1. Walking on Water
What would the surface tension have to be for you to be able to walk on water? (Hint: read Denny section 12.4.)

2. Vorticity in a Shear Flow
Consider the linear shear flow between two parallel plates that we discussed when we introduced viscosity. Given a constant applied shear stress $F/A$ on the top plate, the flow adopts a linear gradient given by

$$\frac{F}{A} = \eta \frac{dv_x}{dy}. $$

Show that there is vorticity in this flow and calculate its magnitude and direction. Assuming the plates are separated by a distance $w$, calculate the circulation around a rectangular loop of width $w$ and length $\ell$ along the direction of motion. (Do not use Stokes’ theorem, but calculate the circulation directly from a line integral.) Show then that the vorticity and circulation you have calculated are consistent with Stokes’ theorem. What is the physical origin of the vorticity in this flow?

3. Human Blood Flow
(a) The volume flow rate of blood as it leaves the human heart and enters the aorta is about $10^{-4} \text{ m}^3\text{s}^{-1}$. Given that the aorta is about 2.5 cm in diameter, what is the average speed of blood flow through the aorta? What is the pressure gradient required to overcome viscous resistance in the aorta? How does that compare to the vertical pressure gradient required to overcome gravity?

(b) In section 5.4.3, Denny discusses Murray’s Law, which states that when a blood vessel splits into two smaller vessels, the sum of the cubes of the radii of the two smaller vessels is equal to the cube of the radius of the larger vessel. This arises because the optimal volume flow rate turns out to be proportional to the cube of the radius of the blood vessel. Given this last fact, what is the arterial radius below which pressure gradients in arteries will be dominated by viscous drag rather than by gravity?

(c) Human capillaries have average radii of about 6 $\mu$m. Assuming Murray’s Law, what is the average speed of blood flow through a capillary? Using continuity, into how many capillaries does the aorta eventually split?

4. Damping of water waves. How important is viscous damping for deep water waves in the ocean?
(a) Estimate the damping time by calculating how long it takes for momentum to diffuse one wavelength. Express your answer in terms of the wave period using cgs units and the appropriate viscosity for water.

(b) Is this damping ever important? Compare the damping time to the time to travel over the surface of the Earth for a $P = 10$ sec wave.