1. **Equilibrium temperatures for the inner planets.** Assume the inner planets and the Sun are blackbodies (i.e. they radiate an energy flux \( F = \sigma T^4 \)). Compute the equilibrium temperatures for the inner planets using a solar radius and temperature \( R_{\text{sun}} = 7 \times 10^8 \) m, \( T_{\text{sun}} = 5785 \) K, and distances to the planets \( a_{\text{Mercury}} = 0.4 \) AU and \( a_{\text{Venus}} = 0.7 \) AU, where 1 AU = \( 1.5 \times 10^{11} \) m. How accurate do you expect your answers to be for each case?

2. **Diurnal Heating of the Ocean and Atmosphere**

Thermal conduction of heat obeys a diffusion-type equation:

\[
\frac{\partial T}{\partial t} = \frac{\kappa}{C_v \rho} \nabla^2 T,
\]

where \( \kappa \) is the thermal conductivity (W m\(^{-1}\) K\(^{-1}\)) and \( C_v \) is the specific heat per unit mass at constant volume (J kg\(^{-1}\) K\(^{-1}\)).

(a) Given that radiative heating by the sun only significantly affects the surface layers of the oceans, estimate to what depth the temperature changes significantly on a daylight time scale due to the effects of thermal conduction. (Table 8.2 on page 147 of Denny gives useful numbers.)

(b) Repeat part (a) for air, using the numbers in Table 8.1 on page 146 of Denny. Does conduction explain the warmth of our atmosphere?

(c) Explain why heating of the ground can drive convection in the air, but heating of the surface layers of the ocean will not drive convection into deeper layers.

3. **Lapse rate for very hot surface.** In class we derived the lapse rate for a dry adiabat \( dT/dz \simeq -10^\circ\text{C}/\text{km} \), and we discussed how condensation of water vapor in upwelling air can reduce this value to the “wet” adiabat \( dT/dz \sim -5^\circ\text{C}/\text{km} \). These two limits bracket the conditions of the lower atmosphere most of the time, which on average have \( dT/dz = -6.5^\circ\text{C}/\text{km} \). In both limits, the density of air is decreasing as you go upward. However, sometimes the ground is so strongly heated that the air near the ground becomes lighter than the air above it, giving rise to a strong convective overturn instability (Rayleigh-Taylor!). This is the cause of various phenomena, like dust or sand whirls in the desert, and the mirage effect caused by the changed index of refraction of the air.

(a) Using hydrostatic balance and the ideal gas law, compute the lapse rate implied by setting the density to be a constant.

(b) Is this more or less steep than the lapse rates for the dry and wet adiabats?
4. Atmospheric gravity waves. When the temperature gradient is not as steep as the adiabatic value, the atmosphere is stable, and will oscillate if perturbed.

(a) Using the class lecture notes on convection, show that if a fluid element is displaced upward a distance $dz \equiv \xi z$, then it will have a density relative to its surroundings of

$$\delta \rho = \frac{\rho}{T \xi z} \left[ \frac{dT}{dz} - \left( \frac{dT}{dz} \right)_{ad} \right]$$

(b) From this density perturbation, compute the buoyancy force (per unit volume) on the fluid element. It should have the form of a harmonic oscillator force $f_z = -\rho N^2 \xi z$, where $N^2$ is the oscillator frequency (called the Brunt-Väisälä frequency). What is the expression for $N^2$?

(c) For a temperature gradient 10% smaller than the adiabatic value (assume dry air), compute a numerical value for $N$. 
