Superfluidity and Superconductivity

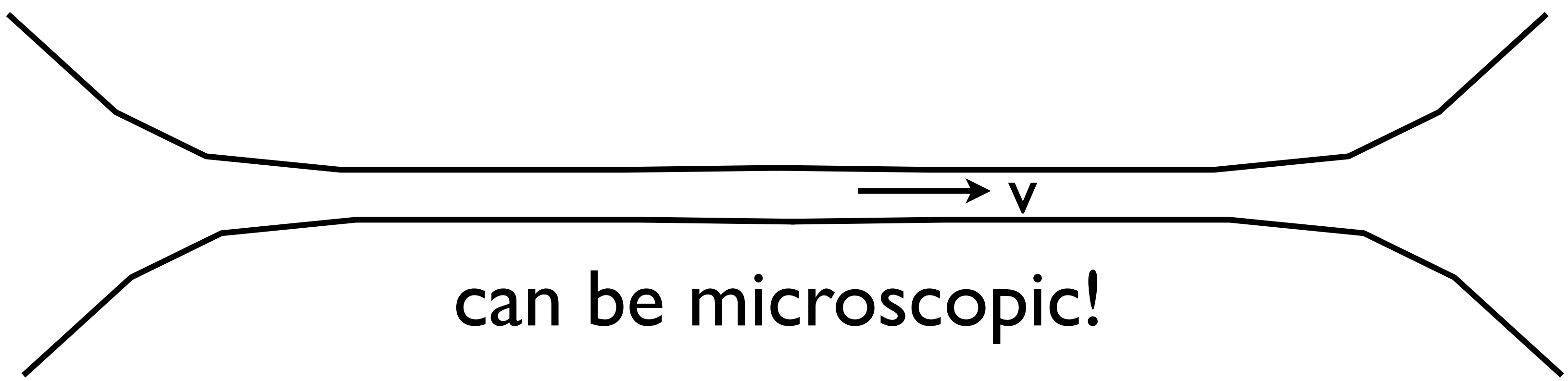
- These are related phenomena of flow without resistance, but in *very* different systems
- Superfluidity: flow of helium IV *atoms* in a liquid
- Superconductivity: flow of *electron charge* inside a solid
- History:
    - Superconductivity discovered in 1911 by Kamerlingh Onnes
    - Superfluidity discovered in 1938 by Kapitza
    - delay due to "critical temperature": T_c of Hg is 4.2K, T_c of He IV is 2.17K, so superconductivity could be discovered by helium refrigeration

Superfluidity

- We start with it, because it is conceptually simpler than superconductivity
- It is however, much more rare: only helium is superfluid (except in ultra-cold nanoK atomic traps), but many metals are superconducting
- Why?
  - Superflow is a quantum effect, and lighter particles are more quantum
  - Helium is (almost) the lightest atom, but it is still much heavier than an electron!

Superfluid properties

- Frictionless flow: helium will flow through a narrow channel without friction (no pressure drop) up to a critical velocity



can be microscopic!

Superfluid properties

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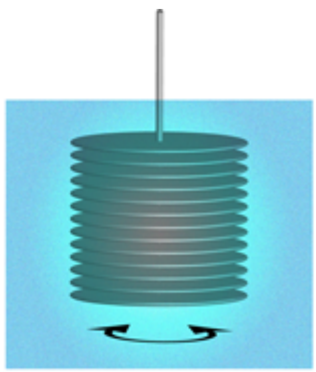
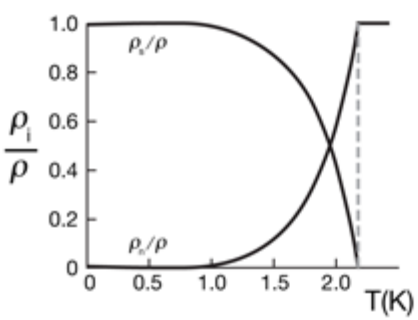


Andronikashvili experiment:

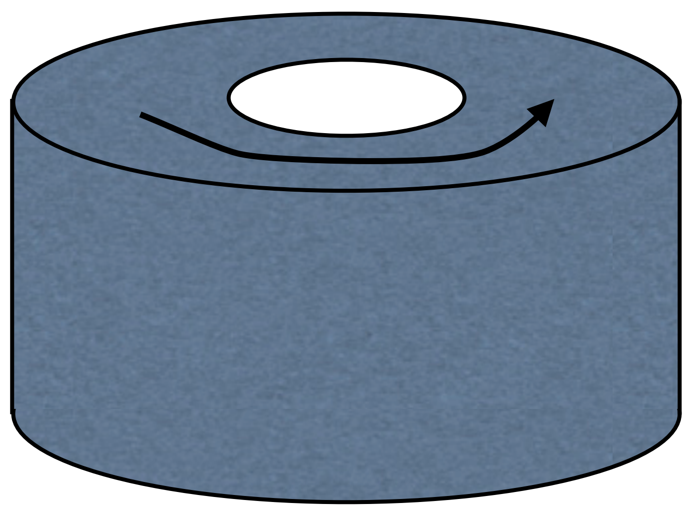
apparent mass of torsional oscillator drops below superfluid temperature

superfluid fraction



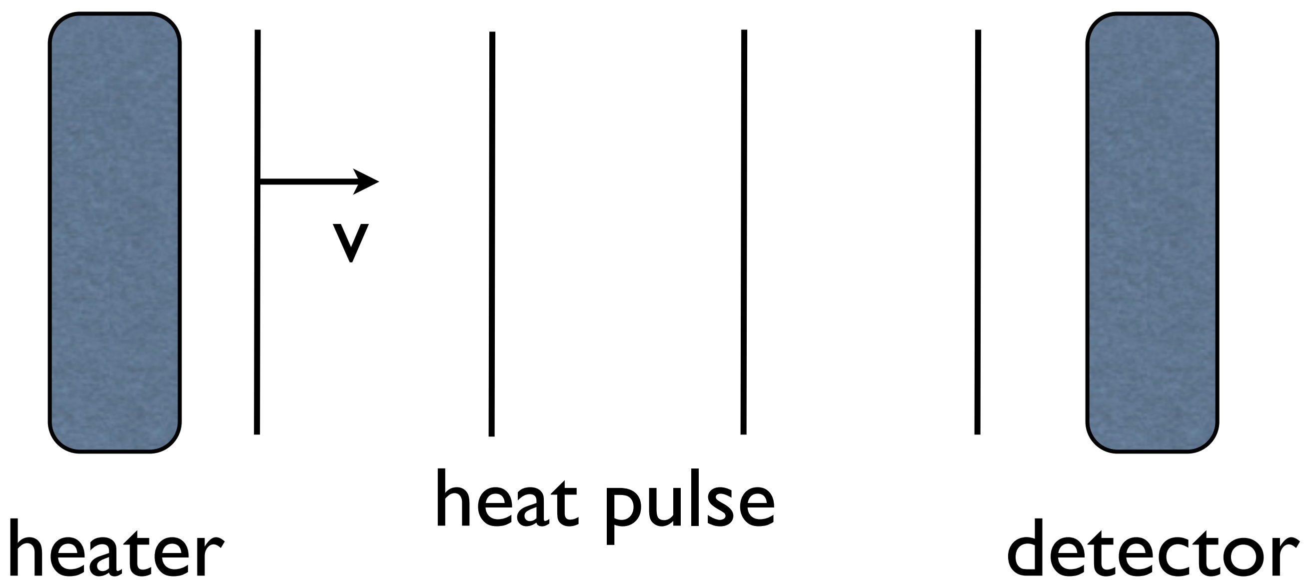
Superfluid properties

- Persistent current
  - Superfluid in an annulus (ring) will flow “forever”



Superfluid properties

- Second sound
  - Heat pulse in superfluid propagate ballistically like a wave, instead of diffusing



Bose-Einstein Condensation

- Superfluidity is a manifestation of *macroscopic quantum coherence*
- Starting picture: BEC
  - Unlike fermions, bosons *like* to occupy the same quantum state (explains why hydrogen is not superfluid. Also it is remarkable that He III - just a different *isotope* - behaves totally differently than He IV. One neutron in the nucleus matters!)
  - Free (non-interacting) bosons are described by the Bose-Einstein distribution:

$$n_b(\epsilon) = \frac{1}{e^{\beta(\epsilon-\mu)} - 1}$$

average number of bosons in a state with energy ε

$$\beta = 1/(k_B T)$$

Bose-Einstein Condensation

- Free bosons

$$\epsilon(k) = \frac{\hbar^2 k^2}{2m} \qquad \mu \leq 0$$

- Total number

$$N = \sum_k \frac{1}{e^{\beta(\frac{\hbar^2 k^2}{2m} - \mu)} - 1} \approx V \int \frac{d^3k}{(2\pi)^3} \frac{1}{e^{\beta(\frac{\hbar^2 k^2}{2m} - \mu)} - 1}$$

$$\rho = \frac{1}{2\pi^2} \int dk \frac{k^2}{e^{\beta(\frac{\hbar^2 k^2}{2m} - \mu)} - 1} = \frac{1}{2\pi^2 \hbar^3} \left(\frac{m}{\beta}\right)^{3/2} I(\overline{\mu})$$

$$I(\overline{\mu}) = \int_0^\infty dk \, \frac{k^2}{e^{\frac{k^2}{2} - \overline{\mu}} - 1} \leq I(0) = \sqrt{\frac{\pi}{2}} \zeta(3/2) \qquad \overline{\mu} = \beta\mu$$

Bose-Einstein Condensation

- Minimum density

$$\rho \leq \frac{1}{2\pi^3\hbar^3}\left(\frac{m}{\beta}\right)^{3/2}\sqrt{\frac{\pi}{2}}\zeta(3/2)$$

- At high temperature, i.e. small β, this is always satisfied. But below some T it is not. Defines

$$k_B T_c = \frac{2\pi\hbar^2}{m}\left(\frac{\rho}{\zeta(3/2)}\right)^{2/3} \approx 3.31\frac{\hbar^2\rho^{2/3}}{m}$$

- What happens for $T<T_c$?

BEC

- Below T_c, the chemical potential gets "almost" to zero. Then n(k=0) becomes *macroscopic*
- So for $T<T_c$, a macroscopic fraction of bosons occupies *one* quantum state
- This one state extends over the whole system, and basically corresponds to the superfluid
- Caveat: BEC as described really applies to non-interacting bosons. He IV atoms are very strongly interacting. But still the BEC starting point is qualitatively good. And we mostly just need the idea.

Landau-type theory

- Idea: the key property of the superfluid is a macroscopically occupied quantum state. We describe it with a *macroscopic wavefunction*

$$\psi(r) = \sqrt{n_s(r)}e^{i\theta(r)}$$

- Here n_s is the *superfluid density* and θ is the phase of the superfluid
- In equilibrium n_s is constant and so is θ. But there may be very stable non-equilibrium states where they vary : states with flow!

Free energy

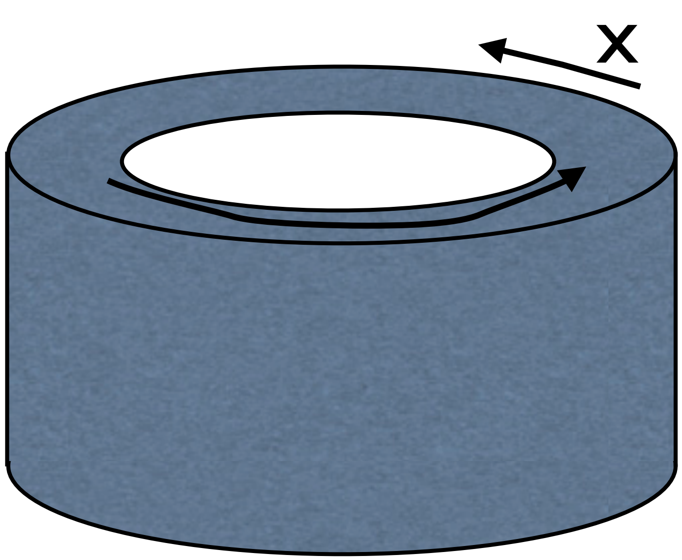
- We assume that the free energy of the system (which tends to its minimum in equilibrium) can be written in terms of n_s and θ:

$$F = \int d^3r \left[\frac{c}{2}(n_s - \overline{n}_s)^2 + \frac{\hbar^2 n_s}{2m}(\nabla\theta)^2 \right]$$

- Understand second term by noting that $\theta = \mathbf{k}\cdot\mathbf{r}$ is like putting all the "condensed" atoms into a state with momentum $\hbar\mathbf{k}$
- Superfluid velocity $\mathbf{v}_s = \frac{\hbar}{m}\nabla\theta$

Persistent current

- Obvious free energy is minimized by v_s=0.
  But suppose we set up a superflow?



narrow cylinder of
circumference L

$$v_s^x = \frac{\hbar}{m}\partial_x \theta \quad \Rightarrow \quad \theta = \frac{m v_s}{\hbar} x$$

single valued wavefunction

$$\theta(L) - \theta(0) = \frac{m v_s}{\hbar} L = 2\pi N$$

"quantized circulation"

$$v_s = \frac{h}{mL} N$$

it is very hard for circulation to decay
because it cannot change continuously!

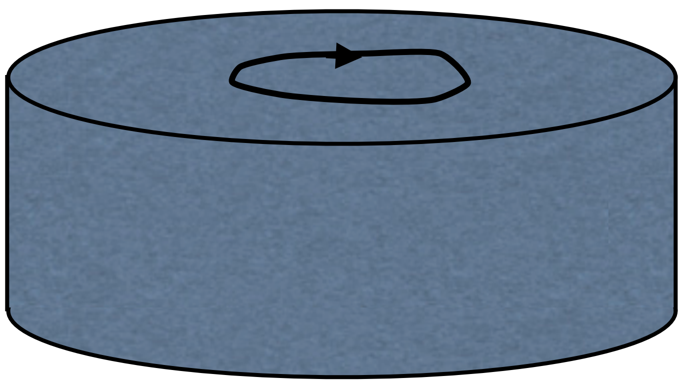
Vortices

- What happens if you rotate a bucket (not an annulus) above T_c, then cool it, then stop the bucket?
    - above T_c fluid has friction so will rotate
    - below T_c, has to keep rotating by conservation of angular momentum
- But if there is no center of the bucket, then there is a problem

Vortices

- Stoke's theorem

$$\oint \nabla\theta \cdot dr = \int \nabla \times \nabla\theta \cdot dA = 0$$

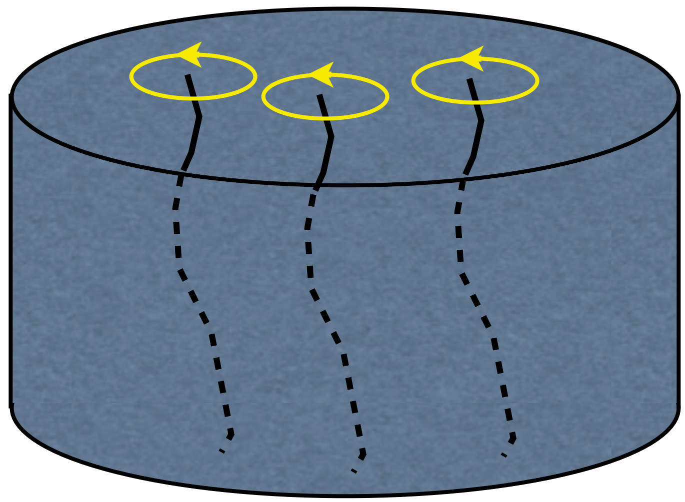


no circulation?

- In fact, circulation exists because *vortices* form. These are places in the liquid where $n_s \to 0$, and so θ is not well-defined!

Vortices

$\theta \rightarrow \theta + 2\pi$



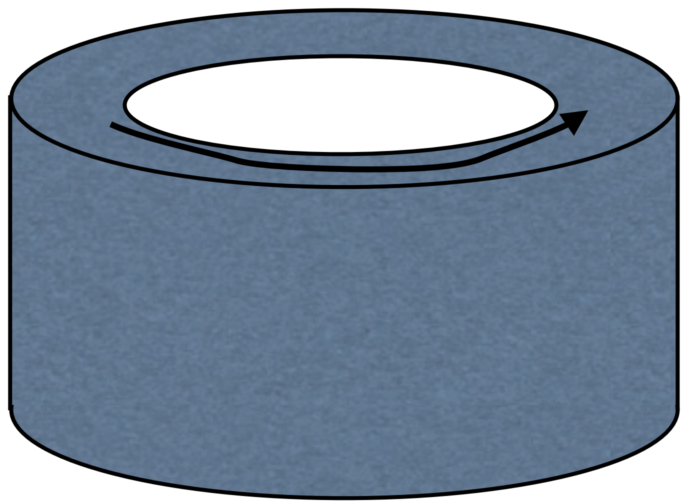
an array of vortices simulates rigid rotation of the fluid

$\oint \nabla\theta \cdot dr = 2\pi$ around a single vortex

- Vortex must escape the system - moving into the wall - to lower the circulation
- They will escape, because the vortex costs free energy - especially the core where $n_s = 0$, but also the $v_s \neq 0$ outside

Persistent current

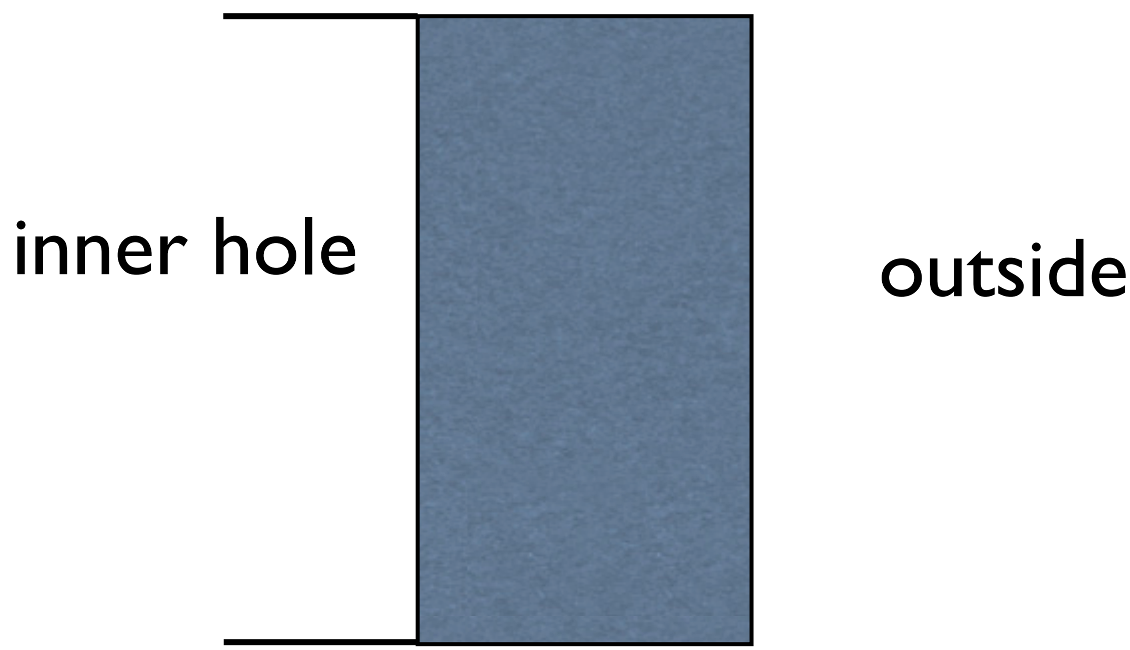
- In an annulus, one has a “giant vortex” in the hole: no core energy



- To decay, the circulation must escape *one vortex at a time*

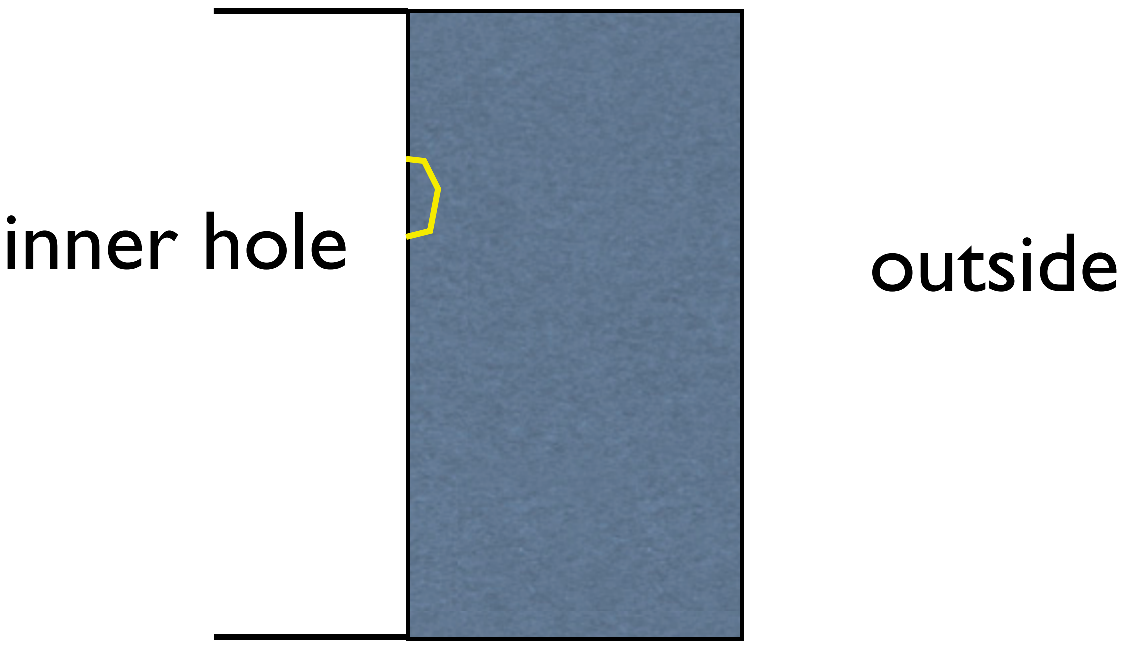
Vortex escape

- radial cut of annulus



Vortex escape

- radial cut of annulus



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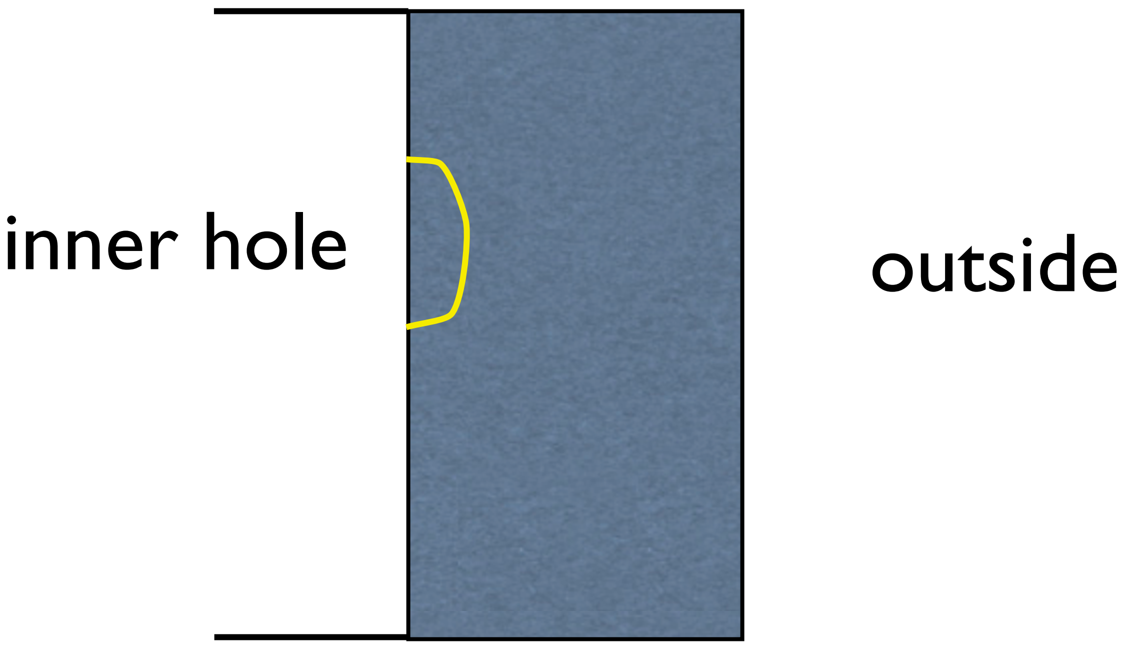

Vortex escape

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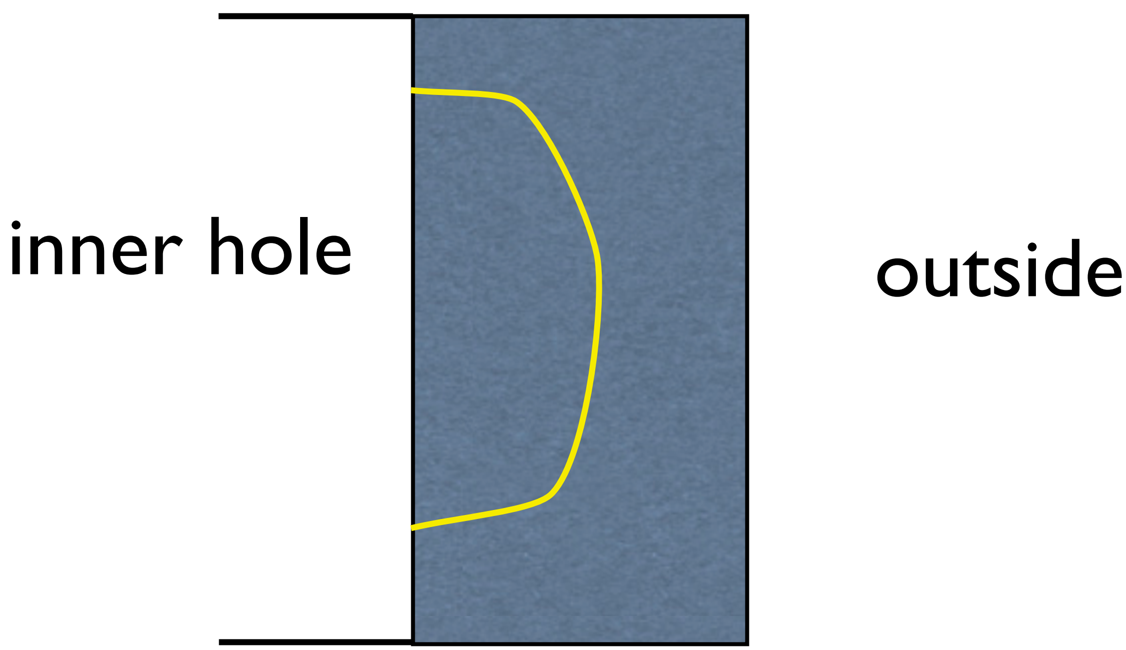
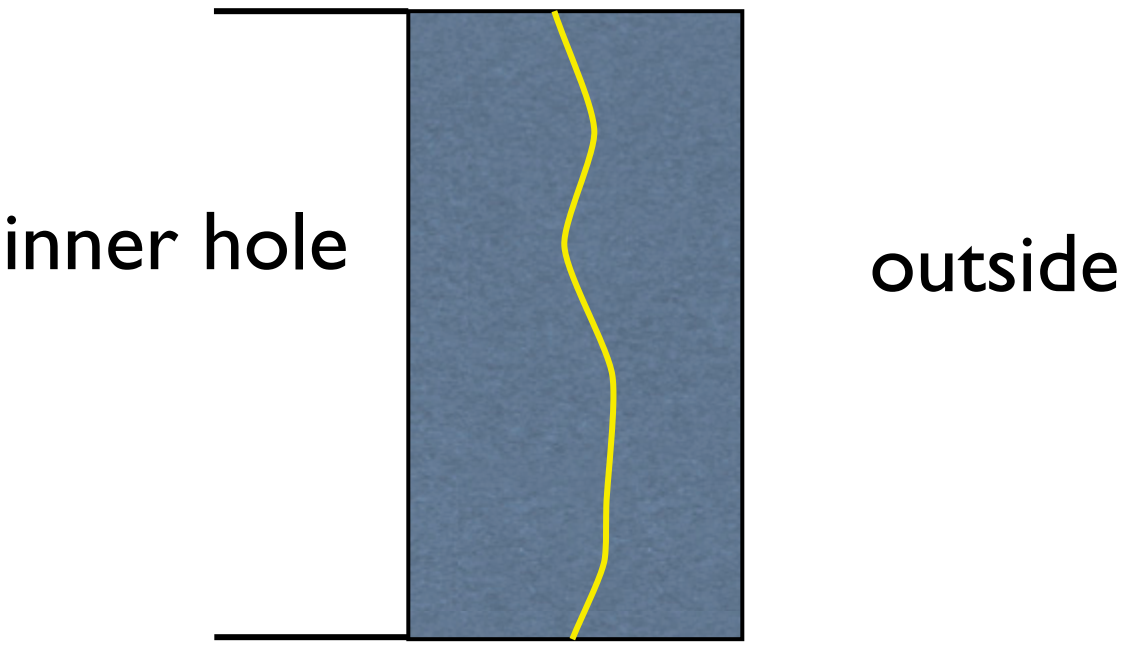


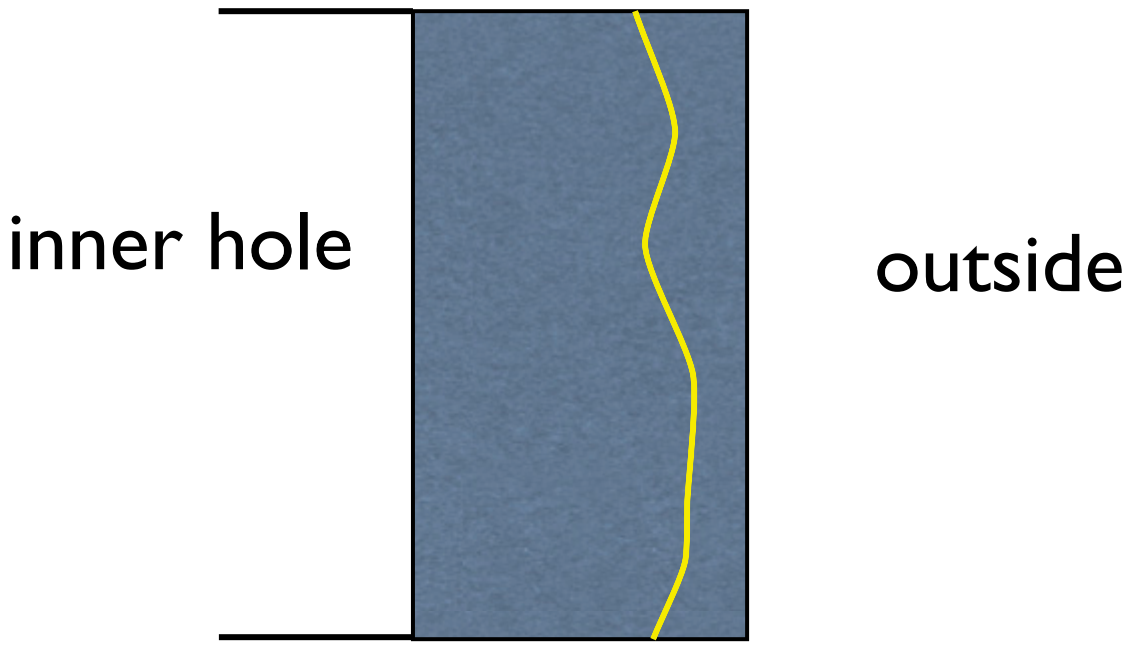
Vortex escape

- radial cut of annulus



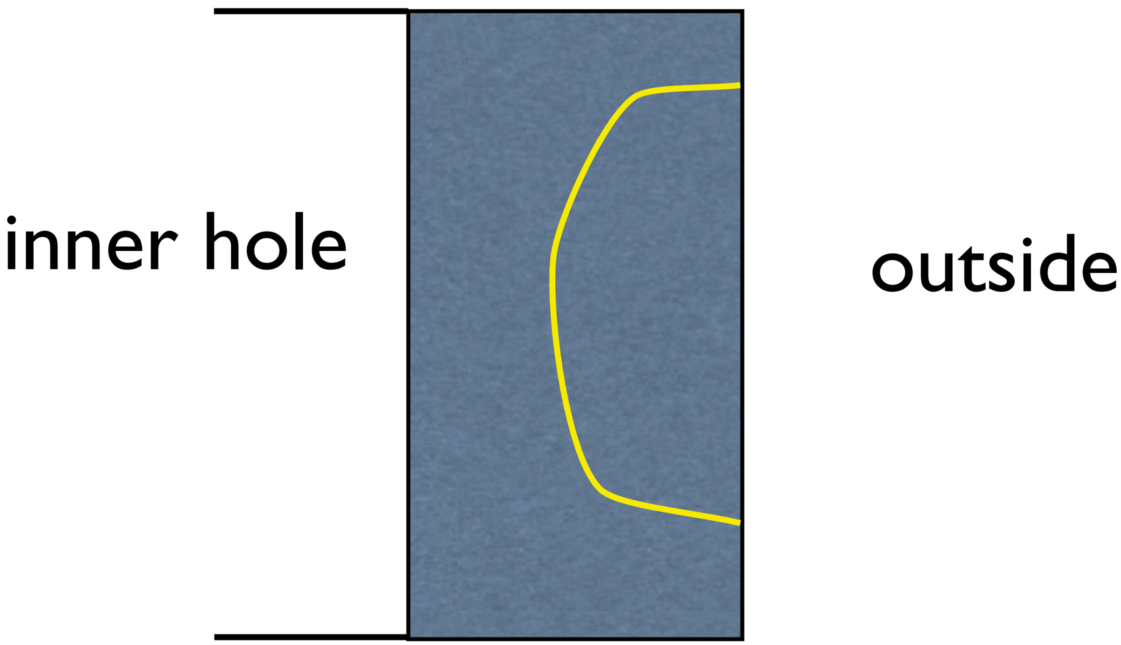
Vortex escape

- radial cut of annulus



Vortex escape

- radial cut of annulus



Vortex escape

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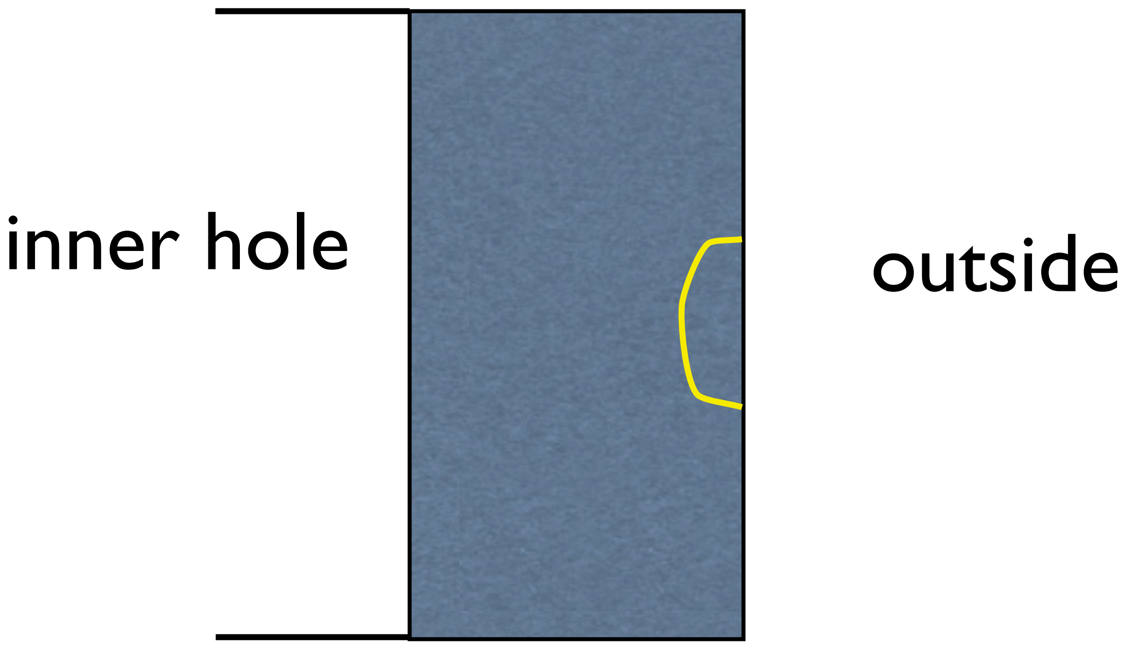
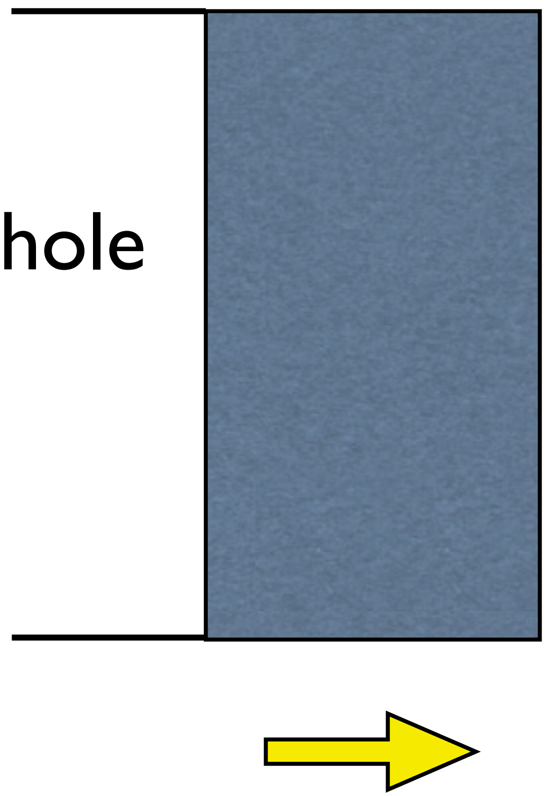


Vortex escape

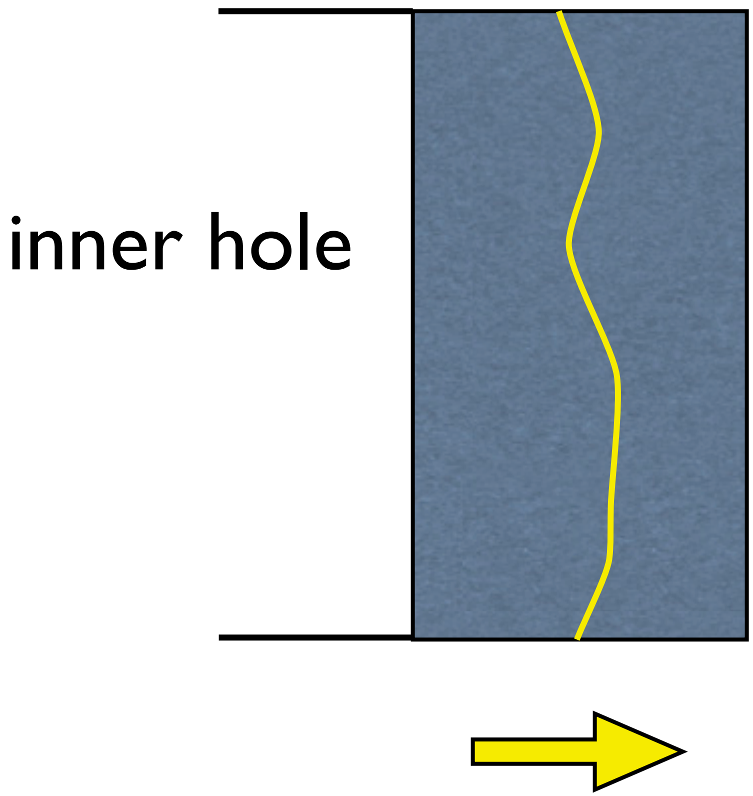
- radial cut of annulus



after this process, one quantum of circulation has been removed

Vortex escape

- radial cut of annulus

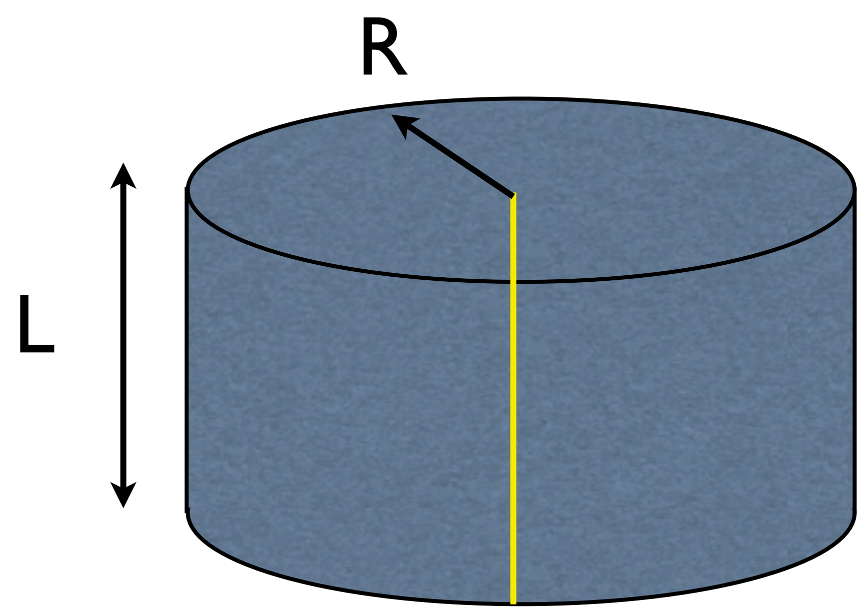


the intermediate state contains a long vortex line, which costs free energy. This free energy barrier requires thermal activation to overcome

after this process, one quantum of circulation has been removed

Vortex free energy

- Crude estimate $$F = \int d^3r \, \frac{\hbar^2 n_s}{2m} (\nabla\theta)^2 \qquad |\nabla\theta| = 1/r$$



$$F \approx L \int_a^R dr 2\pi r \frac{\hbar^2 n_s}{2m} \left(\frac{1}{r}\right)^2$$

$$\approx \frac{\pi\hbar^2 n_s L}{m} \ln(R/a)$$

even neglecting the log, this is of order 0.1K/Å

rate of thermal activation ~ $\exp(-F/k_B T)$

since T < 2.2K, this is negligible even for sub-micron dimensions

Second sound

- How does superfluid helium carry heat so quickly?
- Usually, heat diffuses if there is no fluid flow
- But superfluid helium behaves like *two fluids*: a normal and superfluid part
- Second sound = opposite flow of superfluid and normal fluids: no net mass flow but net heat flow
  - This is because the normal fluid convects heat, but the superfluid has *zero entropy* because it is a *single quantum state!*