HOLOGRAPHY
OCTOBER, 2006

OVERVIEW

Since the last writing (2002), many changes and improvements have been made during the past few years. Holography is now offered in both sessions of PHYS 128A/B, allowing more students to perform the lab. The optics table has been divided into two partitions which allows a specific set-up to remain intact. There is enough equipment to permit two simultaneous labs to run without daily tear-down and set-up. The lab now has two electromechanical shutters. All the lower division accessories have been replaced with professional accessories. We stress using the table's mounting capabilities. All the accessories mount to the table. Magnetic mounts have been eliminated (except for the shutter assembly). Each lab group has their own equipment drawer outfitted with all the accessories needed to perform the lab. This includes both hardware and optical components.

The processing of the film has also changed. Standard developing techniques and chemicals are used.

INTRODUCTION AND OUTLINE

Introduction

In traditional photography, objects are transformed into a two-dimensional image onto the surface of a film or CCD via a lens. This image is a latent image. It becomes visible after processing (developing). The finished product (photograph) has some noticeable limitations. One, it lacks depth of field, that is you can not "see" behind the subject. Secondly, if the photo is torn into shreds, you do not have a complete photo.

Holography, on the other hand does not utilize a lens to form a latent image. It relies on the principle of interference to produce an image. The interference pattern is captured by the film and processed. The hologram is then illuminated with laser light (transmission) or white light (reflection). A holographic image is then observed.

To produce an interference pattern, a single (coherent) source is split and then re-combined. Any path differences are manifested as dark and light rings or patterns. This basic requirement is also a characteristic of holograms. A laser beam is split or caused to be divided. One beam is sent directly to the photographic plate or film. This is the reference beam. The other beam illuminates an object. The scattered light is then sent to the film. This is the object beam. There, the constructive and destructive interference patterns are captured. These patterns are not visible to the naked eye. However you may notice "bull's-eye" patterns or lines on holograms. These are interference patterns usually produced by the optical system and/or by dust on the surfaces of optical components. This is called "noise."

After processing, laser light or white light is directed on or through the hologram depending on which type is being viewed. The reflected or transmitted light is scattered by the interference patterns into your eye. At this point, the eye-brain system interprets the light rays and reconstructs the image of the original object. It is able to do this because all the space-time information has been captured by the interference patterns on the hologram. Hence, the eye-brain mechanism cannot distinguish any difference between the light scattering off the original object and the light being redirected (via the diffraction pattern) by the hologram. In either case the "image" is the same.

Outline

      1. Holographic media, specifically, plates and film.

      2. Viewing a reflection hologram.

      3. Creating a reflection hologram.

      4. Viewing and creating a transmission hologram.

      5. Making an interferometer.

      6. Making a reflection hologram

      7. Making a transmission hologram.

      Appendices
            A. Theory
            B. Wet lab procedures

Deliverables – What is expected?

You will make a Michelson interferometer, at least two reflection holograms, and at least two single side-referenced transmission holograms.

1. Film

The film we will be using is an all purpose film for reflection and transmission holograms. The type is PFG-01. It is a fine-grained red sensitive holographic film designed for transmission or reflection hologram recording. Average grain size is 40 nm, resolving power more than 3000 lines/mm, spectral sensitivity range 600-680 nm (including 633 nm, 647 nm), and emulsion thickness 7-8 um.

To identify the emulsion side, lightly moisten your lips and place the film between your open lips. Compress your lips onto the film. Do not drag film through your lips. Now open your lips. What do you experience? Have you found the emulsion side?

Photographic film is a sheet of plastic (polyester, celluloid (nitrocellulose) or cellulose acetate) coated with an emulsion containing light-sensitive silver halide salts (bonded by gelatin) with variable crystal sizes that determine the sensitivity and resolution of the film. Sometimes the substrate is a glass plate. When the emulsion is subjected to sufficient exposure to light (or other forms of electromagnetic radiation such as X-rays), it forms a latent (invisible) image. Chemical processes can then be applied to the film to create a visible image, in a process called film developing.

In black-and-white photographic film there is usually one layer of silver salts. When the exposed grains are developed, the silver salts are converted to metallic silver, which block light and appear as the black part of the film *negative*.

We will be using holographic (plastic) film because of cost issues and ease of use.

Q 1.1 What is the difference between a latent image and a holographic image?

2. Viewing a reflection hologram

Locate the box of reflection holograms in the lab. Select one and hold it in such a manner as to have the room light reflect off the surface. Reorient the film until you view an image.

Now turn on the slide projector. Place the reflection hologram in the white light beam. Orient the film to view the image. Go outside (if applicable) and let sunlight reflect off the hologram.

3. Creating a reflection hologram

Reflection holograms have a few rare subtleties. First they are very inefficient. Second, they can be viewed with white light. Let's take a closer look at the film (Figure 1).
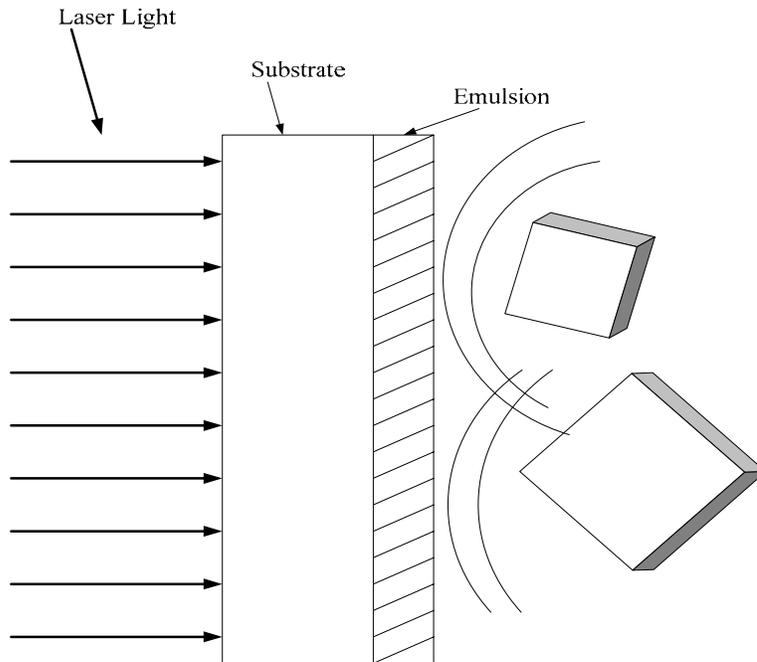

Figure 1. Film during reflection hologram

The laser light passes through the substrate and then through the emulsion. In this trek a Bragg-plane (standing wave) is established by the incoming beam and the reflected light. As the light exits, it proceeds toward the objects that now scatter light back toward the emulsion where it interferes with the light that is passing through the emulsion. Hence a standing wave pattern and an interference pattern are generated. These holograms are also known as volume holograms. To view the hologram, illuminate from the front and move the film around to observe the reconstructed image. The reason that the reflection hologram can be viewed under white light is that the Bragg planes are spaced in such a manner to allow the wavelength of the original light source to be generated in the emulsion and hence be reflected back with the interference pattern for the eye-brain to interpret.

Two things to note here. One, the emulsion faces the object(s) and two; the objects are close, if not touching, the film.

4. Viewing a transmission hologram

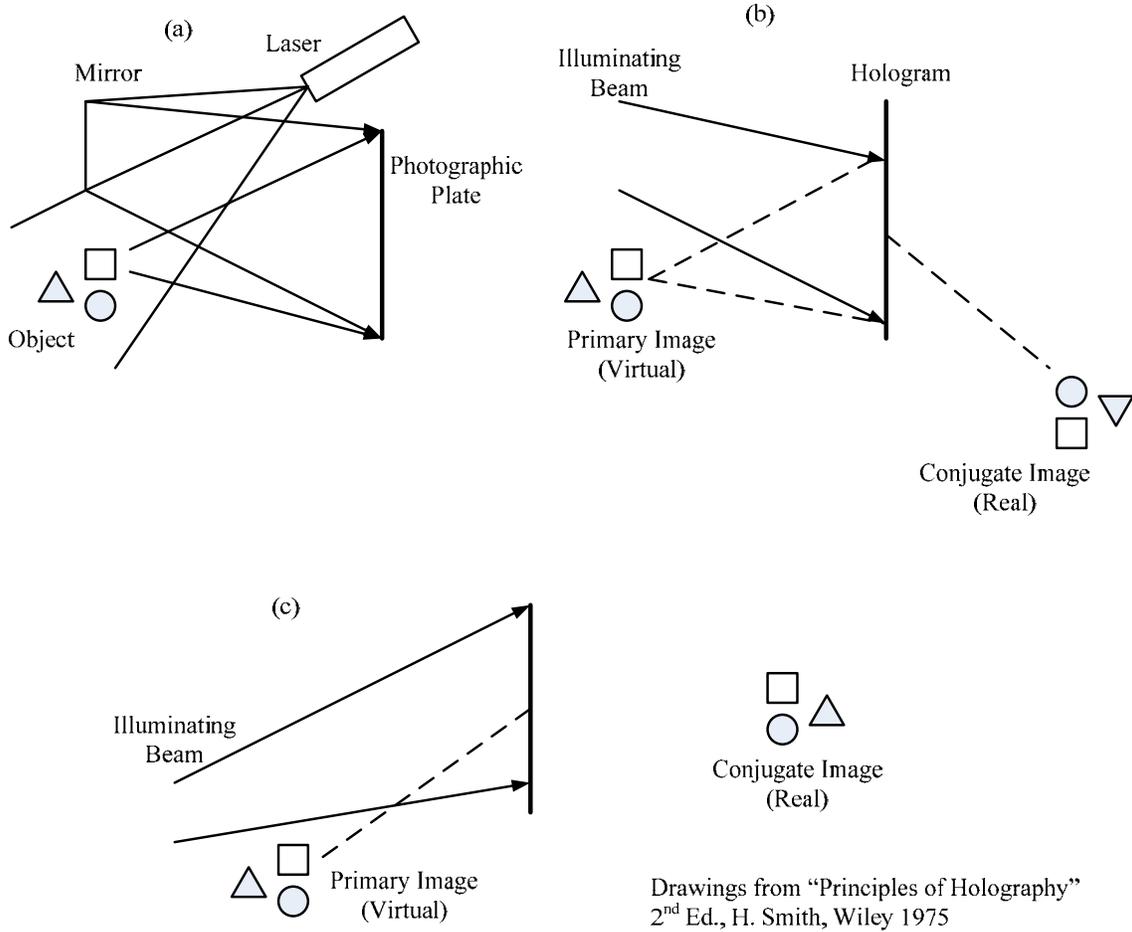Refer to the figure (2) below.

Figure 2.

Figure 2(a) shows one possible layout to make a transmission hologram. Note that part of the beam is directed to the photographic plate (reference beam) and the remainder illuminates the object. Light that scatters off the object onto the film interferes with the reference beam producing the hologram.

Figure 2(b) shows how to view the hologram. Be sure the emulsion side faces the laser. Essentially, the hologram is illuminated with light of the same wavelength that was used to make it. This laser light strikes the hologram at the same angle that the reference beam struck the film during the exposure (2a). Move the film to obtain the best image of the object. See if you can locate both images. The first image is a replica of the object. A second image can be seen usually upside-down, distorted, and having an inverted depth

of field. This image is the conjugate image also sometimes called a "pseudoscopic" image.

If you re-orient the film (2c) so as to illuminate the hologram at the mirror-image angle you will be able to view the conjugate image as a real image. Use a screen to find the image.

Activity I

Take one of the transmission holograms from the container and mount it in the clamp-ring stand assembly. Be sure the beam spreader (modified microscope objective) is installed. Turn on the laser. Orient the hologram so that the beam covers the entire hologram. Turn and rotate the hologram to get maximum brightness. <u>Perform the above operations to find the images.</u> *Caution: The transmission holograms in the box are on glass substrates. Handle with care.*

There are two interesting properties of transmission holograms. The first is that there is depth of field. By repositioning the hologram or your head, you can see behind and around objects. Second, the entire hologram contains information from all points of the original object. This means any part of the hologram contains information to reconstruct the *entire* hologram.

Activity II

Set up a transmission hologram for viewing. Peak for maximum amplitude. Find the black card with a hole in the center. Place the hole anywhere on the hologram and peek through the hole. What do you see?

> Q 4.1 If a hologram shattered and a piece were to be illuminated, what would you see and why?

> Q 4.2 What is sacrificed (lost) as you view your broken piece?

5. Making a Michelson Interferometer

The basic schematic for a Michelson Interferometer is shown in Figure 3.
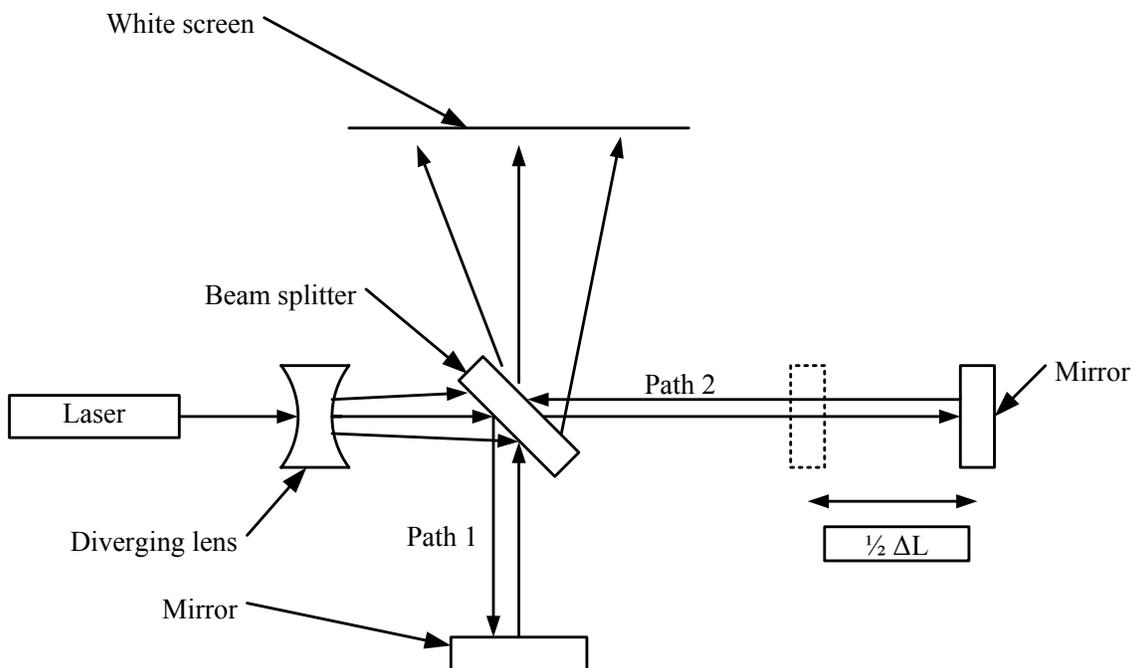

Figure 3. Michelson Interferometer Schematic

Rather than use a diverging lens, we will use a microscope objective to open the beam. The microscope objective is used in the entire lab as a beam expander.

Procedure: Refer to Figure 4

      A. The lasers will be on as you enter the lab room. The M/W and T/R groups each have an assigned area on the optics table. They are labeled on the optics table. The optics table is a dynamically stabilized platform. DO NOT LEAN ON THE TABLE. Try to perform activities without leaning on the table.

Note: All mechanical and optical components are located in the drawers below the hutch. Each group has their own cabinet assigned to them. Do NOT borrow any items from the other drawer or from the storage cabinet.

Place the electromechanical shutter (mounted on a switching magnet base) a few centimeters in front of the laser aperture. CAUTION: The beam is on. The shutter is set to block the beam in the "off" mode. If you need to perform adjustments, simply slide the shutter out of the way. When you are finished, slide the shutter back into position. DO NOT HOLD THE "ON" BUTTON FOR BEAM ADJUSTMENTS. This switch is used only during exposures.

B. Mount the beam splitter/prism-holder accessory with the necessary hardware to the table. The beam splitter is a cube splitter which will onto a smaller table. That assembly is then mounted on the optics table. Mount the hardware on the beam line of the laser and about 15-20 centimeters beyond the shutter. See Figure 4 below.
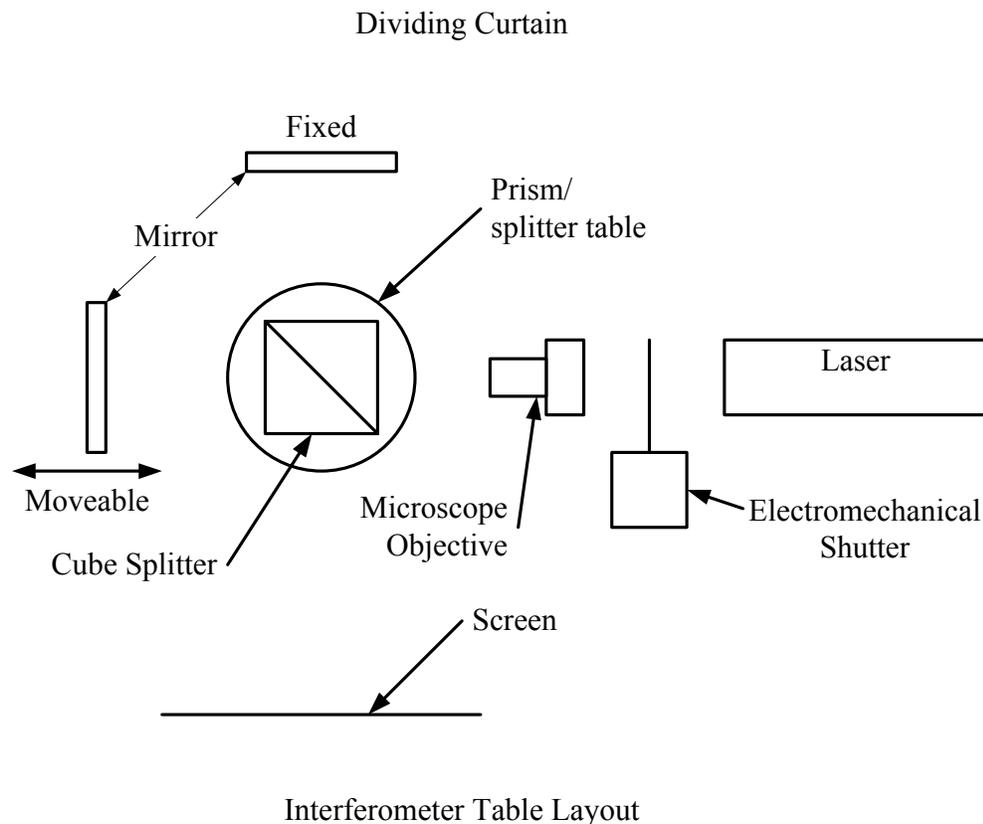

Figure 4. Interferometer layout on the optics table

C. Mount the microscope objective onto the table with the appropriate hardware from your kit. It is mounted between the shutter and the cube splitter.

D. Mount a first surface mirror about 10 cm. away from the splitter toward the curtain. This will be the fixed mirror. Note: the optics table has a matrix of 1/4-20 holes spaced on 1 inch squares. Your instructor will show you how to identify the first surface.

Caution: Never dry wipe first surface mirrors. The instructor will show you the proper method of cleaning first surface mirrors.

E. Mount a second first surface mirror to an adjustable base. Mount the base to the table. Try to get the adjustable second mirror about the same distance away from the splitter as the fixed mirror.

F. Mount a viewing screen on the side opposite the fixed mirror.

ALIGNMENT AND TUNING

A. Unscrew the microscope objective. Place it in a secure place to prevent it from Crashing to the ground.

B. Slide the shutter assembly out of the beam. Caution: The beam is now exposed. You should now see multiple laser beams.

C. First, using the adjustment controls, align the splitter assembly so that the first reflection from the cube bounces back to the laser near but not into the laser. DO NOT ALLOW THE REFLECTION TO ENTER THE LASER APERATURE. THIS WILL CAUSE IRREGULAR MODING IN THE LASER. The beam is now (nearly) perpendicular to the cube surface. This technique is called "auto-collimation."

D. Find a beam blocker (piece of cardboard) and block the light path between the adjustable mirror and splitter. You should see a single dot on the screen. Adjust the fixed mirror to auto-collimate the beam on the cube's surface.

E. Remove the beam blocker (step D). You should now see several dots. Two of them are quite bright. Adjust the second (moveable) mirror and try to hit the first dot. Do not translate the mirror at this time.

F. As you are moving the second ray to overlap the first, you may see the dot-combo twinkle. When you do, STOP. If not, try to physically overlap the two dots.

G. Carefully screw the microscope objective into its holder. View the screen.

H. After you have observed the interference pattern, translate the moveable mirror a centimeter or so. Re-align the instrument and observe the pattern. Be prepared to discuss your findings.

Q 5.1. Theoretically, under perfect lab conditions, if Δp (difference in path length) is zero λ, what would you expect to see on the screen? If $\Delta p = 1/2\ \lambda$? If $\Delta p = 3/2\ \lambda$?

Q 5.2. Why are you unable to achieve the theoretical screen pattern even though you achieved $\Delta p = $ zero λ?

Q 5.3. What scientific application(s) can this instrument be used for?

6. CREATING A REFLECTION HOLOGRAM

The reflection hologram involves exposing a plate or film with laser light as it passes through the film onto the objects. Light reflecting back produces an interference pattern in the emulsion. After processing, the hologram may be viewed upon reflection with either white light or monochromatic light of the wavelength used in the original exposure.

The diagram below is a suggested starting point. There are a many way to make a hologram as there are grains of sand on the local beaches. You are encouraged to alter the geometry and see what happens.
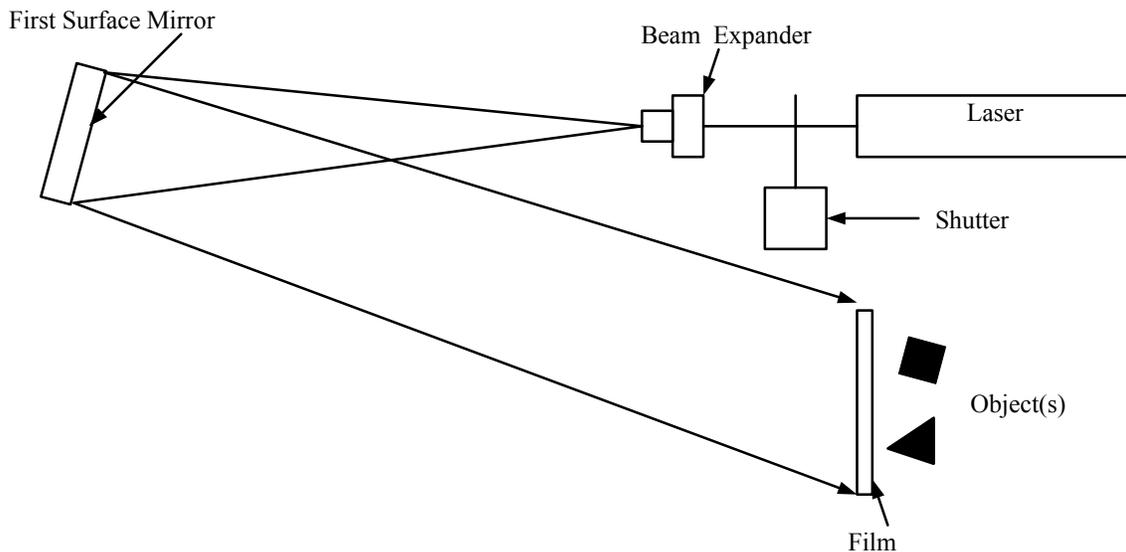

Figure 5. Typical reflection hologram layout

Note that this layout has some elements of the interferometer. Here we have only one main beam striking the film plane. The interference takes place as the light bounces off the objects and produces and interference pattern in the emulsion. Helpful hints:

1. When you set up the array, be sure the metallized surface of the mirror is the first surface.

2. Be sure to place the emulsion so it faces the objects.

3. Use metallic items as objects. The objects should be as close as possible to the emulsion, touching if possible. **DO NOT CEMENT OR EPOXY ANYTHING IN THIS LAB!** We have modeling clay and tacky stuff to prevent objects from moving during the exposure if you feel it should be necessary..

4. Start you exposure at 15-20 seconds. Be sure air pressure to the optics table is turned on. The valve is in room 3215. Your instructor will show you the location.

6.1 LOADING THE FILM

Prior to loading the film, the lab must be made as light-proof as possible.

       a) Lock the outside door so no one can walk in during an exposure.

       b) Pull the black cloth so that it covers the door. Check bottom for light leaks.

       c) Turn on GREEN safe light in the wet lab area. Close door in wet lab. Be sure it is also locked.

       d) Turn off all white light sources. From this point forward, until otherwise advised, you will be working in near dark conditions.

       e) Be sure the shutter is blocking the main beam and that it is working. Test shutter by momentarily depressing then releasing the push-button switch.

       f) Turn on green clock light.

As you enter the wet lab, turn right. On the counter you will see a film safe (black plastic box) containing sheets of 4"x 5" film. You will also find two optically flat glass plates. The film is sandwiched between the plates and held in place by two plastic slip-on clips. (Your instructor will have run you through this process, in the light, so you can practice loading the film successfully.)

Next, carefully open the film safe. In it you will find an envelope or plastic bag. Open the flap. Remove the second bag. Open the second bag and take out one sheet of film. Place in a secure (dry) location. Close the inner bag and place it inside the first bag. Place the bag(s) in the film safe. Close the film safe. This procedure is important because if some one barges in, you only lose one sheet of film not the whole package.

Now first identify the emulsion side. Determine if you want the emulsion side up or down. It makes no difference. The only thing is that you have to able to identify it. Let's say you select "down." Position the film in your hand with emulsion down. Place it on top of one of the glass plates. Place the second plate on top of the film. Locate the plastic clip with the words "emulsion side." Hold the clip with the words facing down (same as the emulsion). Slide the clip over the 4" edge. Now slide the other clip over the other 4" edge. With luck the marked clip now coincides with the emulsion side of the film.

Finally, mount the plate-film assembly to the film holder. BE SURE THE EMULSION SIDE FACES THE OBJECT(S).

Wait a few minutes for the air to stabilize then expose the film.

6.2 DEVELOPING THE FILM

There are basically two types of holographic processes. The first is Amplitude Modulation (AM). In this hologram the interference pattern is stored in closely spaced varying density lines. The second is Phase Modulation (PM). In this type, the holographic information is stored in a closely spaced varying thickness much like a 3-D topo-map. In the former (AM), the image is reproduced by the combination of varying amplitudes. In the latter (PM), phase delays produce the interference patterns.

Phase modulated holograms tend to be more efficient as light is not blocked by dark fringes (in AM holograms). Since reflection holograms are not the most efficient, we will use PM processing. The process is outlined in Appendix A.

Chemicals are usually poured out ahead of time in light into their appropriate trays from bottles marked "Working Solution."

Please take a moment to read the Darkroom Etiquette sheet in Appendix A.

It is important that you be neat and organized in a wet lab. Cross contamination can ruin an entire experiment.

The PM process requires that the hologram be completely (air) dried before viewing. Emulsion swelling during the developing process distorts the surface and hence the hologram. Do NOT heat the film above room temperature.

6.3 VIEWING THE HOLOGRAM

Turn on the viewing laser located on the table. Be sure the beam spreader is properly positioned. Place the (dry) reflection hologram into the beam. Position the hologram so that the incidence angle equals the exposure angle. Try both sides.

> Q 6.3. Which side (facing the viewer) produces the best image?

7. CREATING A TRANSMISSION HOLOGRAM

The main difference between the reflection hologram and the transmission hologram is that the laser beam is split into two or more beams that travel different paths and all eventually terminate at the film plane. See Figure 6..
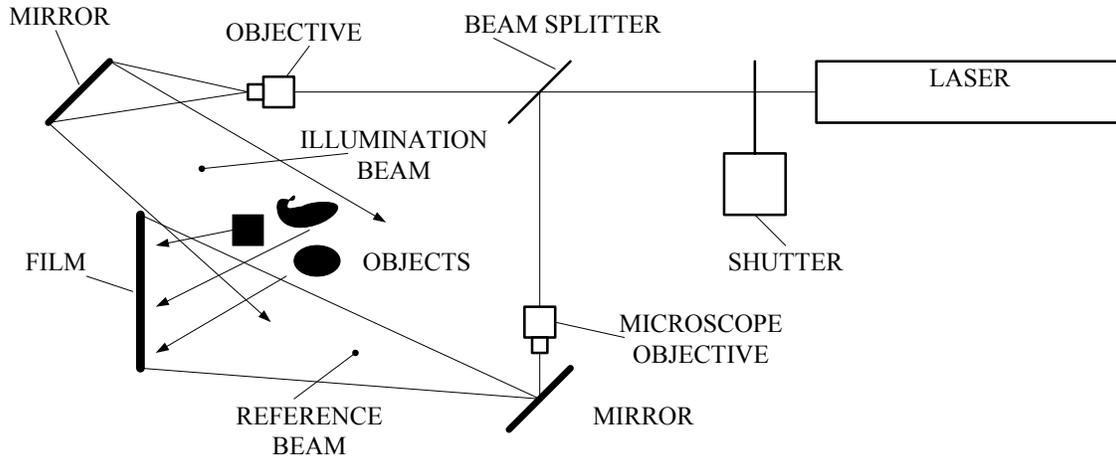
Figure 6. Single illumination, side-referenced layout

Notice the close similarity to the interferometer layout. The main beam is split into two beams that meet at the film.

7.1 DETAILS

The choice of the beam splitter will be determined in the lab through a beam intensity analysis. The beam splitters are of the thin-film type of various ratios. These are part of the optics kits in the drawer.

There are also a few objectives from which to choose from in the kit.

Objects should be diffuse rather than metallic. White chessmen, sea shells, wiffle balls, etc. will work better as objects.

Reference beam intensity-to-film illumination ratio, the beam-balance ratio, of about 4:1 will give best results. You can experiment a bit with this variable. A power meter is available to measure intensities.

Try to keep angles between the reference beam and film and illumination beam and film less than 45°.

Last, but not least, it is important to keep the reference-beam path and the illumination-beam path within a few centimeters. (Why?)

7.2 PROCESSING THE FILM

As mentioned earlier, there are two developing processes that can be used, phase modulation or amplitude modulation. These processes are given in Appendix A.

Transmission holograms are more efficient than reflection ones, so we can develop them using AM processing. After making a few AM holograms, we will take one of them (the one most over-exposed) and bleach it.

You will then make some PM transmission holograms, skipping the AM process.

Compare the holograms using each process. What do you observe?

If time permits, change the beam balance ratio and camera geometry.

Here are some questions for Section 7.

Q 7.1. For transmission holograms, why are diffuse objects better than metallic?

Q 7.2. What is the purpose of "bleach" in reflection and transmission holograms?

Q 7.3. What is the difference between an amplitude-modulated hologram and a phase -modulated hologram? How are they processed (developed)?

Q 7.4. What guideline is essential for the making of a high quality hologram?

Q 7.5. What is a spatial filter and how does it work?

Q 7.6. Why is it important to isolate the camera (table) from the immediate surroundings?

APPENDIX A: Wet-lab chemistry and etiquette.

APPENDIX B: This section deals with the theory of holography.

DEVELOPING PFG-01 HOLOGRAPHIC FILM

TYPE I

<u>REFLECTION</u> HOLOGRAMS: PHASE MODULATION (BLEACHING)

DEVELOPER	TEMP	TIME	AGITATION	COMMENTS
D-19	64º F – 72º F (18º C – 22º C)	4-6 MIN	MODERATE CONTINUOUS	EMULSION SIDE <u>UP</u> DURING DEVELOPMENT TIME WILL VARY DEPENDING ON EXPOSURE AND TEMPERATURE

STOP BATH

KODAK INDICATOR STOP BATH	64º F – 72º F (18º C – 22º C)	30 SEC	MODERATE	EMULSION SIDE UP
RINSE	64º F – 72º F (18º C – 22º C)	1 - 3 MIN		
BLEACH	64º F – 72º F (18º C – 22º C)	UNTIL CLEAR		EMULSION SIDE ***<u>DOWN</u>***
RINSE		4 – 6 MIN		

DEVELOPING PFG-01 HOLOGRAPHIC FILM

TYPE II

<u>TRANSMISSION</u> HOLOGRAMS: PHASE MODULATION (BLEACHING)

DEVELOPER	TEMP	TIME	AGITATION	COMMENTS
D-19	64º F – 72º F (18º C – 22º C)	4-6 MIN	MODERATE CONTINUOUS	EMULSION SIDE <u>UP</u> DURING DEVELOPMENT TIME WILL VARY DEPENDING ON EXPOSURE AND TEMPERATURE

STOP BATH

KODAK INDICATOR STOP BATH	64º F – 72º F (18º C – 22º C)	30 SEC	MODERATE	EMULSION SIDE UP
RINSE	64º F – 72º F (18º C – 22º C)	1 - 3 MIN		
BLEACH	64º F – 72º F (18º C – 22º C)	UNTIL CLEAR		EMULSION SIDE ***<u>DOWN</u>***
RINSE		4 – 6 MIN		

DEVELOPING PFG-01 HOLOGRAPHIC FILM

TYPE III

<u>TRANSMISSION</u> HOLOGRAMS: AMPLITUDE MODULATION

DEVELOPER	TEMP	TIME	AGITATION	COMMENTS
D-19	64º F – 72º F (18º C – 22º C)	4 – 6 MIN	MODERATE CONTINUOUS	EMULSION SIDE <u>UP</u> DURING DEVELOPMENT TIME WILL VARY DEPENDING ON EXPOSURE AND TEMPERATURE

STOP BATH

KODAK INDICATOR STOP BATH	64º F – 72º F (18º C – 22º C)	30 SEC	MODERATE	EMULSION SIDE <u>UP</u>
RINSE	64º F – 72º F (18º C – 22º C)	1 – 3 MIN		

FIX

KODAK RAPID FIXER	64º F – 72º F (18º C – 22º C)	3 – 4 MIN	MODERATE	EMULSION SIDE UP
RINSE		4 – 6 MIN		

Appendix A

DARKROOM ETIQUETTE

1. PLACE PAPER TOWELS UNDERNEATH ALL TRAYS AND RINSE SOLUTIONS TO CONTROL SPILLAGE DURING DEVELOPMENT.

2. FILL DEVELOPING TRAYS 1/4 TO 1/2 FULL ONLY. THIS WILL REDUCE SPILLAGE DURING AGITATION.

3. POUR ALL WORKING SOLUTIONS BACK INTO THEIR RESPECTIVE CONTAINERS.
BE SURE TO RINSE FUNNELS AT THE END OF EACH POUR TO AVOID CROSS-CONTAMINATION.

4. WASH AND RINSE ALL TRAYS.

5. TURN OFF THE SAFELIGHT AND ROOMLIGHTS BEFORE LEAVING THE LAB.

Appendix B: Deriving the Diffraction Pattern

In this appendix, I will derive the "projector effect". Recall that if you illuminate a single spot on a hologram with a laser beam, you can project an image of the original 3-D object on the a screen. (See figure 2.) Here I will not discuss what happens when you illuminating the entire surface of the hologram with a wide beam of light. It is mathematically simpler to consider the rays leaving only a tiny portion of the hologram. It will be shown that these rays will appear to emanate from the original surface of the 3-D object.
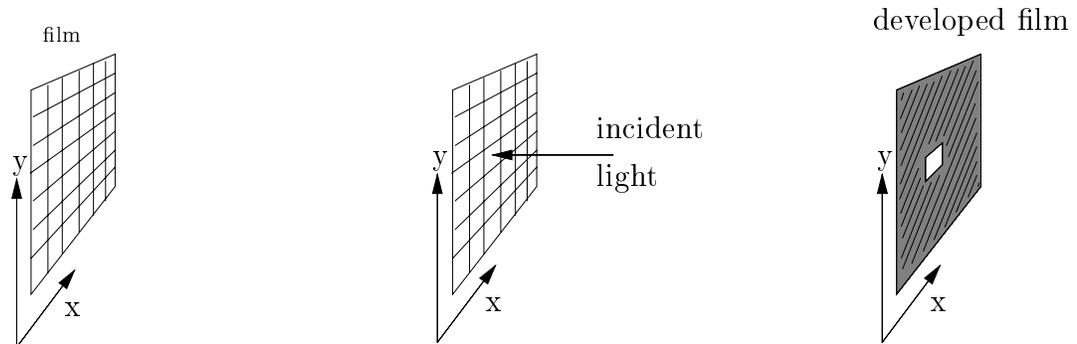
Translucency $t(x, y)$**:**

Figure 8: "Photo-positive" film. Incident light strikes the film and burns a hole in the opaque surface so that in the future, light can get through. This is not how real film works, but it makes the derivation easier.

From now on, I will consider a different kind of film, than the photo-negative film I mentioned in section 1. For this derivation, I will assume we have *photo-positive* film. That is, film which is *initially opaque* and becomes more *transparent* when it is exposed to light. (See figure 8) This opposite the behavior of real film. However, it makes no difference as far a the behavior of the hologram is concerned and it makes the derivation slightly simpler. In principle, if such film existed, you could use either kind of film for making holograms. (In exercise A.a, you are asked to prove this assertion.) In any case, Fowles' derivation of holography [2] also uses photo-positive film, and I will try to keep this derivation compatible with Fowles'.

After the film has been exposed to light, and then developed, portions of the surface will be clear, and other parts will be opaque. I characterize it's translucency by $t(x, y)$, a function which varies between 0 and 1. Later on, when we go to view the hologram, we will later shine uniform monochromatic light onto the surface of the film. $t(x, y)$ represents the fraction of light that is

transmitted at position (x, y) through the film's surface.

$$t(x, y) \quad = \quad \frac{\text{transmitted amplitude traveling through the film at (x,y)}}{\text{incident amplitude of light striking the film at (x,y)}}$$

$$= \quad \frac{|E_t(x, y)|}{|E_i(x, y)|} \tag{1}$$

(This may be different from definitions of translucency given in other textbooks. Some books call $|E_t(x, y)|^2 / |E_i(x, y)|^2$ the translucency.)
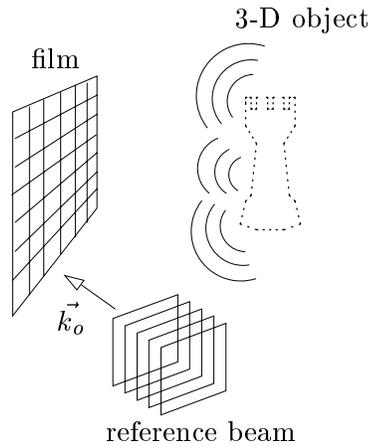
A.1 Creating the hologram:

Figure 9: The light emanating from a solid 3-D object can be approximated by light leaving from many point sources of light on its surface.

To simplify the task, I will approximate the 3-D object by a collection of N *point sources* of light. We can make N very large so that the collection of points of light begins to resemble a solid 3-D object. (See figure 9.) I will show that, when you strike any point of the surface of a hologram with a narrow laser beam of direction $\vec{k}_o$, the resulting diffraction pattern will contain a separate ray of light appearing to emanate from the former location of each point source of light in the original object. (Plus some other unwanted by-products. More on those later.) This is the projector effect.

Let's focus our attention on a small region/pixel on the unexposed film. If we choose a region on the holographic film that is small enough, then the spherical waves emanating from a point source of light on the object appear to be planar by the time they reach the film. [9] I will treat the light emanating from each point source on the object as a perfect plane wave with wave-vector $\vec{k}_i$ and amplitude

---

[9] If you zoom in enough on a spherical wave front, or any smooth surface, it will appear to be flat.
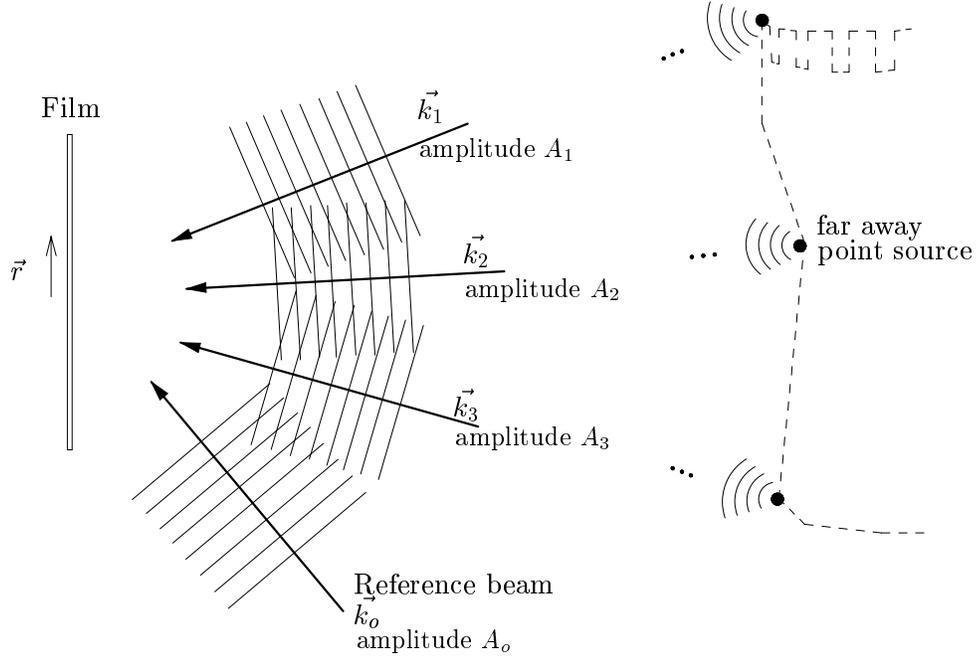
Figure 10: Distant sources of light used to construct a hologram. Note: This diagram is not to scale. The object (a chess piece) is meant to be much larger and much further away than the size of the fragment of film shown in this diagram. We are looking at a magnified side-view of a portion of the film (say, about 0.5mm high). The approaching spherical waves resemble plane waves with direction $\vec{k_1}$, $\vec{k_2}$, and $\vec{k_3}$.

(where it strikes the film) of A_i. By only looking at a small piece of film and assuming the object is a collection of point sources of light, we have simplified the light emanating from the object to a couple plane waves. The reference beam is also a plane wave. Let it have wave-vector $\vec{k_o}$ and amplitude A_o (Note that since all rays of light have the same wavelength, $|\vec{k_o}| = |\vec{k_1}| = \cdots = |\vec{k_N}|$.)

The E-field when it strikes the film will be the superposition of $N+1$ plane-waves:

$$E(\vec{r}) \quad = \quad A_o e^{i\vec{k_o}\cdot\vec{r}} + A_1 e^{i\vec{k_1}\cdot\vec{r}} + \cdots + A_N e^{i\vec{k_N}\cdot\vec{r}} \tag{2}$$

In section A, we talked about the properties of the film. Because the film becomes more translucent as it is exposed to more light, assume that the translucency of the film $t(\vec{r})$ is simply proportional to the intensity of the light striking it. [10]

___

[10] In reality of course, this relationship is not linear. (The translucency cannot exceed 1!) I

16

$$
\begin{aligned}
t(\vec{r}) \quad &\alpha \quad |E(\vec{r})|^2 \\
&= \quad E^*(\vec{r})E(\vec{r}) \quad\quad\quad\quad\quad\quad\quad\quad\quad (3)
\end{aligned}
$$

Substituting in Eq. 2, we get:

$$
= \left| \sum_{n=0}^{N} A_n e^{i\vec{k_n}\cdot\vec{r}} \right|^2 \tag{4}
$$

$$
= |A_o|^2 + \left| \sum_{i=1}^{N} A_i e^{i\vec{k_i}\cdot\vec{r}} \right|^2 + A_o^* \sum_{i=1}^{N} A_i e^{i(\vec{k_i}-\vec{k_o})\cdot\vec{r}} + \sum_{i=1}^{N} A_i^* A_o e^{i(\vec{k_o}-\vec{k_i})\cdot\vec{r}} \tag{5}
$$

(In the last step, I singled out the reconstructing beam, $A_o e^{i\vec{k_o}\cdot\vec{r}}$, from the sum $\sum_{n=0}^{N}$.) This function of $\vec{r}$ represents the opaqueness of the film along its surface. It will be shown later that the third term $A_o^* \sum_{i=1}^{N} A_i e^{i(\vec{k_i}-\vec{k_o})\cdot\vec{r}}$ is responsible for holography.
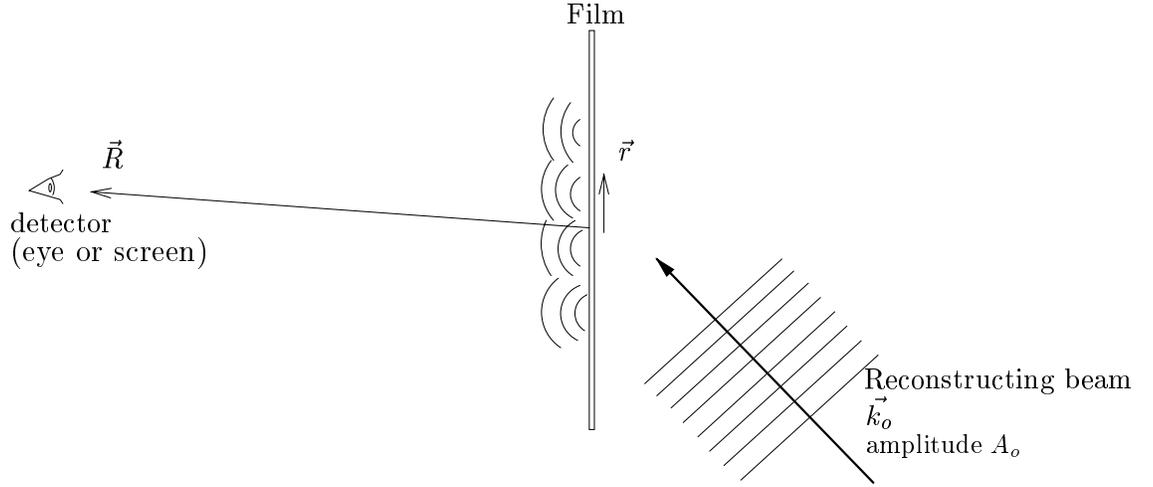
A.2   Viewing the hologram:

Figure 11:

After developing the film, take away the 3-D object, but continue to illuminate the film with the reference beam. (Note that now, the literature calls

chose this gross approximation of translucency in order to make the math easier. Hopefully this is accurate enough to reproduce the same qualitative behavior of the hologram.

it a "reconstructing beam". However it's the same beam.) Let $E'(\vec{R})$ be the electric field just on the other side of the hologram. It is the product of the incident radiation $e^{i\vec{k_o}\cdot\vec{r}}$ (ignoring constant factors), and the translucency of the film $t(\vec{r})$. (See equation 1.)

$$E'(\vec{r}) \quad \alpha \quad t(\vec{r})e^{i\vec{k_o}\cdot\vec{r}} \tag{6}$$

The amplitude of the E-field at position $\vec{R}$ will depend on the various rays of light scattering off the interference fringes in the film and their phases. Using Huygen's principle [2], we have to sum up the contributions of all these rays leaving the surface of the film from different places and striking the detector to find whether or not position $\vec{R}$ is in a region of bright light or not.

Notation: Imagine you place a detector at position $\vec{R}$ on the other side of the hologram. $\vec{R}$ either denotes the position of a point on the screen on which you are projecting, or the position of some other detector (like your eye). $\vec{r}$ is a vector which lies in plane of the film. (See figure 11.) In order to sum over all rays leaving the film, we will integrate over all $\vec{r}$ in the plane of the film. Let the amplitude of the E field at position $\vec{R}$ be denoted $E''(\vec{R})$. Then:

$$\begin{aligned}
E''(\vec{R}) \quad \alpha \quad & \iint \frac{e^{i|k_o||R-r|}}{|R-r|}E'(\vec{r})\mathrm{d}\vec{r}^2 \\
= \quad & \iint \frac{e^{i|k_o||R-r|}}{|R-r|}t(\vec{r})e^{i\vec{k_o}\cdot\vec{r}}\mathrm{d}\vec{r}^2
\end{aligned}$$

To simplify this, I will approximate $1/|R-r|$ as $1/|R|$, and $|R-r|$ as $(|R| - \hat{R}\cdot\vec{r})$ (since $|r| << |R|$).

$$= \frac{e^{i|k_o||R|}}{|R|} \iint e^{-i|k_o|\hat{R}\cdot\vec{r}}t(\vec{r})e^{i\vec{k_o}\cdot\vec{r}}\mathrm{d}\vec{r}^2 \tag{7}$$

For shorthand, I will also denote $|k_o|\hat{R}$ by $\vec{k_R}$. Note that this is a vector with length $|k_R| = |k_o| = |k_j|$ which points in the direction of the detector.

$$= \frac{e^{i|k_o||R|}}{|R|} \iint e^{-i\vec{k_R}\cdot\vec{r}}t(\vec{r})e^{i\vec{k_o}\cdot\vec{r}}\mathrm{d}\vec{r}^2 \tag{8}$$

$$\begin{aligned}
= \quad & \frac{e^{i|k_o||R|}}{|R|} \iint \left[|A_o|^2 e^{i(\vec{k_o}-\vec{k_R})\cdot\vec{r}} + \left|\sum_{i=1}^{N} A_i e^{i\vec{k_i}\cdot\vec{r}}\right|^2 e^{i(\vec{k_o}-\vec{k_R})\cdot\vec{r}} + \right. \\
& \left. \underbrace{A_o^* \sum_{i=1}^{N} A_i e^{i(\vec{k_i}-\vec{k_R})\cdot\vec{r}}} + \sum_{i=1}^{N} A_i^* A_o e^{i(2\vec{k_o}-\vec{k_i}-\vec{k_R})\cdot\vec{r}} \right]\mathrm{d}\vec{r}^2
\end{aligned} \tag{9}$$

(Note: This equation is similar to equation 5.69 of p.146 of Fowles [2]. The first two terms are equivalent to $(a^2 + a_o^2)U_o$. the third term (which is underlined) is equivalent to $a_o^2 U$, and the final term is equivalent to $a^2 U^{-1} * U_o^{-2}$.)

I will now concentrate on the 3rd term in equation 9 (which is underlined) and ignore the other terms. *This is the term which is responsible for holography.* The other terms create unwanted artifacts/ghost-images, which are also present in the hologram. They will be discussed in the next section. For now, let us concentrate on the mathematics which generates the 3-D optical illusion we associate with holography.

Note that if our detector at $\vec{R}$ is placed so that it lies along the direction of a ray leaving one of the point sources of light, $(\vec{k_i} = \vec{k_R} \equiv |\vec{k_o}|\hat{R})$, then $e^{i(\vec{k_i} - \vec{k_R}) \cdot \vec{r}}$ is a constant, 1, independent of $\vec{r}$. All the rays leaving from various places $\vec{r}$ on the surface of the film will have the same phase by the time they reach position $\vec{R}$, and they will all add constructively. Hence there will be a bright ray leaving the film in the direction of $\vec{k_i}$, and it will have an amplitude proportional to A_i. But if it points even $1°$ off of this location, then $e^{i(\vec{k_i} - \vec{k_R}) \cdot \vec{r}}$ will cycle back and fourth, positive and negative, many times as you integrate $\vec{r}$ (say 0.5mm) across the surface of the hologram and the integral will be close to zero.

Hence, when you illuminate a small spot on the surface of the hologram with a laser beam in the direction of $\vec{k_o}$, the many rays of light that leave the hologram will appear to to be a ray emanating from each point the original 3-D object (and traveling through the spot on the film that you are illuminating with the laser beam. Again this is the projector effect.) As mentioned in section 4, when you illuminate a wide portion on the hologram and view the entire hologram with your eye, the pixels on the surface of the hologram will appear to be bright or dark depending on whether or not there was a bright point source of light located behind that spot on the surface of the hologram you are looking at when the hologram was made. It appears you are looking at the original 3-D object behind the surface of the film. Fowles calls this the "virtual image" [2].

Exercises for section A

Exercise A.a) Suppose you were able to make two holograms of the same object using these two different kinds of film. One film contains the photo-negative of the pattern on the other. How would using the new film effect the appearance of the hologram? (Hint: Let $t(x, y) \to 1 - t(x, y)$. Does this substitution change the direction one would have to place $\vec{R}$ in order to see constructive interference?)

Exercise A.b) *Image Resolution:* Suppose you were to make a simple hologram of a point source. (In practice, this could be a tiny reflective metallic sphere. This kind of hologram is sometimes called a "zone plate".) When viewing the hologram under a wide beam of monochromatic light (as shown in figure 12) you should see a sharp bright point in the center of the film coming from where the point source ought to be. However, due to diffraction and the finite size of the wavelength used to create the
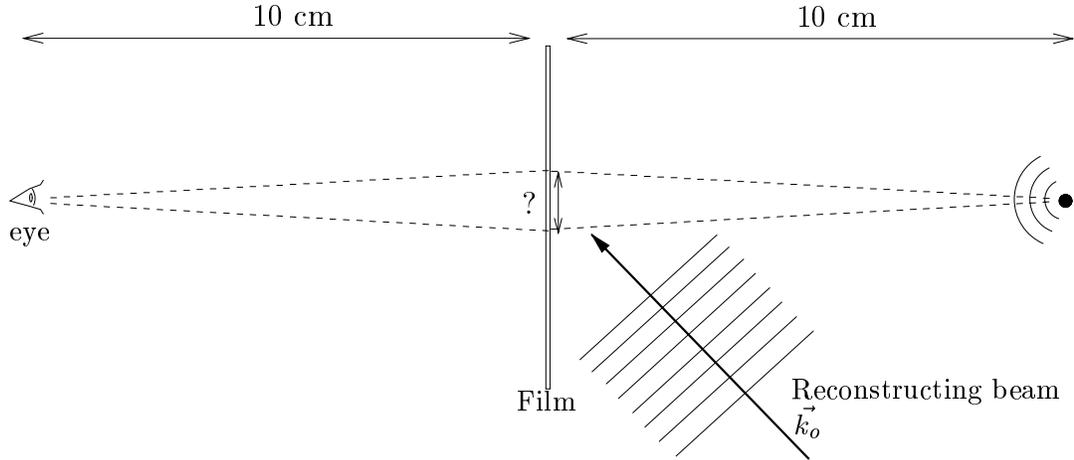
19

Figure 12: Exercise A.b

hologram, this dot will be slightly blurry. It will appear slightly spread out. Assuming no defect in the ideal film, estimate the apparent width of the spot. (Assume $\lambda = 632.8nm$.)

Think A.c) What would happen if you were to change the wavelength of the reconstructing beam? Specifically, suppose you were to illuminate the surface of the hologram with a different light source, one whose wavelength was reduced slightly by, say 5%. What would happen to the diffraction pattern cast on the screen? Would it still be there at all? Would it grow or would it shrink? This question can have a qualitative answer. (Does it grow or shrink? Please don't just say "it would change size". Does it do something else? Also: Don't worry about the depth of the emulsion layer on the film. Assume that this is a transmission hologram and that the interference pattern recorded on the film is completely flat and 2-dimensional.)

Think A.d) Now, if you haven't already, prove the conjecture you made in exercise 5.a Explain (using pictures or equations) why the diffraction pattern reflected from the surface of a hologram is the same as the diffraction pattern emerging through the other side of the hologram. (Up to an overall phase and multiplicative constant. This statement is true for both transmission and reflection holograms, however for this exercise, assume that you have a transmission hologram.)

A.3   Artifacts:

Whenever you see bright light emanating from a point on the surface of the hologram, then you must be viewing the hologram from an angle $(\vec{k_R})$ such that

at least one of the terms in equation 9 has constant phase for all $\vec{r}$. At such an angle constructive interference will cast a bright ray in your direction. (Recall that if the viewing direction is even 1° off, or even 0.1°, then cyclic terms in the integral in equation 9 will cancel themselves out and no light will fall in the direction of $\vec{R}$)

Note that we ignored several other terms in equation 9. These give rise to *other* images. As long as you are careful not to stare into the reference beam, it can be fun to try and look for these other images in the holograms you made. [11] Figures 13 and 14 show the direction where these ghost images are likely to appear.

The first term in equation 9 ($|A_o|^2 e^{i(\vec{k_o}-\vec{k_R})\cdot\vec{r}}$) indicates that there is a ray of constructive interference pointing in the direction of the reference beam $\vec{k_o}$. A large fraction of the light from the reference beam passes right through the film undeflected.

The second term in equation 9 can be expanded:

$$\left|\sum_{i=1}^{N} A_i e^{i\vec{k_i}\cdot\vec{r}}\right|^2 e^{i(\vec{k_o}-\vec{k_R})\cdot\vec{r}} = \left[\sum_{i,j\geq 1}^{N} A_i^* A_j e^{i(\vec{k_j}-\vec{k_i})\cdot\vec{r}}\right] e^{i(\vec{k_o}-\vec{k_R})\cdot\vec{r}} \quad (10)$$

$$= (|A_1|^2 + \cdots |A_N|^2) e^{i(\vec{k_o}-\vec{k_R})\cdot\vec{r}}$$

$$+ \sum_{i\neq j\geq 1}^{N} A_i^* A_j e^{i(\vec{k_o}-\vec{k_R}+\vec{k_j}-\vec{k_i})\cdot\vec{r}} \quad (11)$$

The $(|A_1|^2 + \cdots |A_N|^2) e^{i(\vec{k_o}-\vec{k_R})\cdot\vec{r}}$ terms contribute more light to the bright ray in the direction of $\vec{k_o}$. However the rays of constructive interference generated by the remaining cross-terms in equation 11 (which are not discussed in Fowles) in theory should generate a very wide blurry image centered around the reconstructing beam $\vec{k_o}$ and spreading broadly in either direction. This is shown in figure 14.

If this pattern actually exists, then it is subtle and difficult to make out. [12] It would be an interesting experiment to try and detect this pattern. I have not noticed it yet, and I'm not aware of where it is discussed in the literature.

Finally there is another ghostly image coming from the last set of terms in equation 9 which contain $e^{i(2\vec{k_o}-\vec{k_i}-\vec{k_R})\cdot\vec{r}}$ Fowles [2] calls this the "real im-

---

[11] Again, in this appendix I've assumed that you are using thin "transmission holography" film. However you may have to make many of your holograms on thick "reflection holography" film. I don't think this will matter, but I'm not 100% sure. If in doubt, use one of the true transmission holograms lying around lab, for example the "Gears" hologram Rich made.

[12] Perhaps it is because the rays (whose amplitudes are $A_i^* A_j$) are weaker than the rays which make up the "virtual image" and "real image" (whose amplitudes are $A_o^* A_i$ are $A_i^* A_o$ respectively), since $|A_o| \gg |A_i|$, (however there are many more of them). Perhaps it is because the rays are spread out over such a large area and the human eye is designed to be more sensitive to sudden transitions in brightness present in the original image than to large diffuse blobs. And it could also be that the approximation we made in section A when we said $t(\vec{r}$ was linearly proportional to $|E(\vec{r})|^2$. were too crude.

age". [13] Whenever the viewing direction $\vec{k_R}$ is chosen so that $2\vec{k_o} - \vec{k_i} - \vec{k_R}$ is perpendicular to the hologram surface (ie. to $\vec{r}$), you will see a bright ray of constructive interference. However, figure 13 explains why this image is difficult, if not impossible to see.

Under normal viewing conditions, shining the reconstructing beam at the original direction $\vec{k_o}$, the "real image" is difficult to see. Under these conditions, you must view the hologram from a very steep angle, so that the rays leaving it and striking your eye are almost parallel to the surface of the film. If you do manage to see it, the "real image" should appear to be upside-down and backwards (and squished), and it's perspective will appear strange and inverted. Usually when students say they can see the "real image", it really is because they are holding the hologram upside down, or the reconstructing beam is hitting the wrong side of the film. Doing this will also generate an image that is upside down and backwards. You may have seen this already if you did the "Stop" exercises 4.b at the end of section at the end of section 4, [14] If you do manage to see an upside down and backwards image, try to make sure you are viewing it under conditions when the reconstructing beam hits the film at the same angle (and the same side) as it struck the film when when the hologram was made.

B Developing Procedure

(as of 1/23/2002.)

UNDER THE SAFE GREEN LIGHT:

1. Put film in developer

   a. Emulsion side up.

   b. Agitate by rocking tray.

   c. Developing time depends on exposure time.

      REFLECTION: Develop until lines are fairly dark. (About 30% light transmission. There is a light intensity meter you can use to measure the opaqueness of your hologram *after you have developed it first.*)

      TRANSMISSION: Develop until lines are light. (About 70% light transmission.)

2. Stop bath for 30 seconds. Then rinse for 1 minute.

3. Bleach until lines are gone. Film will turn pink. Rinse for 1 minute.

---

[13] Counter-intuitively, the picture of the 3-D object you see when you are viewing the hologram correctly is called the "virtual image".

[14] Its probably be fruitless to argue about whether this is "real image". I suppose it *is* the real-image, under strange viewing conditions, ie. once you have rotated the angle of the reconstructing beam from $\vec{k_o}$ to $-\vec{k_o} + 2(\vec{k_o} \cdot \hat{z})\hat{z}$. (See figure 16.)

4. ~~Put film in ascorbic acid with bright light on until the film turns light brown. Rinse for 1 minute.~~

5. ~~Dry with heat gun placed 1 meter from the film.~~

References

[1] "Black & White Film"

http://www·sunspotphoto·com/film/blackwhite·html

[2] Fowles, G.R., "Modern Optics" 2nd Ed. (1989), pp. 145-146 *There are many books that talk about holography, but I have yet to find one that actualy derives it. In general Fowles is a pretty good optics theory book but it has the most unsatisfying, terse explanation of holography. I have not read any of the other references, but I collected them from various recommendations. I hope they are relevant.*

[3] Collier, R.J., Burckhardt, C.B., Lin, L.H., "Optical Holography" Academic Press (1971), Chapters 1, 3, and 7. *This book takes a very mathematical approach.*

[4] Smith, H., "Principles of Holography", 2nd Edition Wiley-Interscience (1969), Chapters 1, 2, 3, 8

[5] Outwater, C., "A Guide to Practical Holography" Pentagonal Press (1974)

[6] Leith, E.N., Upatnieks, J. "Photography by laser", Scientific American (June 1965), vol. 212, no. 6

[7] Kock, W.E., "Lasers and Holography: An Introduction to Coherent Optics"

[8] Saxby, G., "Manual of Practical Holography", Focal Press (1991)

[9] Berner, J., "The Holography Book", Avon Books (1980)

[10] Butters, J.N., "Holography and Its Techniques"

[11] Caufield, H.J., Sun Lu, "The Applications of Holography" Wiley and Sons (1970)

[12] Lehman, M., "Holography, Technique and Practive", Focal Press (1970)

[13] Lengyel, B., "Introduction to Laser Physics", J. Wiley & Sons, Inc (1966), Chapters I, II, V, and VI

[14] Smith, W.V., "Laser Applications", Artech House, Inc, (1970)

[15] "Holography", UCB Physics 111 laboratory manual, (2002)
http://ist-socrates·berkeley·edu/ phylabs/writeupPDF/HOL·pdf
.

General references on coherent light and optics:

[16] Siegman, "Lasers" University Science Books, (1986)

[17] Karlov., N.V., "Lectures on Quantum Electronics", Moscow: Mir Publishers; Boca Raton, Fla: CRC Press, (1993)

[18] Demtroder, W., "Laser Spectroscopy: Basic Concepts and Instrumentation", Springer-Verlag

[19] Kuhn, K.J., "Laser Engineering", Prentice-Hall (1998)

[20] Lipson, S.G., Lipson, H., Tannhauser, D.S., "Optical Physics" 3rd Ed., Cambridge University Press, (1998)

[21] Young, M., "Optics and Lasers", Springer-Verlag, (1993)

[22] Davis, C.C., "Lasers and Electro-Optics. Fundamentals and Engineering" Cambridge, (1996)
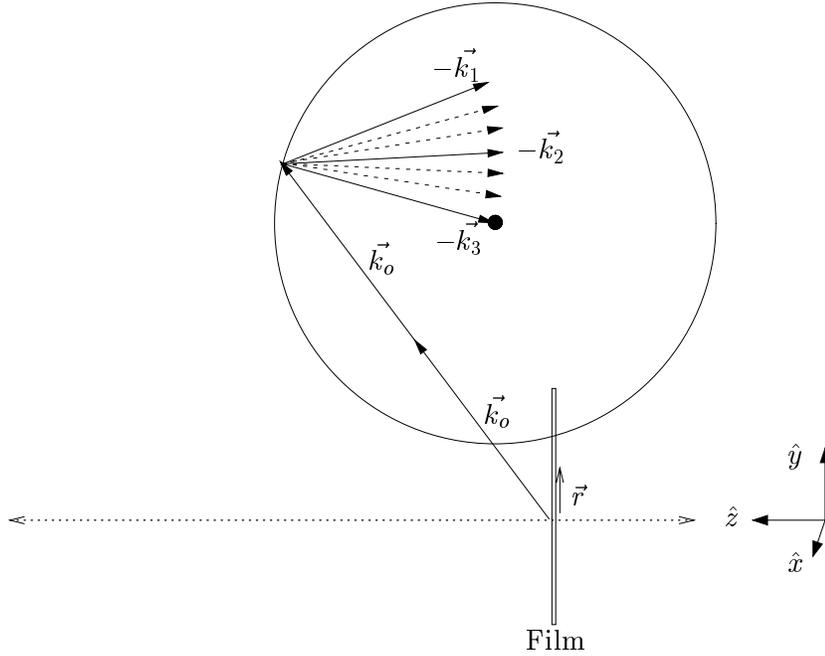
Hooray I'm done!

Figure 13: *No Real Image?* Depending on the angle of the reference beam $\vec{k}_o$, and the size of the object, it can be impossible to see the "real image" discussed by Fowles [2] and other books. In this example, the image is too small, and the reconstructing beam too steep. The "real image" comes from the terms containing $e^{i(2\vec{k}_o - \vec{k}_i - \vec{k}_R)\cdot\vec{r}}$. In order to see constructive interference from these terms, for some choice of $\vec{k}_R$ and $\vec{k}_i$, the phase of $e^{i(2\vec{k}_o - \vec{k}_i - \vec{k}_R)\cdot\vec{r}}$. must not vary across the surface of the hologram $\vec{r}$. For this example, $\vec{k}_1$, $\vec{k}_2$, and $\vec{k}_3$ denote rays which emanate from the top, middle, and bottom of the object. (See figure 10.) Note that in this case, it is impossible satisfy the condition for constructive interference. In this case, $2\vec{k}_o - \vec{k}_i - \vec{k}_R$ cannot be perpendicular to $\vec{r}$ (ie. lie on the horizontal dotted line). Even if you choose the most downward pointing vector $-\vec{k}_3$, the sum $2\vec{k}_o - \vec{k}_3$ lies too far away from the dotted horizontal line (as indicated by the circle) to be reached by subtracting $\vec{k}_R$, no matter what direction $\vec{k}_R$ is pointing. Hence there will be no "real image" for this hologram. You might imagine that if $\vec{k}_o$ had been chosen with a more shallow slope, and/or the object was closer, it may have been possible to see this ghost image, but it would be projected at a very steep angle. $\vec{k}_R$ would have to be pointing almost straight up.
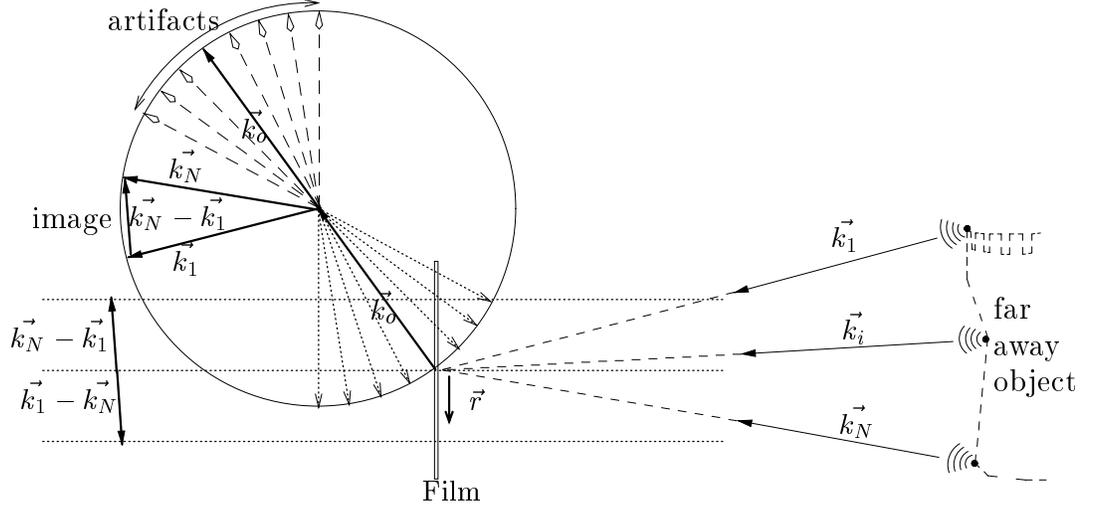
Figure 14: Artifacts from the $e^{i(\vec{k_o} - \vec{k_R} + \vec{k_i} - \vec{k_j}) \cdot \vec{r}}$ cross-terms. The picture of the 3-D object will be projected in the directions ranging from $\vec{k_1}$ to $\vec{k_N}$ (emanating from the top and bottom of the original object, respectively). $\vec{k_o}$ is the wave-vector of the reconstructing beam and $\vec{k_R}$ is the wave-vector of the rays traveling in the direction of the detector which is located at position $\vec{R}$. (See figure 11.) $\vec{r}$ represents any point which lies on the surface of the film. (We are integrating over $\vec{r}$.) The *dashed* arrows pointing to the *upper-left* represent *artifacts*. Looking in these directions (ie. if $\vec{k_R}$ points in any of these directions), you will see rays of constructive interference due to the cross terms. These rays will generate a diffuse wide picture which does *not* resemble the original image. Why does constructive interference produce rays of light in the direction of the *dashed* arrows? Note that, since real objects generate infinitely many rays leaving their surface, there is a continuous distribution of possible values of $\vec{k_i} - \vec{k_j}$ ranging from $\vec{k_N} - \vec{k_1}$, to $\vec{k_1} - \vec{k_N}$. When $\vec{k_R}$ points along the direction of the dashed arrows, then $\vec{k_o} - \vec{k_R}$ (indicated with the *dotted* arrows pointing to the lower right) lies in between this range (indicated by the three dotted horizontal lines). Within this range of directions it *is possible* to choose a *pair* of rays $\vec{k_i}$ and $\vec{k_j}$ (in between $\vec{k_1}$ and $\vec{k_N}$) such that the vector sum $\vec{k_o} - \vec{k_R} + \vec{k_i} - \vec{k_j}$ is perpendicular to the plane of the film (ie. lies on the central dotted line). Hence $(\vec{k_o} - \vec{k_R} + \vec{k_i} - \vec{k_j}) \cdot \vec{r} = 0$ for all points on the surface of the film, $\vec{r}$, and constructive interference will occur for that direction of $\vec{k_R}$ (due to the pair of rays $\vec{k_i}$ and $\vec{k_j}$). In practice these artifact rays of light are difficult to see.
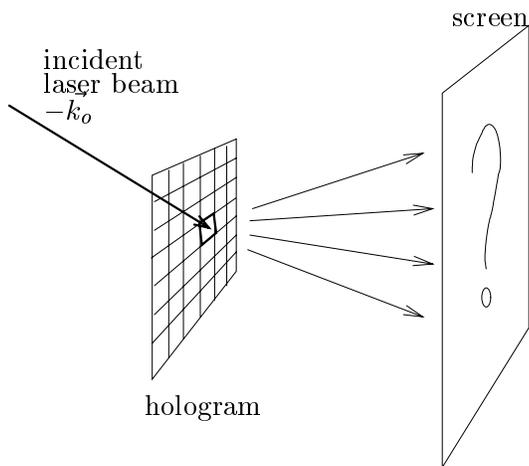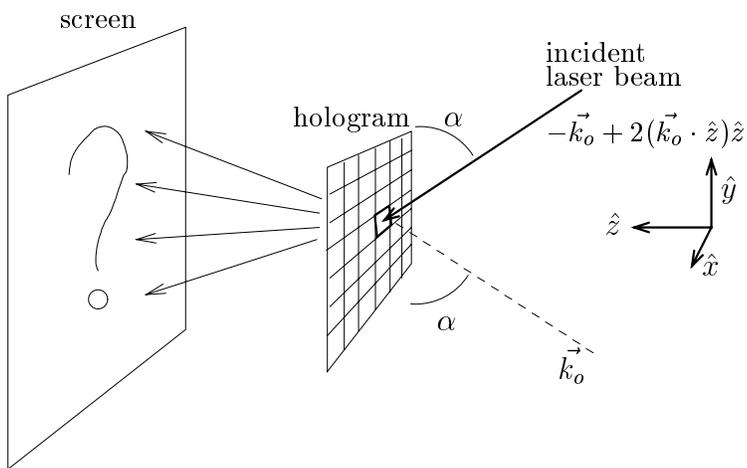
26

Figure 15: Exercise 3.a

Figure 16: Exercise 3.b