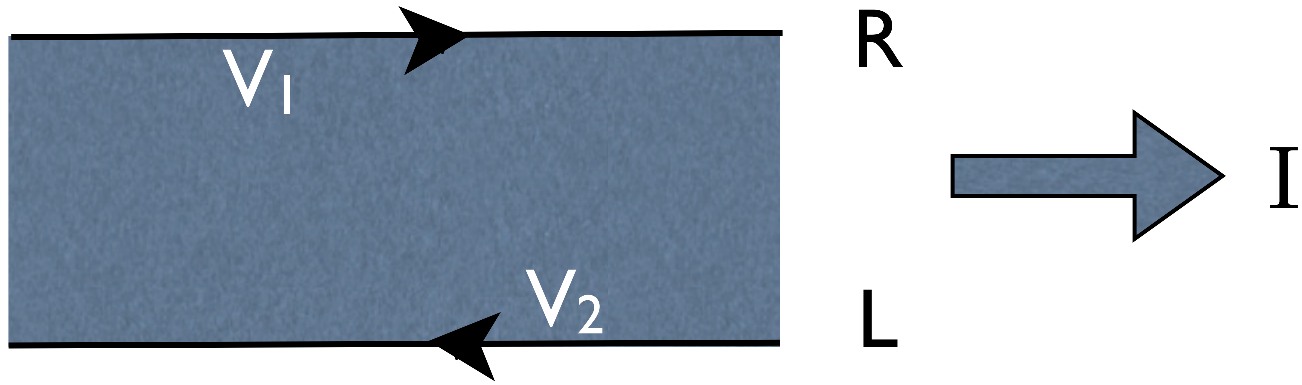
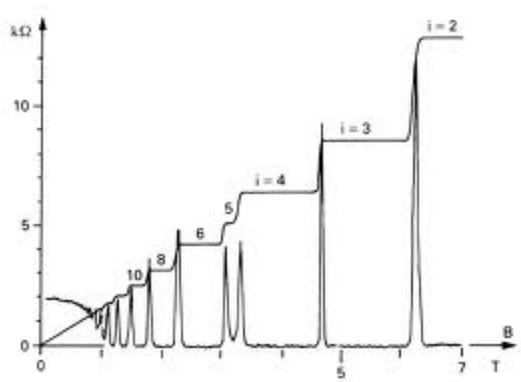
Quantum Hall effect



$$I_x = N\frac{e^2}{h}V_y \qquad G_{xy} = N\frac{e^2}{h}$$

n.b. h/e^2 = 25 kOhms

$$V_x = 0$$



Quantization of Hall resistance is incredibly precise: good to 1 part in 10^{10} I believe.

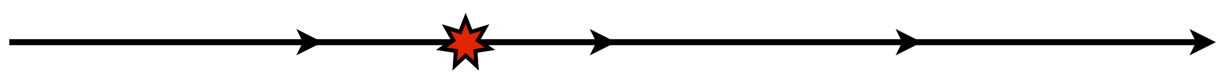
WHY??

Robustness

- Why does disorder not mess it up more?
- Answer: this is due to the *macroscopic separation* of *protected chiral edge states*
- Two ingredients:
  - Scattering at a single edge does not affect it
  - Scattering between edges is exponentially suppressed

Single edge

- Scattering at one edge is ineffective because electrons are chiral: they cannot turn around!



electron just picks up a phase shift

In equations: $H = \hbar v q_x \to -i\hbar v \partial_x$

$$\left[-i\hbar v \partial_x + V(x)\right]\psi = \epsilon\psi$$

$$\psi(x) = e^{-\frac{i}{\hbar v}\int_0^x dx'\, V(x')} e^{i q_x x} \qquad \epsilon = \hbar v q_x$$

Robustness

- To backscatter, electron must transit across the sample to the opposite edge. How does this happen?
- Think about bulk states with disorder

$$\frac{d\mathbf{k}}{dt} = -e\frac{\mathbf{k}}{m} \times \mathbf{B} + \mathbf{F}$$

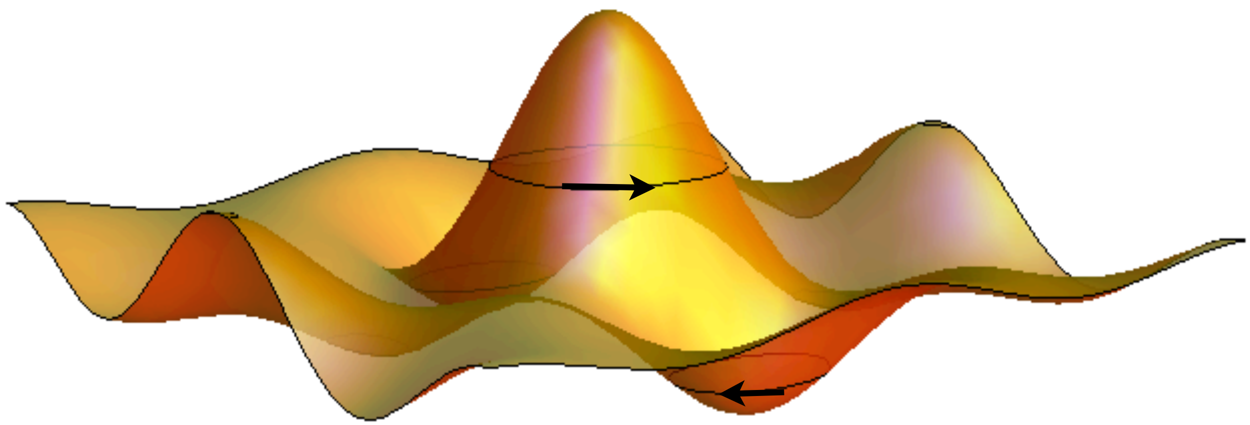
$$\mathbf{F} = -\nabla U(\mathbf{r})$$

high field $\mathbf{k} \approx \frac{m}{e}\mathbf{F} \times \mathbf{B}$

describes center of mass drift of small cyclotron orbits - along equipotentials

Robustness

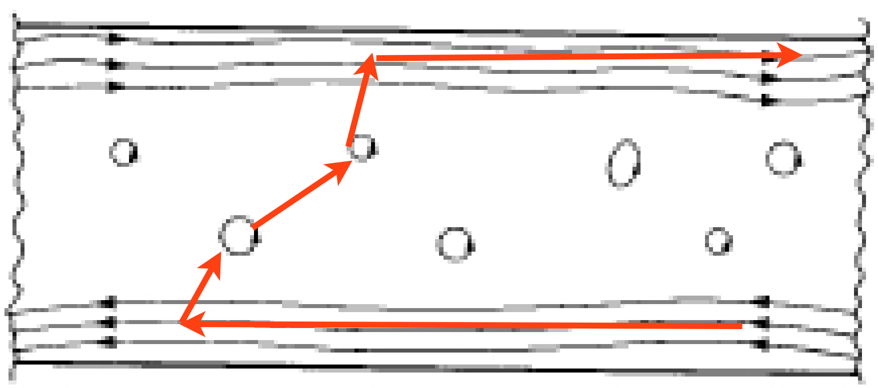
- Bulk orbits $\mathbf{k} \approx \frac{m}{e}\mathbf{F} \times \mathbf{B}$



Note: near edge where force is normal to boundary, this just gives the edge state

Robustness

- In real samples, disorder is important, and splits the degeneracy of the bulk LL states
- BUT...it cannot "back-scatter" from one edge to another - "protected" edge state



In the center of a Hall plateau, it looks like this

Electrons need to quantum tunnel from one localized state to another to cross the sample and backscatter

Robustness

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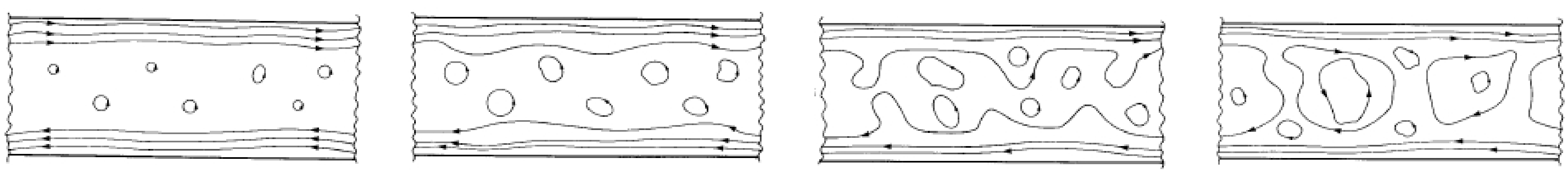
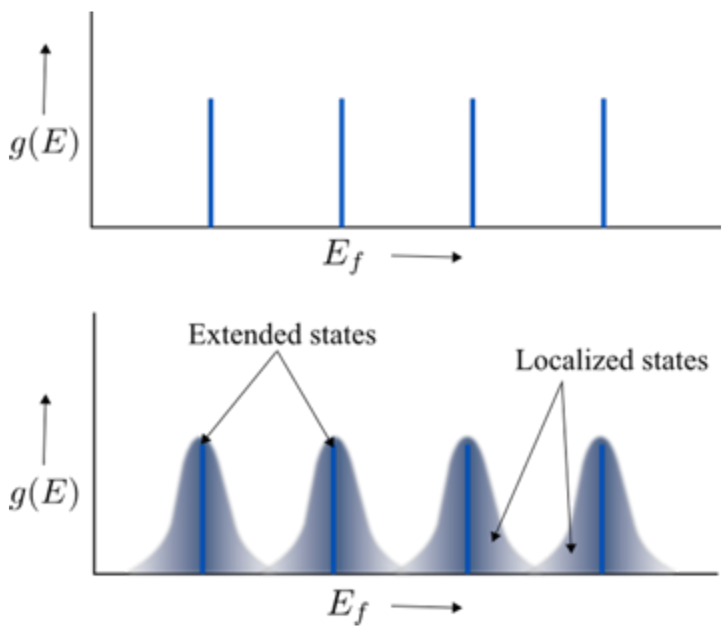
- BUT...it cannot "back-scatter" from one edge to another - "protected" edge state



somewhere here an edge state "peels off" from boundary and crosses the bulk

Robustness

- Quantum picture



it turns out truly delocalized states occur only at one energy...this is NOT obvious

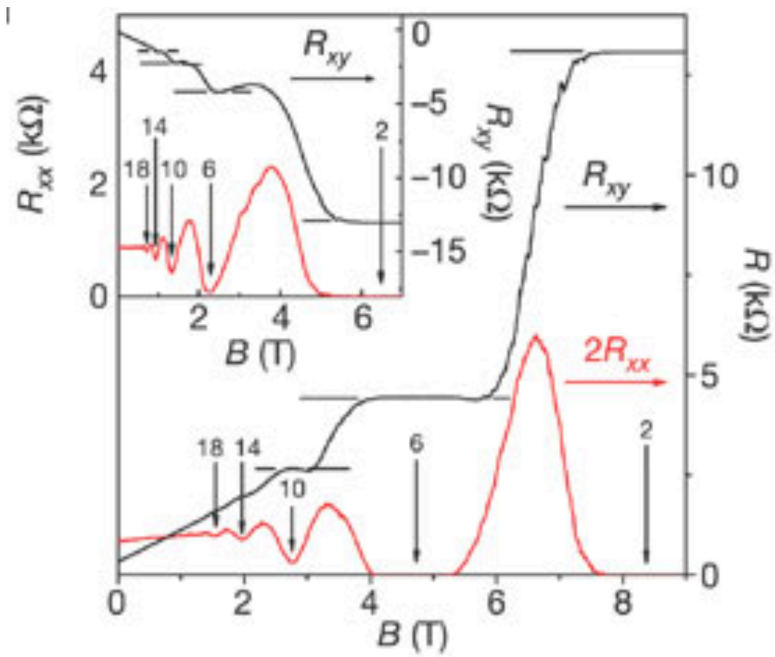
consequently, IQHE “steps” in Hall conductance are expected to be *infinitely sharp* at T=0, for a large sample

IQHE phases

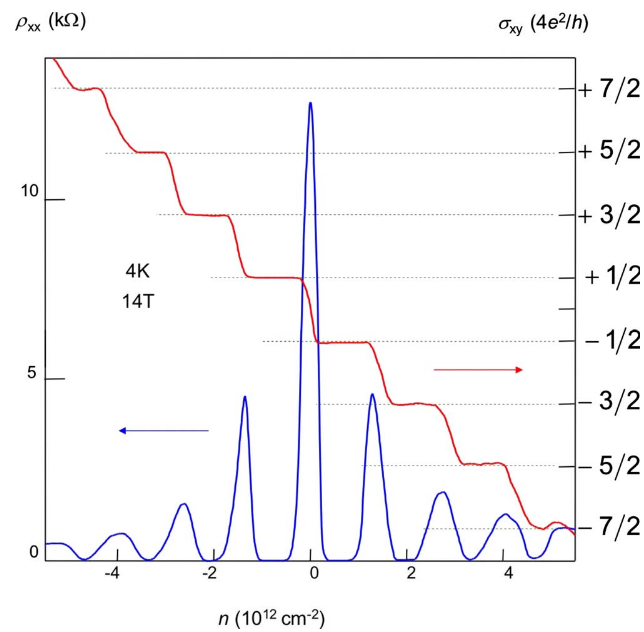
- Actually, the states with different integer quantum Hall conductivity are *different phases of matter* at T=0: they are sharply and qualitatively distinguished from one another by σ_{xy}
- This means that to pass from one IQHE state to another requires a quantum *phase transition*: this corresponds to the point at which the edge state delocalizes and "percolates" through the bulk
- However, unlike most phases of matter, IQHE states *break no symmetry*
- They are distinguished not by symmetry but by "topology" (actually the Hall conductivity can be related to topology...a bit of a long story)

Graphene IQHE

What first experiments saw...



Zhang *et al*, 2005



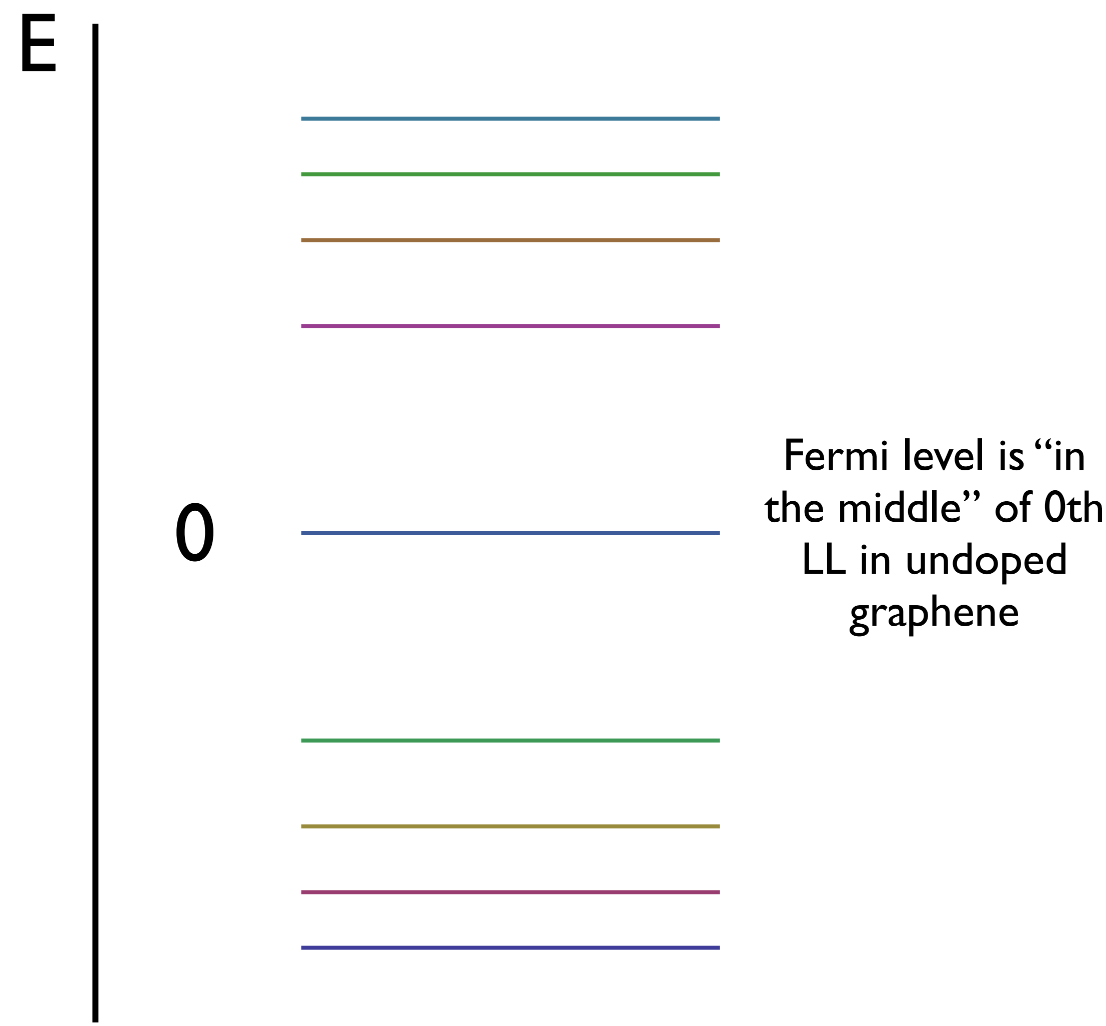
Novoselov *et al*, 2005

$$\sigma_{xy} = \frac{e^2}{h} \times (\pm 2, \pm 6, \pm 10, \cdots)$$

Relativistic vs NR LLs

- We expect σ_{xy} to change by $\pm e^2/h$ for each filled Landau level
- Each Landau level is **4-fold** degenerate
  - 2 (spin) * 2 (K,K')

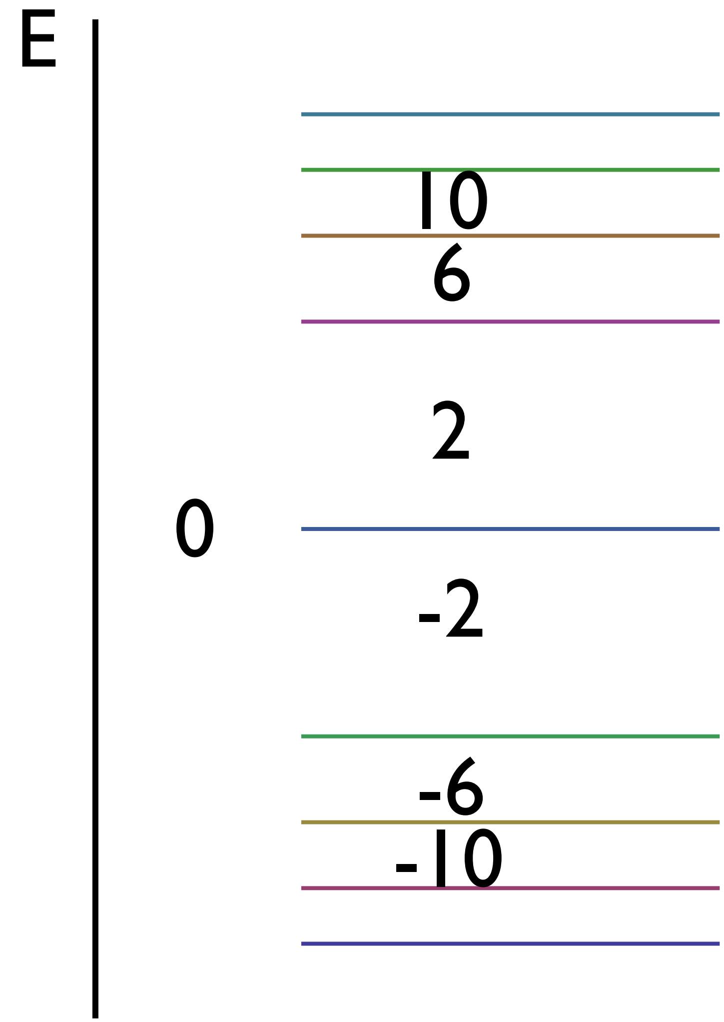
Consistent with observations, but zero is not fixed



Relativistic vs NR LLs

- Guess: when $E_F=0$, there are equal numbers of holes and electrons: $\sigma_{xy}=0$

Then we get the observed sequence

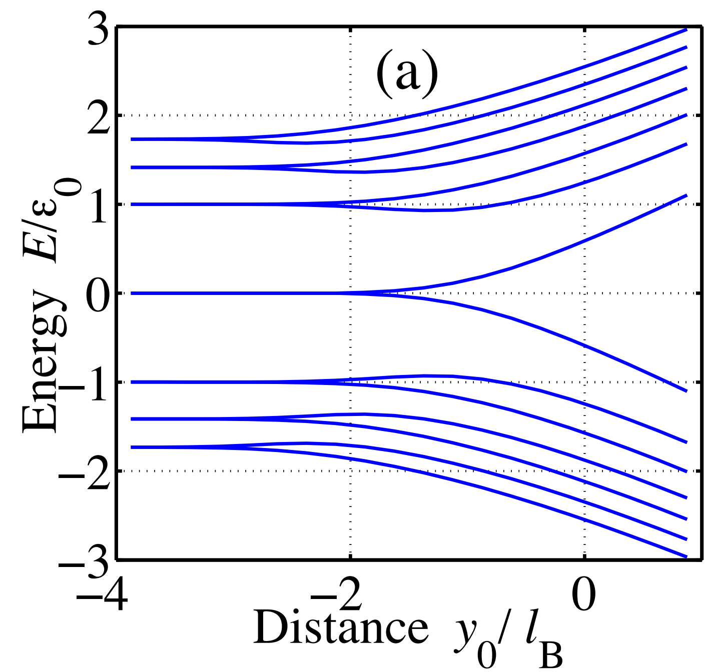


Fermi level is "in the middle" of 0th LL in undoped graphene

Edge state picture

We will not work this out here, but one can show that levels at the edge bend as shown here

The downward bending of half the levels naturally explains the observed quantization, and why the filled valence states do not contribute to σ_{xy}



the splitting of n != 0 levels shown is due to lifting of the spin/valley degeneracy, which we have not talked about.

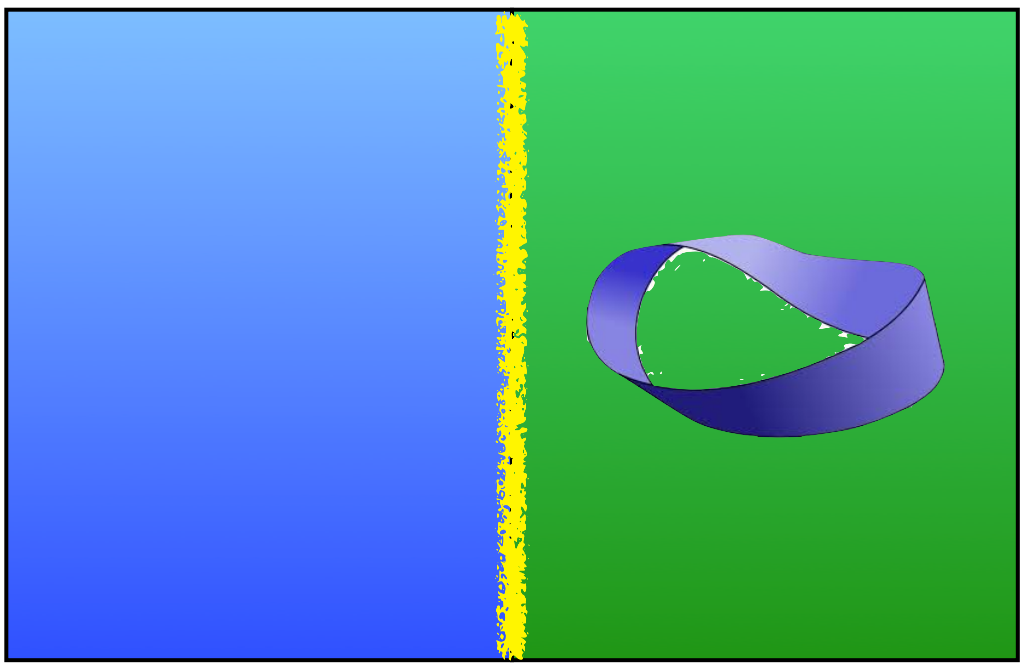
This leads to additional integer states at higher fields.

Topological Insulators

- So the IQHE states are examples of what we now call “Topological Insulators”: states which are distinguished by “protected” edge states
- Until very recently, it was thought that this physics was restricted to high magnetic fields and 2 dimensions
- But it turns out there are other TIs...even in zero field and in both 2d and 3d!

Topological insulators

- General understanding: insulators can have gaps that are "non-trivial": electron wavefunctions of filled bands are "wound differently" than those of ordinary insulators



"unwinding" of wavefunctions requires gapless edge states

Topological insulators

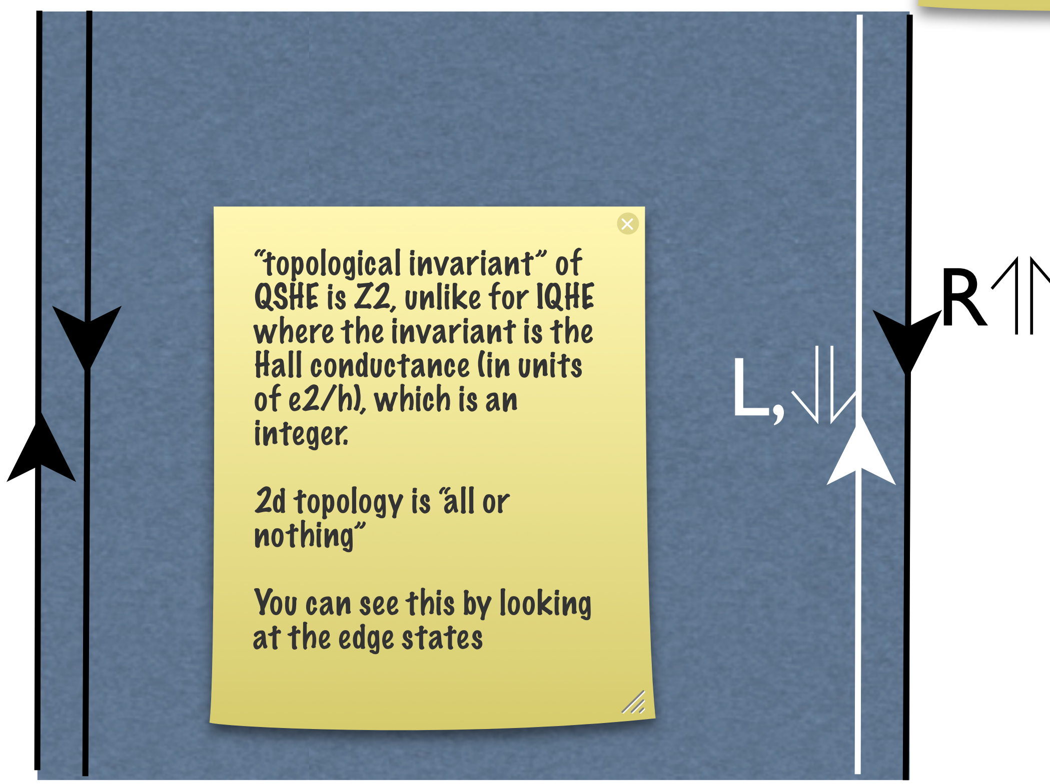
- 2005: Kane+Mele "Quantum spin Hall effect": 2d materials with SOC can show protected edge states *in zero magnetic field*
- 2007: QSHE - now called 2d "Z_2" topological insulator - found experimentally in HgTe/CdTe quantum wells
- 2007: Z_2 topological insulators predicted in 3d materials
- 2008: First experiments on $Bi_{1-x}Sb_x$ start wave of 3d TIs

Since then there has been explosive growth

QSHE

- Edge states!

To preserve time-reversal symmetry, there *must* be counter-propagating edge states, and no net spin (magnetization)



"topological invariant" of QSHE is Z2, unlike for IQHE where the invariant is the Hall conductance (in units of e2/h), which is an integer.

2d topology is "all or nothing"

You can see this by looking at the edge states

like IQHE but with counter-propagating edge states for opposite spin