# Phys 105A Solutions # 6

1. (a) We need to solve:

$$\ddot{x} + \omega_o^2 x = \frac{F_o}{m}\cos\omega_o t \qquad (*)$$

The general solution to the homogeneous eq. is

$$x_h(t) = A\cos(\omega_o t - \delta)$$

To find a solution to inhomogeneous eq. we try (note similarity to critically damped case)

$$x(t) = B t^p \sin\omega_o t$$

Sub into $(*)$ + find that this is a solution if $p = 1$, $B = \frac{F_o}{2m\omega_o}$. Complete soln is

$$\boxed{x(t) = A\cos(\omega_o t - \delta) + \frac{F_o}{2m\omega_o} t \sin\omega_o t}$$

(b) Initial cond:

$$x(0) = 0 \Rightarrow \delta = \pi/2, \quad \dot{x}(0) = 0 \Rightarrow A = 0$$

So the soln is just the second term:

$$\boxed{x(t) = \frac{F_o}{2m\omega_o} t \sin(\omega_o t)}$$

(c) At the breaking point $|F| = k|x_b| = 5F_o$. This is reached when

$$k\frac{F_o}{2m\omega_o}\left|t\sin(\omega_o t)\right| = 5F_o$$

or

$$\boxed{\left|t_b \sin\omega_o t_b\right| = \frac{10 m\omega_o}{k} = \frac{10}{\omega_o}}$$

(5.43) 2.(a) If four 80 kg men cause the four springs to compress 2 cm, the weight on each spring is

$(80\,kg)(9.8\,m/sec^2) = 784\,N = K\,x$

Setting $x = .02\,m$ yields $K = 3.92 \times 10^4\,N/m$

$\approx 4 \times 10^4\,N/m$

(b) For two springs in parallel, the net spring constant is double: $K_{axle} = 8 \times 10^4\,N/m$

So the frequency (not angular frequency) is

$$f = \frac{1}{2\pi}\sqrt{\frac{K_{axle}}{m}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{8 \times 10^4}{50}} = \frac{40}{2\pi} \approx 6.3\,Hz \approx 6\,Hz$$

(c) Resonance requires the time between bumps on road equal the period of oscillation of spring

Period $T = 1/f = .16\,sec$

Car needs to travel $d = 80\,cm$ ~~apart~~ in this time.

So speed is $v = \frac{d}{T} = \frac{.80\,m}{.16\,sec} = 5\,m/sec$

(5.47) 3. Use $\cos\theta\cos\phi = \frac{1}{2}[\cos(\theta+\phi)+\cos(\theta-\phi)]$

$\sin\theta\sin\phi = \frac{1}{2}[\cos(\theta-\phi)-\cos(\theta+\phi)]$

$\sin\theta\cos\phi = \frac{1}{2}[\sin(\theta+\phi)+\sin(\theta-\phi)]$

So if $m \neq n$: $\int_{-\tau/2}^{\tau/2} \cos(n\omega t)\cos(m\omega t)\,dt$

$$= \frac{1}{2}\int_{-\tau/2}^{\tau/2} [\cos(n+m)\omega t + \cos(n-m)\omega t]\,dt$$

$$= \left.\frac{\sin(n+m)\omega t}{2(n+m)\omega}\right|_{-\tau/2}^{\tau/2} + \left.\frac{\sin(n-m)\omega t}{2(n-m)\omega}\right|_{-\tau/2}^{\tau/2}$$

But $\omega\tau = 2\pi$, so all terms vanish.

If $m = n \neq 0$:

$$\int_{-\tau/2}^{\tau/2} \cos^2(n\omega t)\,dt = \frac{1}{2}\int_{-\tau/2}^{\tau/2} [\cos(2n\omega t)+1]\,dt$$

The $1^{st}$ integral again vanishes, but the second is $\tau/2$.

The same result holds for $\int_{-\tau/2}^{\tau/2} \sin(n\omega t)\sin(m\omega t)\,dt$

since sign between the two cosines didn't matter.

Finally:

$$\int_{-\tau/2}^{\tau/2} \sin(m\omega t)\cos(n\omega t) = \frac{1}{2}\int_{-\tau/2}^{\tau/2} \sin(n+m)\omega t + \sin(m-n)\omega t$$

$$= -\left.\frac{\cos(n+m)\omega t}{2(n+m)\omega}\right|_{-\tau/2}^{\tau/2} - \left.\frac{\cos(m-n)\omega t}{2(m-n)\omega}\right|_{-\tau/2}^{\tau/2} = 0$$

since $\cos(\theta)$ is even function: $\cos(-\theta) = \cos\theta$.

(6.1) 4. In spherical coordinates

$$ds^2 = dr^2 + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

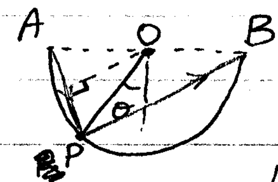
For the curve $r = R$, $\phi = \phi(\theta)$ we have

$$ds^2 = R^2 \left(1 + \sin^2\theta \, \phi'^2\right) d\theta^2$$

So the length is

$$L = R \int_{\theta_1}^{\theta_2} \sqrt{1 + \sin^2\theta \, \phi'^2} \, d\theta$$

(6.5) 5.




Let the hemisphere have radius $R$. If $d$ is the distance traveled by light ray, the time is $t = d/c$ ($c$ = speed of light). Need to compute $d$:

Angle $AOP$ is $90 - \theta$, so $R \sin \frac{1}{2}(90 - \theta)$ is half the distance $AP$. So $d(AP) = 2R \sin \frac{1}{2}(90 - \theta)$

Angle $BOP$ is $90 + \theta$, so $d(PB) = 2R \sin \frac{1}{2}(90 + \theta)$

The total distance is

$$d = 2R\left[\sin \frac{90-\theta}{2} + \sin \frac{90+\theta}{2}\right], \quad t = d/c$$

Find extremum:

$$\frac{dt}{d\theta} = \frac{R}{c}\left(-\cos \frac{90-\theta}{2} + \cos \frac{90+\theta}{2}\right) = 0 \Rightarrow \theta = 0$$

Is it max or min?

$$\left.\frac{d^2t}{d\theta^2}\right|_{\theta=0} = \frac{R}{2c}\left(-\sin(45) - \sin(45)\right) < 0$$

So time is <u>maximized</u>.