Superconductivity

- Discovered earlier (1911) than superfluidity due to higher T_c (Hg, 4.2K?)
- Many similar properties to superfluidity
  - zero resistance $T<T_c$
  - persistent currents
- Differences
  - Perfect diamagnetism: the Meissner effect
  - Energy gap - for measurements involving single electrons, a SC often behaves like a semiconductor

Superconductivity

- Similarity to superfluidity suggests BEC
- But electrons are fermions!
- What happens is that electrons *bind* into *Cooper pairs*. A pair of fermions is a boson, so Cooper pairs can condense.
- Why should they bind? Electrons repel by Coulomb force! This is the question of the "mechanism" of superconductivity

Mechanisms

- There is no *one* mechanism
- BUT most superconductors arising from simple metals (i.e. which are simple metals above T_c) are understood from the BCS theory of pairing due to *electron-phonon coupling*
- Roughly, this arises because an electron distorts the lattice, and this distortion lasts a relatively long time, so that it can attract a second electron, even after the first has left
    - "Retardation": two electrons bind but do not occupy the same position at the same time, so their Coulomb repulsion is minimized.

London theory

- Once we accept that Cooper pairs form, we can study their condensation the same way we study BEC

$$\psi(r) = \sqrt{n_s^*(r)}e^{i\theta(r)}$$ Pair wavefunction

- Similar to superfluid, $\mathbf{p} = \hbar\nabla\theta - q\mathbf{A}$
    - The difference arises from the charge of Cooper pairs

London Theory

- This implies *screening*: check Maxwell eqns
- Pairs: $n_s^*=n_s/2$, $q=-2e$, $m^*=2m$
- Hence the current is

$$\mathbf{j} = -\frac{qn_s^*}{m*}\mathbf{p} = -\frac{\hbar n_s e}{2m}\left(\nabla\theta + \frac{2e}{\hbar}\mathbf{A}\right)$$

- This is often called the "London equation"
- Use with Maxwell equation to describe screening

London theory

- Maxwell

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$$

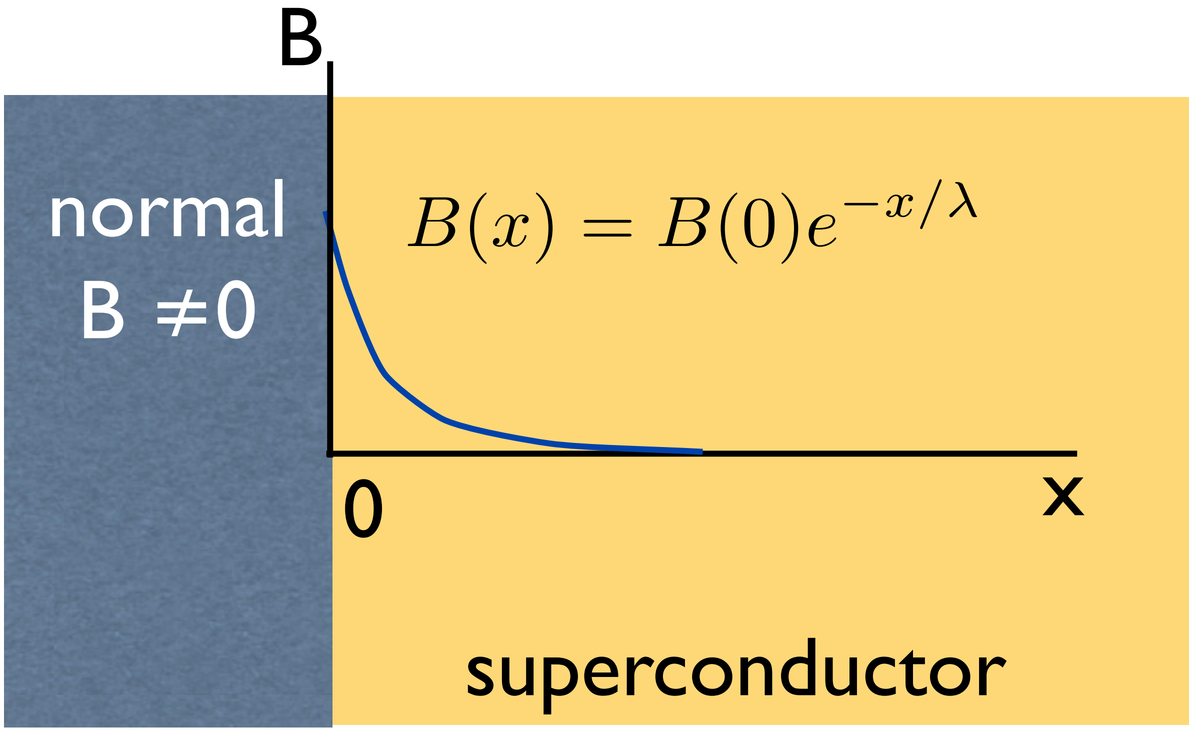
$$\nabla \times \nabla \times \mathbf{B} = \mu_0 \nabla \times \mathbf{j}$$

$$\cancelto{0}{\nabla(\nabla \cdot \mathbf{B})} - \nabla^2 \mathbf{B} = \mu_0 \left(-\frac{n_s e^2}{m} \nabla \times \mathbf{A} \right)$$

$$\boxed{\nabla^2 \mathbf{B} = \frac{1}{\lambda^2} \mathbf{B} \qquad \lambda = \left(\frac{m}{n_s e^2 \mu_0} \right)^{1/2}}$$

- These two equations describe screening

London theory



λ is called London penetration depth

$\lambda \sim$ 10-100nm typically

Meissner Effect

- The above argument suggests screening of magnetic fields is due to currents which flow because of infinite conductivity
- If there was "only" infinite conductivity, then we would expect that an applied field would not penetrate, but that if we *started* an experiment with a field applied, and then cooled a material from the normal to superconducting state, the field would remain
- This is in fact not true: magnetic fields are actively *expelled* from superconductors

Meissner effect

- Expulsion of an applied field occurs because in the superconducting state, the field *costs free energy*, i.e. is thermodynamically unfavorable
- Free energy

$$F = \int d^3r \left[n_s^* \frac{p^2}{2m^*} + a(n_s - n_s^{\mathrm{eq}})^2 + \frac{B^2}{2\mu_0} \right]$$

Meissner Effect

$$F = \int d^3r \left[\frac{n_s}{8m} |\hbar \nabla \theta + 2e\mathbf{A}|^2 + a(n_s - n_s^{\mathrm{eq}})^2 + \frac{B^2}{2\mu_0} \right]$$

gauge: we can always choose A
to cancel ∇θ

$$F = \int d^3r \left[\frac{n_s e^2}{2m} |\mathbf{A}|^2 + a(n_s - n_s^{\mathrm{eq}})^2 + \frac{|\mathbf{B}|^2}{2\mu_0} \right]$$

key point: if B≠0, A must vary linearly with r, which implies $|A|^2$ diverges. Superconducting kinetic energy becomes infinite!

Meissner effect

$$F = \int d^3r \left[\frac{n_s e^2}{2m} |\mathbf{A}|^2 + a(n_s - n_s^{\mathrm{eq}})^2 + \frac{|\mathbf{B}|^2}{2\mu_0} \right]$$

- Instead, superconducting state *expels* the field.
- Eventually, if a large enough field is applied to a superconductor, the superconductivity is destroyed