Conservation of momentum: $\boxed{m_1 \vec{V} = m_1 \vec{V}_1 + m_2 \vec{V}_2}$ $(*)$

Conservation of kinetic energy:

$$\boxed{\frac{1}{2} m_1 V^2 = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2} \qquad (**)$$

Using $(*)$: $\frac{1}{2} m_1 V^2 = \frac{1}{2} m_1 \left(\vec{V}_1 + \frac{m_2}{m_1} \vec{V}_2 \right)^2$

$$= \frac{1}{2} m_1 V_1^2 + m_2 \vec{V}_1 \cdot \vec{V}_2 + \frac{m_2^2}{2m_1} V_2^2$$

Setting this equal to right hand side of $(**)$:

$$m_2 \vec{V}_1 \cdot \vec{V}_2 + \frac{m_2^2}{2m_1} V_2^2 = \frac{1}{2} m_2 V_2^2$$

$$\Rightarrow \boxed{\vec{V}_1 \cdot \vec{V}_2 = \frac{V_2^2}{2} \left(1 - \frac{m_2}{m_1} \right)}$$

So if $m_1 > m_2$, $\vec{V}_1 \cdot \vec{V}_2 > 0 \Rightarrow \theta < \pi/2$

but if $m_1 < m_2$, $\vec{V}_1 \cdot \vec{V}_2 < 0 \Rightarrow \theta > \pi/2$

(5) The solution for a damped harmonic oscillator w/ $\beta < \omega_0$ is

$$x(t) = A e^{-\beta t} \cos(\omega_1 t - \delta)$$

with $\omega_1 = \sqrt{\omega_0^2 - \beta^2}$. The velocity is

$$\dot{x}(t) = -A e^{-\beta t}\left[\beta \cos(\omega_1 t - \delta) + \omega_1 \sin(\omega_1 t - \delta)\right]$$

So

$$\dot{x}(0) = 0 \Rightarrow \tan\delta = \beta/\omega_1$$

Since $\beta \ll \omega_0$, $\omega_1 \simeq \omega_0$, and $\boxed{\delta \simeq \beta/\omega_0}$

Also, $x(0) = A$ to 1st order in δ, so $\boxed{A = A_0}$

The energy

$$E = \tfrac{1}{2} m\dot{x}^2 + \tfrac{1}{2} k x^2 \qquad (k = m\omega_0^2)$$

is not constant due to the damping. We get

$$E = \tfrac{1}{2} A_0^2 e^{-2\beta t}\Big[\underbrace{m\omega_0^2}_{k} \sin^2(\omega_0 t - \delta)$$

$$+ 2\omega_0 \beta m \sin(\omega_0 t - \delta)\cos(\omega_0 t - \delta) + k\cos^2(\omega_0 t - \delta)\Big]$$

$$\boxed{E = \underbrace{\tfrac{1}{2} k A_0^2 e^{-2\beta t}}_{\text{leading term}} + \underbrace{\tfrac{1}{2} A_0^2 e^{-2\beta t} m\omega_0 \beta \sin(2\omega_0 t - 2\delta)}_{\text{small correction}}}$$