Practice problems for Midterm #2
(The actual midterm could contain questions similar to these, but not so many of them. This is just intended to be extra practice and not a representative exam.)

Question 1:

a) Show that \( \left. \frac{\partial S}{\partial T} \right|_P = \frac{C_p}{T} \), with \( C_p \) defined as \( C_p = \left. \frac{\partial H}{\partial T} \right|_P \).

b) Using part a and the appropriate Maxwell Relation, show that: \( dS = \frac{C_p}{T} dT - \beta V dP \)

c) Starting with the definition of the Helmholtz free energy, show that for an ideal gas: \( P = T \left( \frac{\partial S}{\partial V} \right)_T \)

d) Show that \( \left. \frac{\partial C_p}{\partial P} \right|_T = -T \left( \frac{\partial^2 V}{\partial T^2} \right)_P \)

Question 2:

Consider two small interacting systems. System A is a paramagnet consisting of 20 elementary dipoles each of which can store either 0 or 1 units of energy. System B is an Einstein solid consisting of 4 elementary oscillators each of which can store any number of units of energy of the same size as those for system A.

<table>
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<tr>
<th>PARAMAGNET</th>
<th>EINSTEIN SOLID</th>
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<td>20 dipoles</td>
<td>4 oscillators</td>
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If the two systems share 2 units of energy, what is the most likely division of energy between the two systems?
**Question 3:**

Even an “evacuated” box is filled with a photon gas due to the electromagnetic radiation that is thermally emitted by the interior walls. A photon gas is a gas-like collection of photons, which has many of the same properties of a conventional gas like hydrogen or neon - including pressure, temperature, and entropy. At thermal equilibrium the entropy $S$ of a photon gas can be shown (using quantum mechanics) to depend only on its energy $U$ and the volume $V$ of its container.

$$S(U,V) = aU^{3/4}V^{1/4}$$ where $a$ is a constant.

Use the expression for $S(U,V)$ to find both the temperature and pressure of the photon gas as a function of both $U$ and $V$ (and possibly $a$).

**Question 4:**

An object with constant pressure heat capacity $C_P$ and temperature $T_1$ is brought in contact with a thermal reservoir with constant temperature $T_2$ (different from $T_1$) until the object reaches equilibrium (at constant pressure). What is the total change in entropy for this process (including both the object and reservoir)?

**Question 5:**

Adsorption is the process of particles sticking to the surface of a solid (rather than getting absorbed inside of it). To model adsorption, consider a surface that consists of $M >> 1$ discrete sites; each site has two possible states: vacant (with energy $\varepsilon = 0$), or bound to a single atom (with energy $\varepsilon_o$). If $N >> 1$ atoms are bound (with $M >> N$), find expressions for the following parameters of the system of adsorbed atoms:

(a) The energy, $U(N, \varepsilon_0)$.
(b) The multiplicity, $\Omega(M, N)$.
(c) The temperature $T(M,N)$.

**Question 6:**

Suppose that you want to know how much heat it would take to boil water at 473 K and 1atm pressure, rather than 373 K. At $T = 373$ K, $\Delta H_{vap} = 40.7$ kJ mol$^{-1}$ is the enthalpy of vaporization. Assuming that the heat capacities of the liquid ($C_p,_{\text{liquid}} = 75$ J K$^{-1}$ mol$^{-1}$) and the vapor ($C_p,_{\text{vapor}} = 3.5$ J K$^{-1}$ mol$^{-1}$) are constant over this temperature range, calculate the enthalpy of vaporization at 473 K (for 1 mole).