Curie Susceptibility

- Magnetic moment in general is proportional to spin

spin S quantum spin
$S^2 = S(S+1)\hbar^2$

$$\boldsymbol{\mu} = g\mu_B \mathbf{S}/\hbar$$

g-factor
(could be a tensor)
$g \approx 2$ for pure spin moment

Bohr magneton

$$\mu_B = \frac{e\hbar}{2m_e} = 9.3 \times 10^{-24} J/T$$
$$= 0.671 K/T$$

- Magnetic dipole interaction

$$\mathcal{H} = -\boldsymbol{\mu} \cdot \mathbf{H}$$

Curie Susceptibility

- Thermodyamics $$M = -\frac{\partial F}{\partial H} \qquad F = -kT \ln Z$$

$$Z = \sum_n e^{-E_n/kT}$$

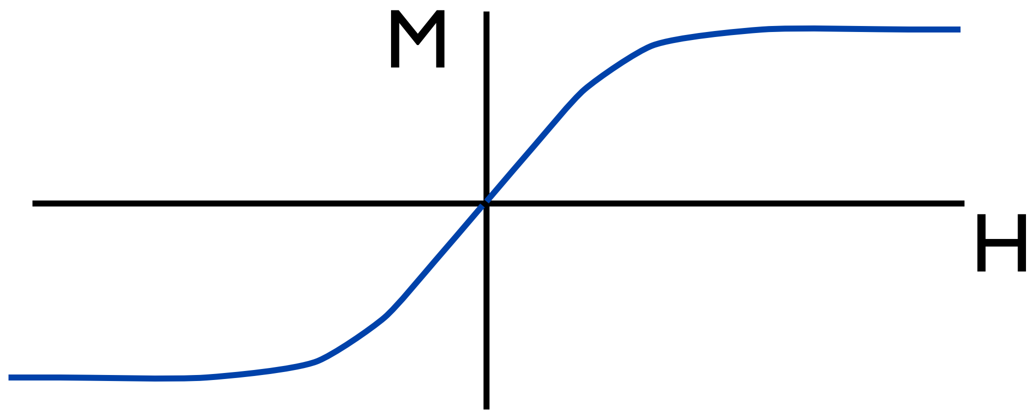
- For example, S=1/2 $$E_{\uparrow/\downarrow} = \mp g\mu_B H/2$$

$$Z = 2\cosh\left(\frac{g\mu_B H}{2kT}\right) \qquad M = \frac{g\mu_B N}{2}\tanh\left(\frac{g\mu_B H}{2kT}\right)$$

Curie Susceptibility

- Magnetization

$$M = \frac{g\mu_B N}{2} \tanh\left(\frac{g\mu_B H}{2kT}\right)$$



- Susceptibility

$$\chi = \frac{(g\mu_B)^2 N}{4k_B T} = \frac{A}{T}$$

Curie Law:
indicates local moment

Curie Susceptibility

- Thermodyamics $M = -\frac{\partial F}{\partial H}$ $F = -kT \ln Z$

$$Z = \sum_n e^{-E_n/kT}$$

- General result

$$\chi = \frac{N(g\mu_B)^2}{3} \frac{S(S+1)}{kT}$$

plot χ^{-1} versus T to extract “effective moment”

Contrast with metals

Curie Law

$$\chi = \frac{N(g\mu_B)^2}{3}\frac{S(S+1)}{kT}$$

Pauli paramagnetism

$$\chi = V\frac{(g\mu_B)^2}{4}D(\epsilon_F)$$

basically $kT \rightarrow \varepsilon_F$

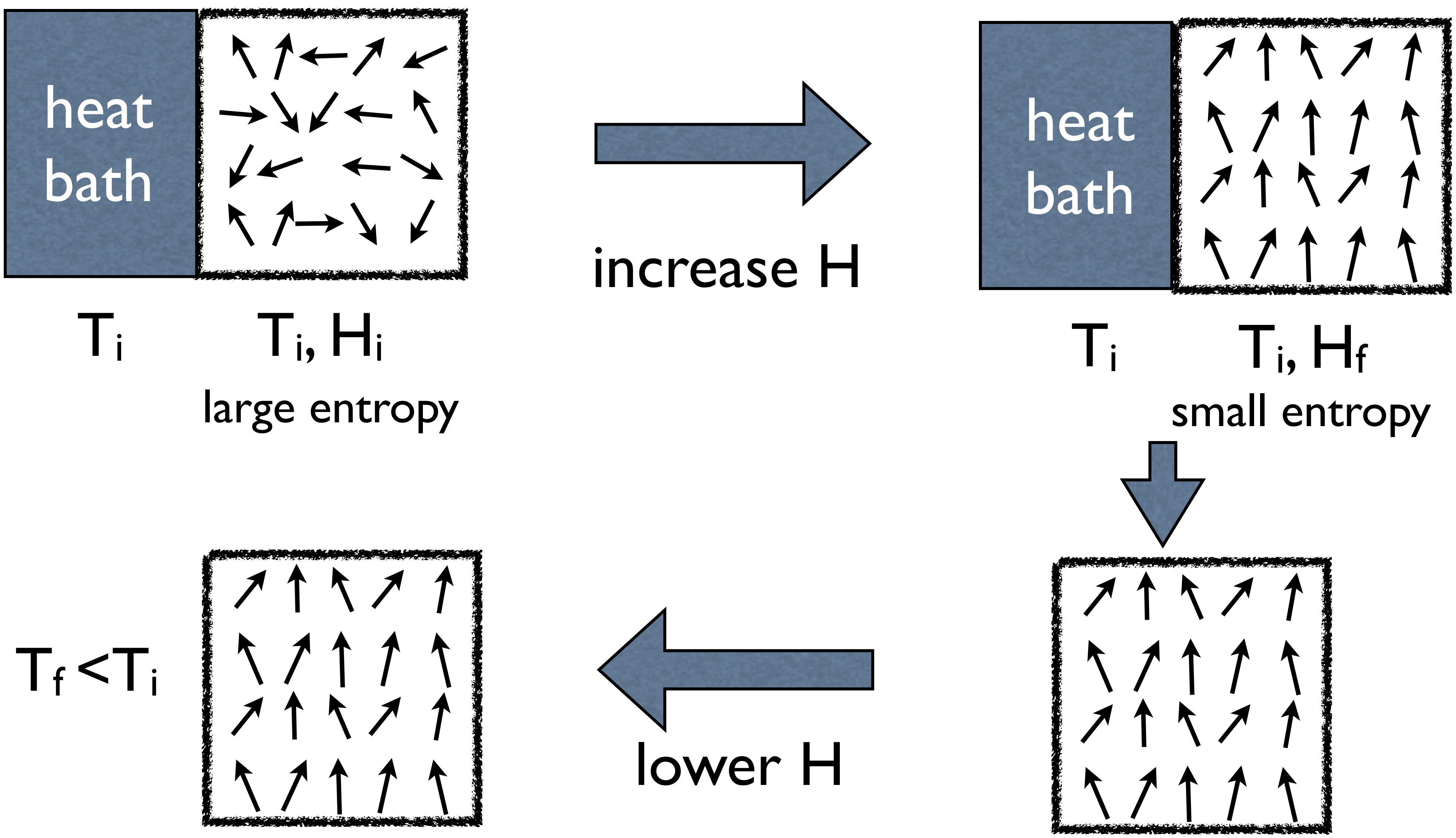
Does χ really diverge for local moments?

NO, because they interact

Magnetic cooling

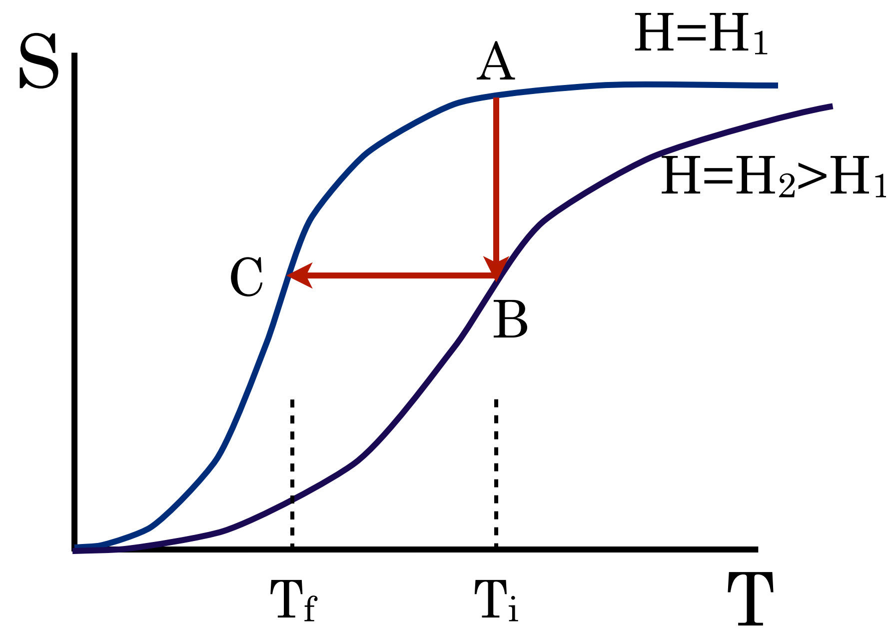
- The large susceptibility of free spins at low temperature means they are easily aligned by small magnetic fields
- This alignment corresponds to a drastic reduction of entropy. One can use this control over entropy to remove entropy from another system, thereby cooling it.

Magnetic Cooling



Magnetic Cooling

- A→ B: isothermal step - raise field, lower entropy
- B→ C: adiabatic step - lower field, same entropy: lower temperature
- For paramagnetic spins, S = S(H/T)
  - Hence $H_1/T_f = H_2/T_i$



Exchange

- How do spins interact?
  - Magnetostatic dipole-dipole coupling

$$H_{d-d} = -\frac{\mu_0}{4\pi r^3}\left[3(\mathbf{m}\cdot\mathbf{r})(\mathbf{m}'\cdot\mathbf{r}) - \mathbf{m}\cdot\mathbf{m}'\right]$$

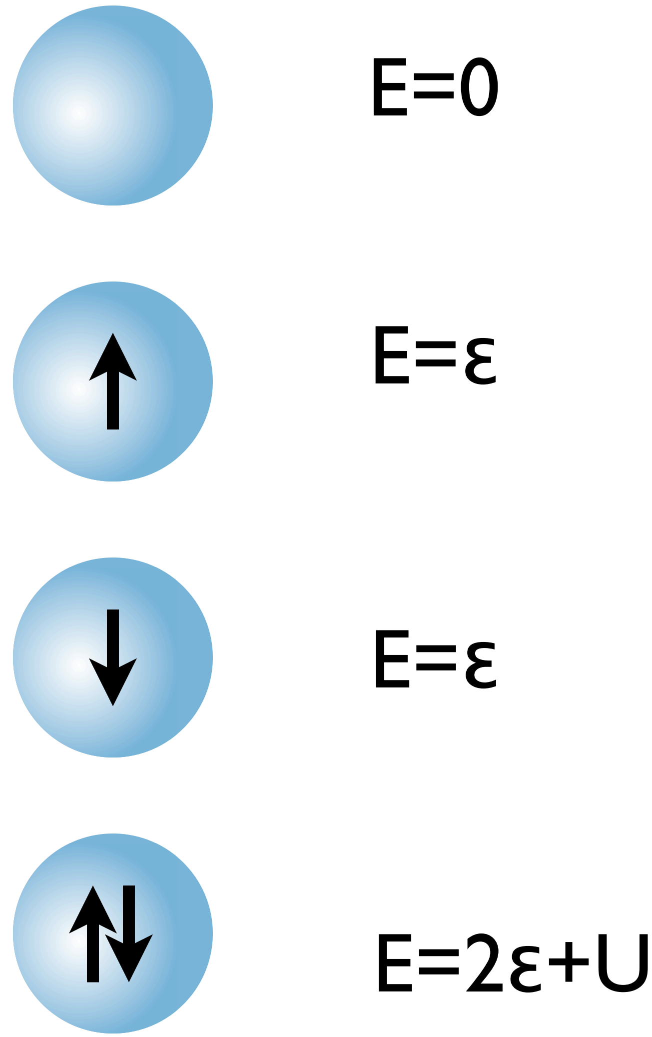
    - This is rather weak, $\lesssim$ 1K for even large spins
  - Electrostatic interaction usually dominates, just as it does inside atoms
    - Indirectly leads to spin coupling through Pauli principle

Exchange

- Generally think of exchange interactions between spins as due to hopping of electrons from one orbital to another
- Many types of exchange, complicated by varieties of orbitals involved, including those on non-magnetic atoms
- We will just discuss the simplest case

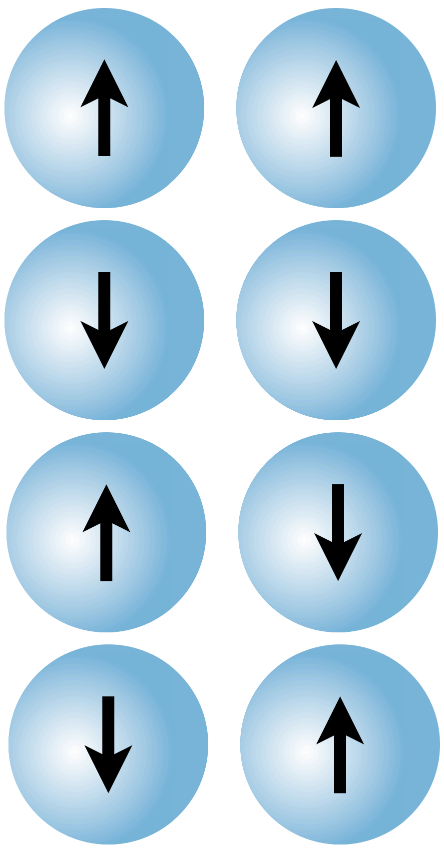
Direct exchange

Assume individual
S=1/2 ions



Direct exchange

- Two ions, 2 electrons. With no contact:

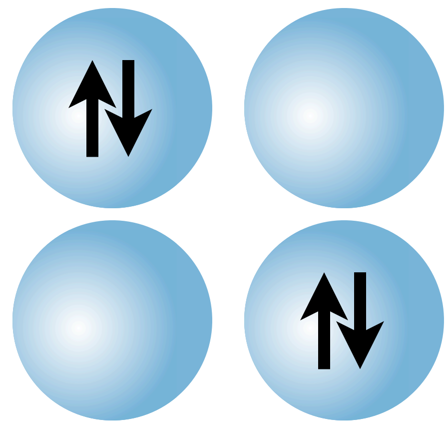


E=2ε

E=2ε

E=2ε

E=2ε



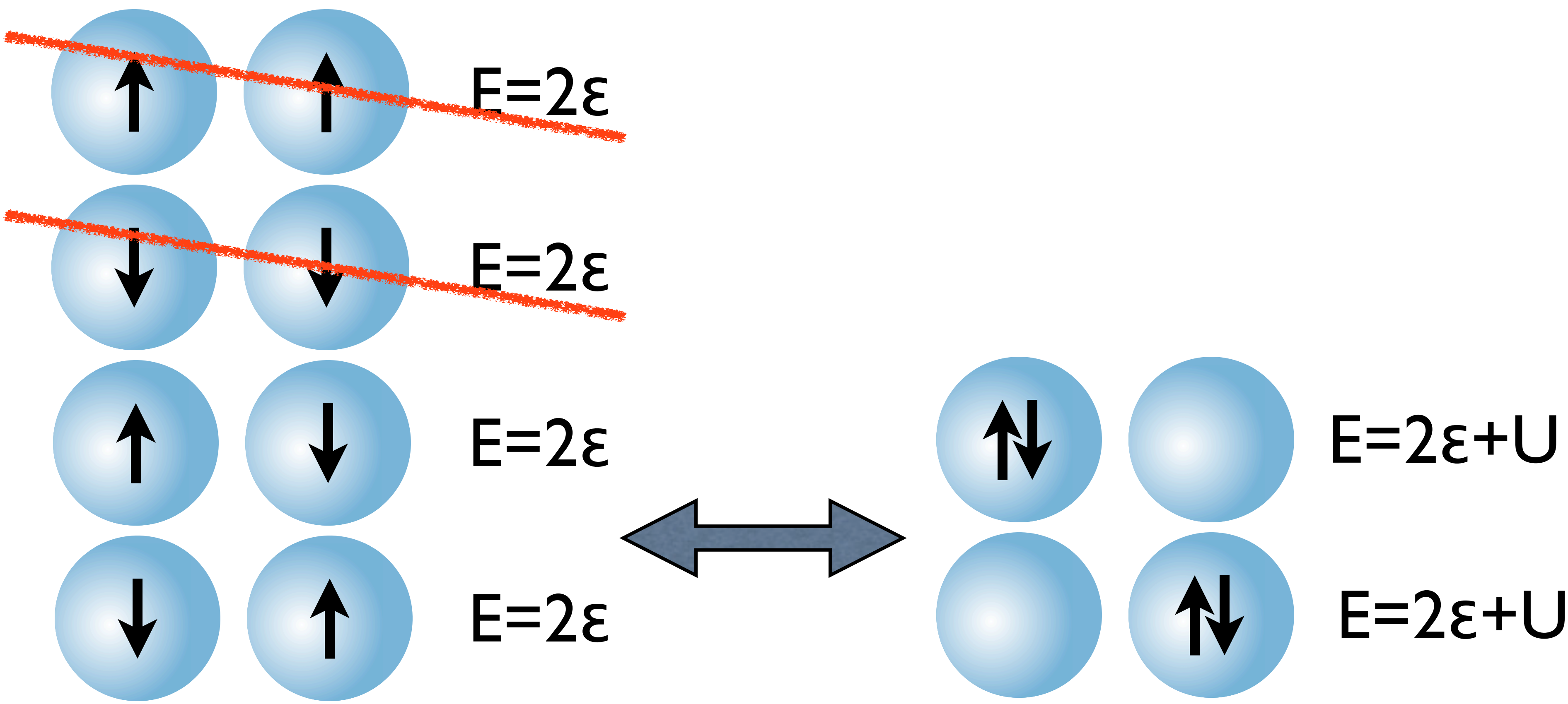
E=2ε+U

E=2ε+U

Now include
electron hopping

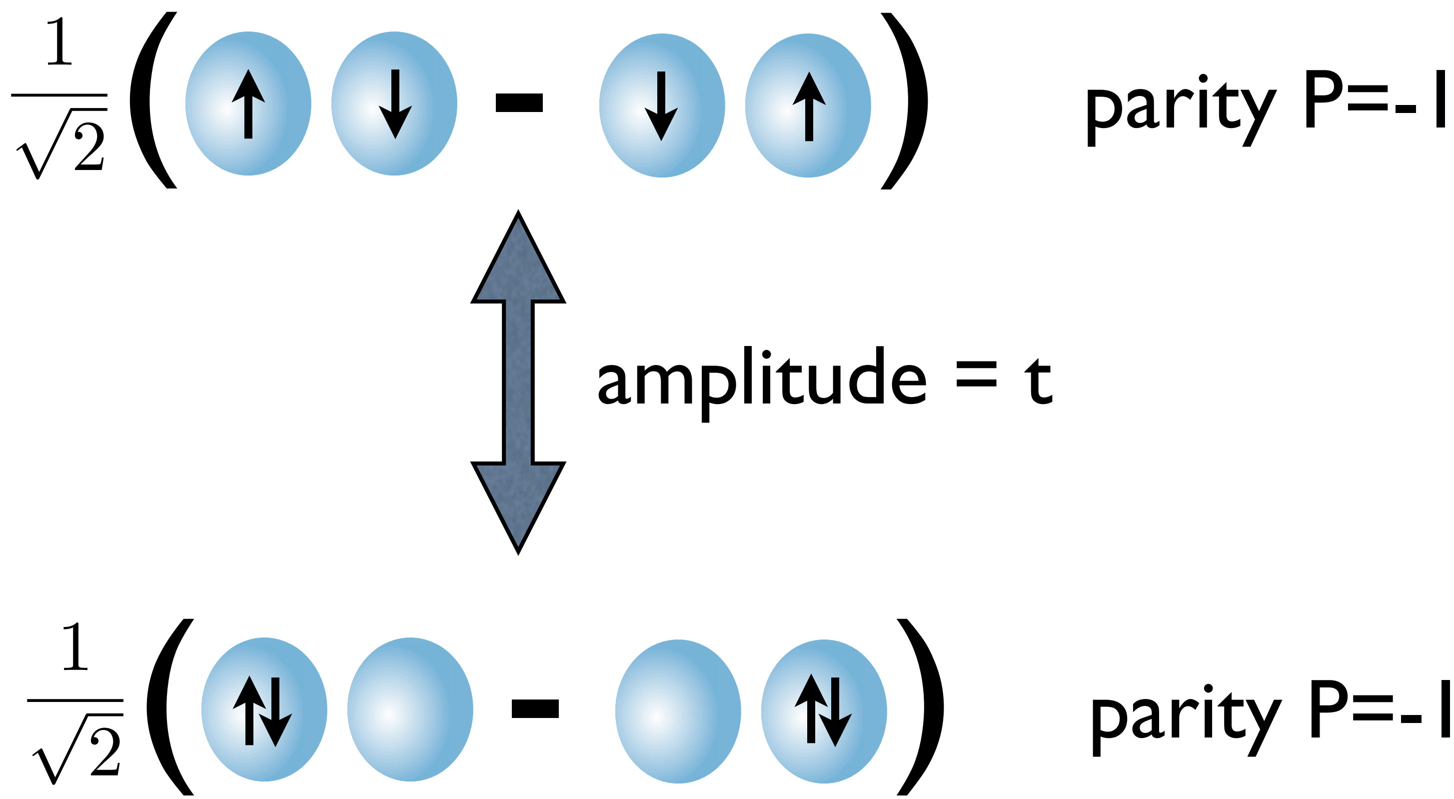
Direct exchange

- Hopping? Prohibited by Pauli for parallel spins (S=1)



Direct exchange

- Since spin is conserved, only S=0 state can hop



Direct exchange

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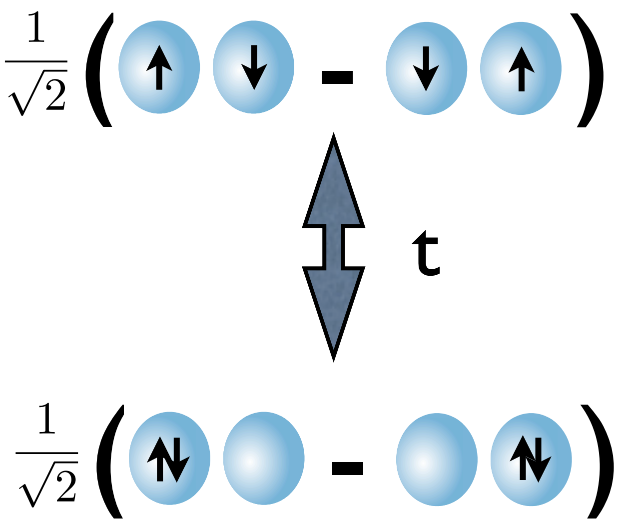


$E_0 = 2\varepsilon$

$E_0 = 2\varepsilon + U$

$$H = \begin{pmatrix} 2\epsilon & t \\ t & 2\epsilon + U \end{pmatrix}$$

$$E_- \approx 2\epsilon - \frac{t^2}{U}$$

singlet energy lowered by $O(t^2/U)$

Direct exchange

- Write as an "effective Hamiltonian":

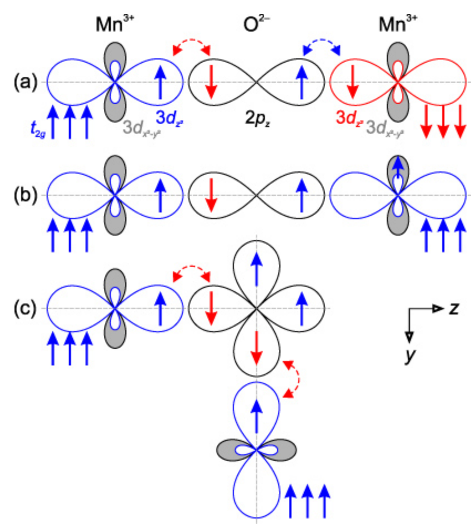
$$H_{\text{eff}} = -\frac{t^2}{U}\hat{P}_{S=0}$$
$$= -\frac{t^2}{U}\left[1 - \frac{1}{2}\left(\mathbf{S}_i + \mathbf{S}_j\right)^2\right]$$
$$= \text{const.} + J\,\mathbf{S}_i \cdot \mathbf{S}_j$$

- With J ~ t^2/U. Note this is typically *antiferromagnetic* interaction: favors singlet/ anti-parallel spins

J can be as large as 1000K, or as small as a few K

Other exchanges

- Super-exchange: exchange due to electrons hopping through an intermediate orbital



usually antiferromagnetic

this can be ferromagnetic

Other exchanges

- RKKY exchange:
  - interaction of spins mediated by delocalized metallic electrons
  - only in a metal, obviously
- Double exchange
  - more exotic exchange in some metals with some localized electrons