Dirac electrons

So we learned that electrons are described by a 2d Dirac equation

$$v q_\mu \gamma^\mu \psi = \epsilon \psi$$

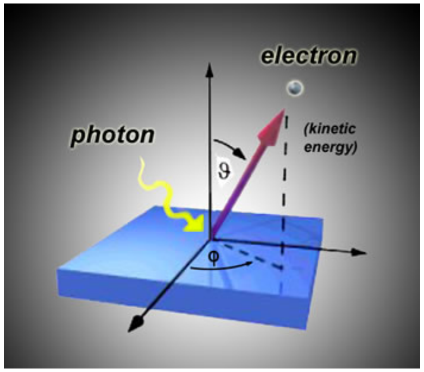
v ≅ 10^6 m/s = c/300

Can we see this in experiments? YES

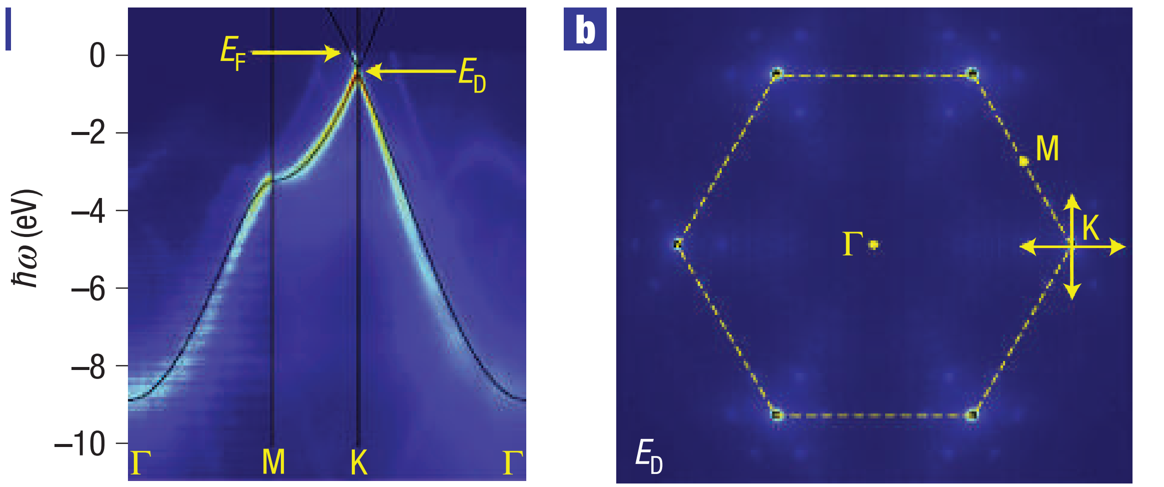
- ARPES
- STM
- Conductivity
- "Klein tunneling"
- Landau levels

Will probably not talk about conductivity as it is a bit more subtle and we don't have so much time.

ARPES



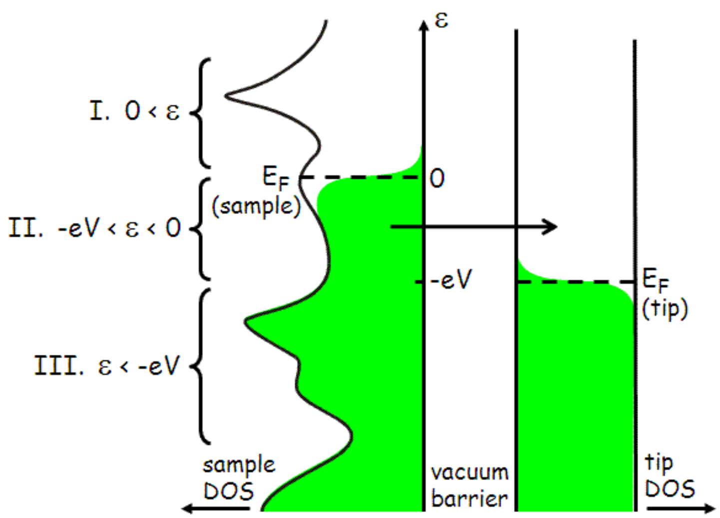
Bostwick *et al*, 2006



solid line is a plot of the energy
we calculated in class!

STM

An STM measures the "local" density of states



Current is proportional to the number of states in the sample between E_F and E_F + eV

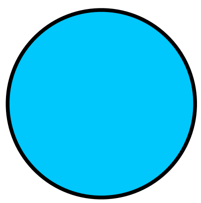
$$I \propto \int_{\epsilon_F}^{\epsilon_F + eV} d\epsilon \, g(\epsilon)$$

$$\frac{dI}{dV} \propto g(eV)$$

STM

DOS of graphene

of states with energy between 0 and E?



2 from spin

$$k_F = \epsilon_F / v$$

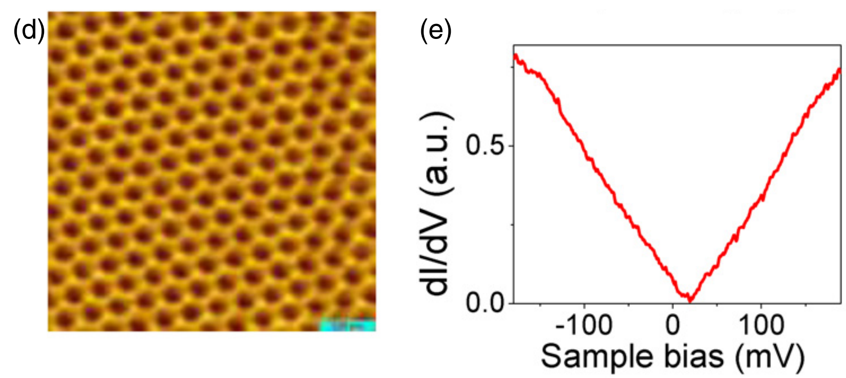
$$N(\epsilon) = 2 \times \frac{\pi k_F^2}{(2\pi)^2} = \frac{k_F^2}{2\pi} = \frac{\epsilon^2}{2\pi v^2}$$

DOS (per unit area)

E. Andrei *et al*, 2012

$$g(\epsilon) = 2 \times \left| \frac{dN}{d\epsilon} \right| = \frac{2|\epsilon|}{\pi v^2}$$

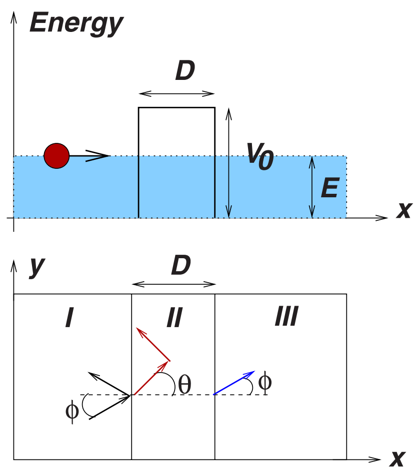
2 from two K points



Klein "tunneling"

Dirac electrons can pass through a barrier without *any* reflection

It is not really tunneling as what happens is they turn from electrons into holes



Just solve Dirac equation for a potential barrier

- In each region solution is a free plane wave
- Match wavefunction at both interfaces x=0,D

Dirac equation

$$\left[v q_\mu \gamma^\mu + V(x)\right]\psi = E\psi$$

$$\begin{pmatrix} V & v(q_x + iq_y) \\ v(q_x - iq_y) & V \end{pmatrix}\psi = \epsilon\psi$$

$$\begin{pmatrix} 0 & v|q|e^{i\theta} \\ v|q|e^{-i\theta} & 0 \end{pmatrix}\psi = (E - V)\psi$$

Solution is:

$$\psi = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ se^{-i\theta} \end{pmatrix} e^{iq_x x + iq_y y}$$

$$\theta = \arctan\left(\frac{q_y}{q_x}\right)$$

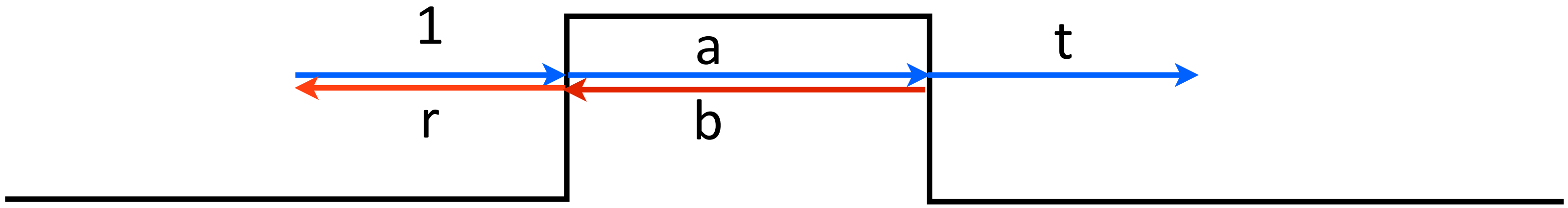
$$E - V = sv|q|$$

$$s = \pm 1$$
$$= \mathrm{sign}(E - V)$$

Scattering problem

Take for simplicity "normal incidence" k_y=0

$k_x = E/v$ $\qquad$ $q_x = (V - E)/v$



$$\psi_I = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}e^{ik_x x} + \frac{r}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}e^{-ik_x x} \qquad \psi_{III} = \frac{t}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}e^{ik_x x}$$

$$\psi_{II} = \frac{a}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}e^{iq_x x} + \frac{b}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}e^{-iq_x x}$$

Matching

$$\psi_I(x=0) = \psi_{II}(x=0)$$

$$\psi_{II}(x=D) = \psi_{III}(x=D)$$

Scattering problem

Matching

$$1 + r = a + b \qquad ae^{iq_x D} + be^{-iq_x D} = te^{ik_x D}$$

$$1 - r = -a + b \qquad -ae^{iq_x D} + be^{-iq_x D} = te^{ik_x D}$$

Solve

$$a = r \qquad te^{ik_x D} = be^{-iq_x D} = e^{-iq_x D}$$

$$b = 1 \qquad t = e^{-i(k_x + q_x)D}$$

$$a = r = 0$$

NO reflection. Perfect transmission.

You can find general solution away from Normal incidence in the Castro Neto review article

Landau levels

Landau levels are a general phenomena
for electrons in magnetic fields

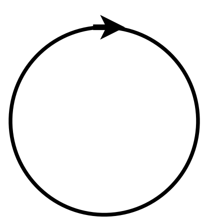
$$\hbar \frac{d\mathbf{k}}{dt} = -e\mathbf{v}_n(\mathbf{k}) \times \mathbf{B}$$

Which implies

$$\mathbf{k} \cdot \mathbf{B} = \text{const}$$

$$\epsilon_n(\mathbf{k}) = \text{const}$$

So electrons just circle in constant energy
contours in k space



electron interferes with itself
and orbit becomes *quantized*

This works differently for ordinary "Schrödinger" and Dirac electrons