1 HW4 Problem 1

a) Use Gauss’s law in integral form, and appropriate symmetries and boundary conditions, to find the electric field $\vec{E}$ as a function of position inside each of these charge distributions:

- A solid sphere of radius $R$ and charge density $\rho_0$, at $r < R$.
- A solid cylinder of radius $S$ and charge density $\rho_0$, at $s < S$.
- A solid slab of thickness $D$ and charge density $\rho_0$, at $z < D/2$.

In each case, sketch the charge density and the Gaussian surface that you use. Show which faces (if any) of that surface contribute nothing to the integral (in other words, where $\vec{E} \cdot d\vec{a} = 0$). Give magnitude, direction, and dependence on position in each of these cases.

b) Use Gauss’s law in integral form to show that the electric field outside of the shapes in the previous section is the same as the electric field outside of:

- A hollow spherical shell with surface charge $\sigma_S$ and radius $R_S$.
- A line charge $\lambda$.
- A thin sheet of surface charge $\sigma_P$.

In each case, sketch the figure and your Gaussian surface. Find the relationships between $\sigma_S$, $R_S$, $\lambda$, $\sigma_P$ on the one hand; and $\rho_0$, $R$, $S$, and $D$ on the other.
2 HW5 Problem 2

a) A long, rectangular tube has sides of length \( L \) (long sides) by \( d \) (short sides). Both long sides and one short side are grounded (\( V = 0 \)). The un-grounded short side has a piecewise linear, continuous potential:

\[
V(x) = \begin{cases} 
V_0 \left( \frac{d}{2} + x \right), & -\frac{d}{2} < x < 0 \\
V_0 \left( \frac{d}{2} - x \right), & \frac{d}{2} > x > 0 
\end{cases}
\]  

(1)

Here, for convenience I have taken the midpoint of the un-grounded side as the origin, with the \( z \)-axis normal to that face. The \( x \)-axis runs along the un-grounded face, perpendicular to the long sides. You may find it useful to recall from lecture that the potential along the ungrounded face is (part of) a triangle wave, the integral of a square wave. The triangle wave has Fourier series

\[
T(x) = \frac{8}{\pi^2} \sum_{k=1, \text{k odd}}^{\infty} \frac{1}{k^2} \cos \left( \frac{2\pi k x}{2d} \right)
\]  

(2)

- Find the potential within the tube, \( V(x, z) \).

\[
V(x, z=0)
\]

b) (Optional, no credit, not graded) You might find it interesting to check out triangle waves on the web: particularly Wolfram’s MathWorld (an offshoot of Mathematica). Why are there sines there, and cosines here? What about the alternating signs there? Is the form here really the integral of a square wave? And what about the difference in arguments \( 2\pi \) vs \( \pi \)?