Physics 123B: Homework 4

due February 13, 4pm to Amanda Jones in Kohn Hall 1220 or by email to Prof. Balents

1. **Vortices in rotating superfluids:** As mentioned in class, a rotating "bucket" of superfluid must contain a non-zero density of vortices. You should assume $T \ll T_c$, which means that the superfluid density equals the full density.
   (a) What vortex density (in cm^{-2}) should be expected for a container of superfluid ^{4}He rotating at $1Hz$?
   (b) What vortex density is expected for a superfluid of Na atoms, rotating at $60Hz$?
2. **Kosterlitz-Thouless transition:** Very thin superfluid helium films, with thickness d in the nanometer range, can be prepared and studied. In such films, the superfluidity can be destroyed with increasing temperature by the proliferation of vortices induced by thermal fluctuations. To study this effect, find the full free energy of a single vortex in a cylindrical sample of radius R and height d, adding to the free energy derived in class the contribution due to the entropy, which equals $-TS$, due to the different possible positions of the vortex. Calculate the entropy using the formula $S = k \ln \Omega$, where Ω is the number of non-overlapping places to put the vortex core (of radius a) inside the circular cross-section of the cylinder. From this, find the temperature, T_{KT} above which the total free energy of the vortex becomes negative at large R. Above this Kosterlitz-Thouless temperature, it becomes favorable for vortices to enter the sample, and superfluidity is destroyed. Estimate T_{KT} for a film of thickness $d = 3\mathring{A}$, assuming the three-dimensional mass density $\rho = 0.14 g/cm^3$.