Physics 123B: Homework 1

due Friday, January 16, 4pm to Amanda Jones in Kohn Hall 1220. Or by email directly to Prof. Balents.

1. **Cuprate Fermi surface:** The important energy band of the cuprate high-temperature superconducting materials can be modeled by considering a tight binding band of a single orbital per site of a square lattice, with lattice spacing a. Assume a hopping γ between nearest neighbor sites and a hopping γ' between second-neighbor sites, which are on opposite corners of the squares (i.e. separated by $\sqrt{2}a$ distance).

   (a) Show/argue that the energy band is described by the function

   $$\epsilon(k) = -2\gamma(\cos k_x a + \cos k_y a) - 2\gamma'(\cos(k_x + k_y)a + \cos(k_x - k_y)a). \qquad (1)$$

   (b) Suppose the Fermi energy (with the above definition of the zero of energy) is zero and $\gamma' = 0$. What is the density of electrons, per site?

   (c) Take $\gamma = 1$ and $\gamma' = -0.1$. Sketch or plot the Fermi surfaces for the Fermi energy $= 0, -0.2, -0.4, -0.6$. Lowering the Fermi energy corresponds to "hole doping".