1. Stability of Swim Bladders in Fish

Most mammals float in water when they hold their breath and sink when they exhale (try it the next time you go swimming!). Roughly 6% of their body’s volume is the lungs. One obstacle that marine animals face in living in water is to maintain a neutrally buoyant state (i.e. have the same density as the local water density) within the range of depths occupied by the animal. In other words, they would like to stay at some arbitrary depth and not expend energy just to remain there. This is an issue since the density of water is not exactly constant - it generally increases slightly with depth due to both temperature and salinity gradients.

A swim bladder is one way to accomplish this and is the way many bony fishes solve the problem. The swim bladder is a thin-walled sac filled with gas. Through slow diffusion processes (many minutes to hours), the fish can add gas to the sac (thus increasing its volume and decreasing the fish’s density) or take it away. Such an arrangement has an intrinsic instability that highlights why some aquatic animals prefer other methods for attaining neutral stability. Let’s assume the fish of mass $M$ has a fixed volume $V$ (the tissues excluding the swim bladder), plus a variable volume $V_s << V$ in the swim bladder. The density of the fish is

$$\rho_F = \frac{M}{V + V_s} \approx \frac{M}{V} \left(1 - \frac{V_s}{V}\right),$$

so that there is a maximum density $\rho_m = M/V$ when $V_s \to 0$, and a minimum density when $V_s$ is a maximum (let’s say $V_s/V = 0.07$ at maximum). Assume that at some location in the ocean, at pressure $P$, the fish has had enough time to set $V_s$ and equilibrate to the local water density, $\rho = \rho_F$. Now, let’s imagine that the fish forgets that it’s supposed to maintain a constant depth and suddenly swims upward by a small distance $\Delta z$.

(a) What is the pressure change $\Delta P$ associated with $\Delta z$?

(b) Since there is no time for the bladder to exchange gases and the gas remains at the same temperature, what is the change in its volume, $\Delta V_s$, due to the pressure change?

(c) What is the density of the fish at the new location? Write it as $\rho_F(\text{new}) = \rho + \Delta \rho$. You should find that the density of the fish has dropped, $\Delta \rho < 0$, when it moves up. What is the sign of $\Delta \rho$ when the fish moves down?

(d) Let’s assume we can neglect the slight change in water density with depth (it is about 1023 kg m$^{-3}$ at the surface and rises with depth to 1026 kg m$^{-3}$ at 1 km). In this case, the density of the fish is now smaller than the sur-
roundings. What is the magnitude of the upward acceleration, \( a \), in terms of the acceleration due to gravity, \( g \)?

(e) Write \( a \) in terms of the depth \( d \). Assuming that \( V_s/V \) does not change too much with depth, where is the acceleration largest - near the surface or at large depths?

2. **Blood Pressure** A doctor tells you that your blood pressure varies between 70 mm of mercury and 120 mm of mercury.

(a) What are these pressures in SI units? (The density of mercury is 13,600 kg m\(^{-3}\).)

(b) To what column of blood does 100 mm of mercury correspond? (Blood is mostly water, so you may assume that the density of blood is approximately 1000 kg m\(^{-3}\).)

(c) If you faint when the blood pressure in your head reaches zero, what is the maximum height your head could be above your heart?

3. **Ideal Pipe Flow** Imagine an ideal liquid (no viscosity) of density \( \rho \) flowing at uniform velocity \( v_1 \) in a horizontal, rigid pipe of cross-sectional area \( A_1 \). Neglect gravity.

(a) What is the rate of mass flow (kg/s) down the pipe?

(b) What is the velocity of the liquid when the pipe shrinks to a smaller area \( A_2 \)?

(c) What is the pressure difference in the liquid between the narrow pipe and the wide pipe?

4. **Falling Water** Turn a faucet on gently so that water streams out in a laminar (rather than turbulent) flow. You should notice that the falling stream of water has a cross-sectional area which decreases downward.

(a) Using the continuity equation, calculate how the cross-sectional area of the stream should vary with height. (Hint: the water is essentially in free fall.)

(b) A new effect arises below some height in the falling stream. Look carefully - what do you observe? What piece of physics which we have neglected so far might be causing this?

(c) Provided the sink is not too deep and the water flow out of the faucet is not too small, we can ignore the new effect in part (b). Put a smooth, horizontal plate below the falling stream. Let’s now calculate how the water spreads cylindrically away from the stream. Using Bernoulli, what is the outward horizontal speed of the fluid as it spreads away?

(d) Go a large cylindrical distance \( r \) from the center of the splash point. How does the thickness, \( d \), of the fluid layer depend on \( r \)? Try doing the experiment to see if you can observe this!