1. Write down the Lagrangian for a particle moving along the $x$ axis subject to a restoring force $F = -kx$. Find the Lagrange equation of motion, and solve it.

2. Consider a mass $m$ moving in the $x - y$ plane with potential energy $V = k(x^2 + y^2)/2$, with $k > 0$. Write the Lagrangian using the $x$ and $y$ coordinates, and find the two Lagrange equations of motion. Describe what the solutions look like.

3. Consider a particle of mass $m$ moving in the $x - z$ plane ($z$ is vertical), constrained to move along the path $z = kx^2$ ($k > 0$) and experiencing gravitational acceleration $g$ in the $-z$ direction. Write the Lagrangian in terms of the coordinate $x$, and find the Lagrange equation of motion. Find the frequency of small oscillations.

4. A sphere of radius $\rho$ and mass $M$ rolls without slipping on the lower half of the inner surface of a hollow cylinder with inner radius $R$; the cylinder is fixed, and gravity also acts on the sphere. Determine the Lagrangian function, the equation of constraint, and Lagrange’s equations of motion. Find the frequency of small oscillations.

5. A particle moves in the $x - y$ plane under the influence of a force $F = Ar^{\alpha-1}$, with $r = \sqrt{x^2 + y^2}$, directed towards the origin, where $A$ and $\alpha > 1$ are constants. With the potential energy at the origin equal to zero, find the Lagrange equations of motion. Show whether angular momentum is conserved, and whether the total energy is conserved.