PROPERTIES OF HELIUM–NEON LASERS

The acronym LASER stands for "Light Amplification through Stimulated Emission of Radiation". It has also become standard to refer to the device itself as a laser. The light emitted by a laser has very special properties which distinguish it from the light given off by an ordinary source of electromagnetic radiation, such as a light-bulb. These special properties make it possible to use laser's for very unusual purposes for which ordinary, even nearly-monochromatic light is not suitable.

I. PROPERTIES OF LASER LIGHT

A. The light is extremely monochromatic with wavelength λ= 632.8 nm

B. Consequently, the light has high "temporal coherence", meaning as you travel along the direction of propagation, the components of the electric field continue to oscillate like a sine-wave with a single wavelength, amplitude, and phase. Of course, no light source generates perfect plane-waves. Real wave-trains have finite length. The distance over which the waveform remains similar to a sine-wave is called the coherence-length of the beam, L_c, and it is typically about 10-30 cm for commercial He-Ne lasers.

C. The light is unidirectional and aligned so as to be parallel to the body of the laser.

D. The light is "spatially coherent". The phase of radiation is nearly constant throughout the cross-sectional width of the beam. This property is a consequence of property C, and is entirely independent of temporal coherence (property B).

E. A Brewster-window is often inserted in the laser by the manufacturer to produce light with a definite state of linear polarization.

II THE LASING MECHANISM

A He-Ne laser consists of a hollow tube filled with 90 percent He and 10 percent Ne gases and fitted with inward-facing mirrors at the ends of the cavity. The combined pressure of the two gases is approximately 1 Torr (1/760 atmospheres).

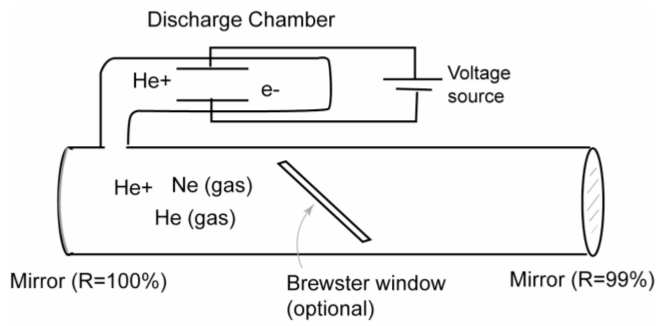

Figure 1. The laser cavity

An He-Ne laser works by exciting Neon atoms in the gas. The 632.8 nm optical light emitted by the laser is generated in the process when the Neon atoms decay from an excited state to an ordinary state.

II A) QUANTUM THEORY

"For the present basic investigation, a detailed determination of the quantum (states) is not required..."

"In quantum theory, a molecule can only exist in a discrete set of states Z_1, Z_2, ...Z_n,... with energies e_1, e_2,...e_n,..." [11] (In this document [#] denotes a citation. Interested readers can find the litterature referrence to which this number refers at the end of this lab manual.)

A molecule can be promoted from state (call it "Z_n") to a higher energy state ("Z_m") by absorbing a photon with frequency, v, proportional to the energy separation between these two states.

$$hn = e_m - e_n \qquad (e_m > e_n) \tag{1}$$

The molecule can decay from a state Z_m to a lower energy state Z_n, by emitting a photon of that same frequency. This is slightly more complicated because there are two ways this can happen:

1) The molecule collides with a photon of frequency $hn = e_m - e_n$ and emits a second photon. The two photons leaving the molecule have identical

frequency, phase, polarization, and travel in the same direction. This is called "stimulated emission". It is the reason why lasers generate light with the unusual properties mentioned above.

2) The molecule can decay on its own, emitting a photon of frequency $hn = e_m - e_n$, seemingly without any prompt from the external world. This is called "spontaneous emission." [1] This photon will travel in a random direction with a random phase.

It is important to mention transition between any pair of states can be forbidden or suppressed (made to happen infrequently) by *selection rules*.

Helium and Neon are noble gases containing only one atom per molecule. Consequently, the states of a Helium and Neon molecule are relatively simple: The different states of an Ne molecule correspond to the different orbitals that electrons that surround the Ne atom can occupy. Radiation striking the Neon atoms causes the electrons circling these atoms to jump from one orbital to another.[2]

An electrical discharge created by strong electric fields ionizes the He gas. Helium in its lowest energy state has an electron configuration of $1s^2$. After the ionized He+ atoms recombine with their electrons some of the Helium atoms end up in the 1s2s-singlet state. Once in this state, the He atoms are forbidden from spontaneously making radiative transitions to lower energy states by quantum selection rules.[3]

However, excited He atoms can decay through collisions with other atoms. The purpose of He atoms is to act as a power source for the Ne atoms, kicking them up into an excited state. If an excited He atom collides with an Ne atom, there is a finite probability that during the time over which the two atoms are in close proximity with each other, the excited He atom can transfer its energy to the

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[1] A theoretical explanation of the mechanism responsible for the emission and absorption of photons is rarely covered in an undergraduate quantum mechanics course. One has to treat the electromagnetic field in empty space as a quantum mechanical system. A thorough explanation of this is given in chapters 21 and 23 of Merzbacher. [12]

[2] By comparison, molecules containing multiple atoms bonded together are more complicated. The bonds connecting the atoms can stretch or bend, and the molecules can rotate. Electrons in these molecules still can jump from orbital to orbital. But in addition, molecules also have different states corresponding to the different ways these molecules can rotate or vibrate. For example, CO_2 gas lasers work by making transitions between the different rotational states of a CO_2 molecule.

[3] Hence this excited state has a very long lifetime (it is *metastable*). One way to understand why is to consider that for the He atom to decay from one energy level to another, a photon must be created, and that photon must travel in a direction. However the $1s^2$ and 1s2s(singlet) electronic states of the He atom are spherically symmetric. It would be surprising if decaying from one isotropic state to another created a photon travelling in a particular direction. This is a hand-wavy, sketchy version of the argument for why this transition is rarely observed. (See Merzbacher, Ch 23 for a full explanation. [12])

Ne atom.[4] This requires that there is a vacant excited state for the Ne atom to jump to. Fortunately, Neon has an excited state whose excitation energy nearly matches the excitation energy of the He atom. (20.5 eV, see Figure 2)

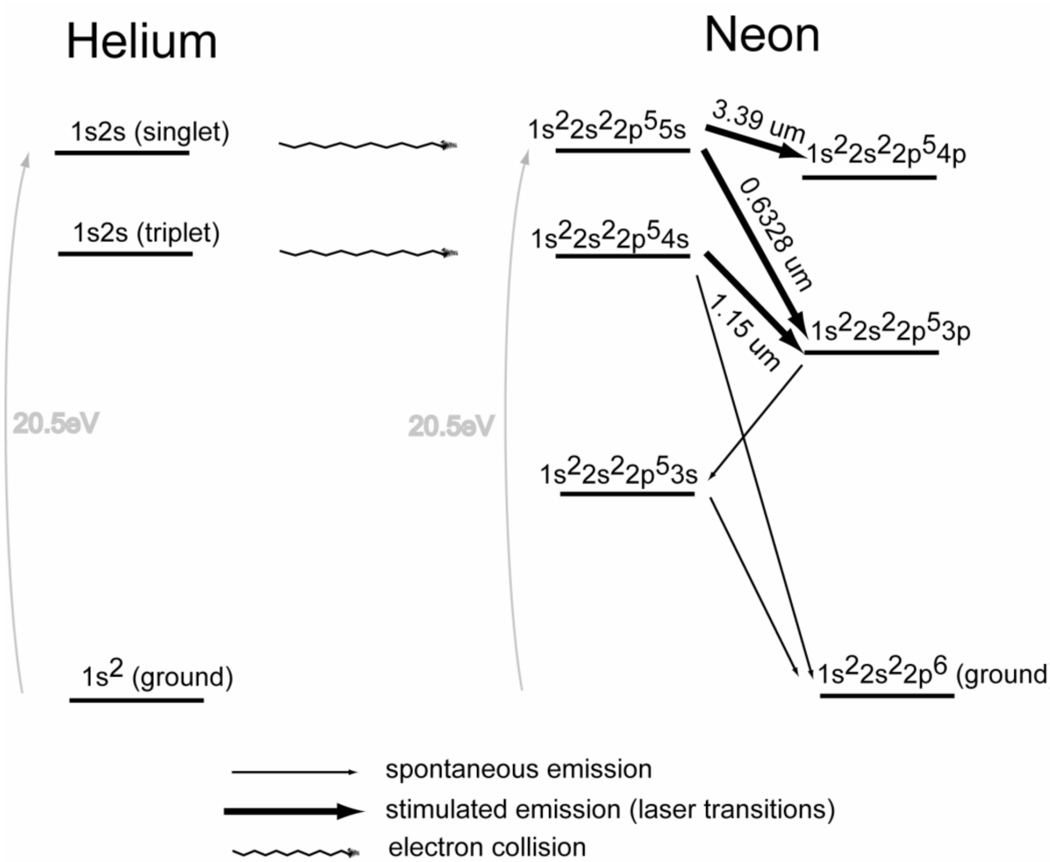

Figure 2. Excited states of the He and Ne atoms (simplified). States are labeled by the orbital configuration of the electrons (instead of using traditional spectroscopic notation).

Note: Neon ordinarily has an eletron configuration of $1s^2 2s^2 2p^6$. The energy necessary to excite the Ne atom to the $1s^2 s2p^5 5s$ state nearly matches the excitation energy of 1s2s (singlet) Helium.

[4] One might think the amount of energy transferred to the Ne atom depends on how fast the two atoms collide with each other. However the kinetic energy of the atoms in the colliding due to thermal motion at room temperature (k_B * *300* K = ($^1/_{40}$ eV)), is negligible compared to the *2* eV needed to excite the outermost electron in the Ne atom. Essentially all of the energy that excites the electron in the Ne atom comes from being in close proximity to and interacting with the excited electron in the He atom.

Furthermore the excited Ne atom *is* allowed to decay by spontaneous emission to a lower energy state, releasing a photon of energy. That lower energy state, in turn may decay to a lower energy state, until the ground state of Ne is reached. Of course, at each transition, a photon is generated, and there are many such transitions, but we can only see the ones which generate light in the visible spectrum. [5] The transition that generates 632.8 nm photons occurs when the outermost Ne electrion decays from the 5s to the 3p orbital. See figure 2.

This starts a chain reaction. As a 632.8 nm photon leaves one Ne atom, it collides with another excited Ne atom stimulating the emission of a second photon with identical direction, frequency, and phase. The light flux has been amplified by a factor of two! These two identical photons collide with other Ne atoms and increase the amplification. Photons emitted parallel to the long axis of the laser strike the mirrors and are reflected back along the axis, further increasing the amplification. (By contrast, photons which don't point down the axis of the cylinder will quickly leave the cavity and have fewer chances to collide with Ne atoms. These photons will not be amplified.) One of the mirrors is about 1% transmissive and this 1% of the laser light is the output of the laser. Imagine how bright it must be inside the tube!

[5] Other transitions between the states of the Neon atom are often either suppressed by selection rules, or they spontaneously decay so rapidly (producing photons that travel in random directions) that stimulated emission (which does produce coherent light along the direction of the cavity) rarely happens. Also the laser cavity can be designed to oscillate preferentially at 632.8nm by putting anti-reflective coatings on each mirror that absorb light at other frequencies. These details are discussed in Garret, CGB [5], and Banwell, CN & McCash, EM [4], among others.

III NON-IDEAL BEHAVIOR:
A) DOPPLER-BROADENING

Consider the fact that the gas atoms inside the laser cavity are not stationary, but move about in thermal motion.

Exercise III.1: From the equipartition theorem of classical statistical mechanics, we know that, on average, the kinetic energy of an atom is about equal to $(1/2)k_BT$. Calculate the ratio of its typical speed, *V*, to the speed of light. (You may take $T \cong 400°K$, and $M_{Ne} \cong 20 \times 1.67 \times 10^{-27}$ kg.)

An atom moving with velocity *V* along the tube axis will emit radiation of frequency $v = v_o(1 + V/c)$, where v_o is the radiation frequency of a stationary atom, and *V* is its velocity.

Exercise III.2: From the wavelength of the He-Ne laser line, $\lambda = 6.328$ x 10-7 m, calculate its frequency.

Thus we get a band of frequencies eligible to participate in stimulated emission and thus to be amplified. The laser light produced then also has a band of frequencies centered about v_o. This effect is called Doppler-broadening.

Exercise III.3: On the basis of the result of Exercise III.1, calculate the width, in Hz, of the Doppler broadened spectral line.

III B) MODES OF OSCILLATION

For the optical cavity to act as an optical resonator, and thus amplify the radiation, the length of the cavity must be equal to an integral number of half wavelengths of the radiation. Thus, the cavity is optically resonant for frequencies:

$$n_n = \frac{nc}{2L} \tag{2}$$

where *n* is a large integer, and *L* is the cavity length.

Exercise III.4: How large is n, typically? (As a rough estimate, take the distance between the mirrors in the laser cavity to be 20cm.)

Any frequency, ν_n, lying within the Doppler-broadened gain-curve will be present in the laser light.

Exercise III.5: Estimate the frequency separation (in Hz) between adjacent longitudinal modes of your laser [i.e., $?\nu = (\nu_{n+1} - \nu_n)$]. In order for a line to be amplified in your laser, it must fall within the Doppler-broadened gain-curve. How many lines should be observable?

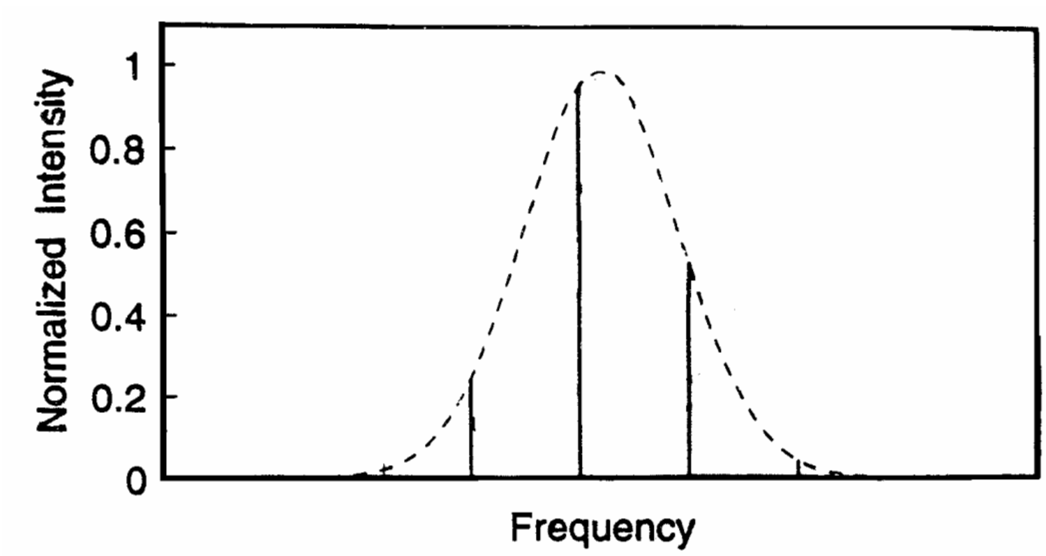

Figure 3. Schematic illustration of the Doppler-broadened gain curve and of the spectral lines of a He-Ne laser.

The above effects account for the departure of gas laser light from monochromaticity.

In addition, laser radiation is not only confined by the mirrors on either end of the cavity, but also by the walls of the glass tube (although to a lesser extent). Consequently, laser light also exhibits transverse-electric and transverse-magnetic modes of oscillation, in which the photons propagate in a direction that is not aligned with the symmetry axis of the laser cavity. This results in transverse-nodes (regions of zero intensity) within the cross-sections of the beam. The modes are traditionally labelled TEM_{pq} modes where p and q are integers giving the number of modes along the x and y axis (which are taken to be perpendicular to the directon of propagation).[6] Figure 4 shows a series of cross-sections of the laser beam for various transverse modes. Many He-Ne lasers have a combination of TEM_{00}, TEM_{10}, and TEM_{01} modes, resulting in a "doughnut" intensity distribution.

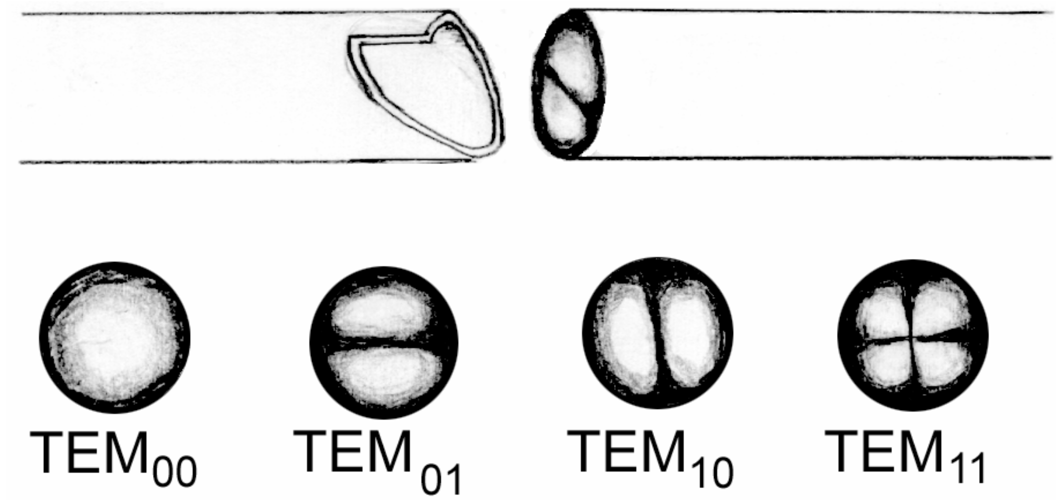

Figure 4. Illustration of the intensity profiles for various transverse modes. (Bright regions correspond to regions of intense light.)

IIII C) TEMPORAL COHERENCE

The fact that there are multiple frequencies contained in a laser beam places restrictions on the suitability of using a particular laser for an interference experiment. Interference patterns are observed when two beams of light take different paths to reach the same spot. But if the distance travelled by two different beams of light is large enough, you cannot produce a visible interference pattern. This is an important consideration when creating holograms, for example (which are created by the process of many different beams of light interfering with each other on the surface of a piece of film. This directly limits the shape and placement of the objects that can appear inside the hologram.)

[6] Minor detail: For circular cylindrical cavities, r and q are the relevant variables to use when solving for the spatial variation of the modes, not x and y. Regardless, the terminology "TEM_{01}", and "TEM_{10}" traditionally refers to the spatial variation of the modes along the x and y directions.

The coherence length of a beam of light (denoted L_c) is defined as the maximum difference in (optical) pathlength between two interfering beams such that an interference pattern is still visible. (A beam with high "temporal coherence" is defined as a beam with a long coherence length.) The coherence length depends on the distribution of the frequencies that make up the beam. The broader the frequency distribution, the shorter the coherence length. More precisely:

$$L_c = \frac{c}{\Delta n} \tag{3}$$

where Δn is the uncertainty in the frequency of the laser beam.

A discussion/derivation of this formula is given in Appendix A, and in [7].

Exercise IIII.6: Estimate the coherence length of your laser.

IV. EXPERIMENTAL ANALYSIS OF LASER LIGHT

The laser takes approximately 30 minutes to warm up.

When working with this laser, note that the polarization of the laser tends to flip repeatedly since the laser switches modes. Therefore, try to make all of your measurements during one polarization in order to eliminate the chances of error.

Never reflect the laser light directly back into the laser itself. It will not only skew your results, but also can damage the laser.

A) Polarization:

The degree of polarization is defined as:

$$r = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \tag{4}$$

It will change as the laswer warms up.

Exercise IV.7: Using a sheet of polaroid linear polarizer, and a pyro-electric laser power meter, measure the intensity $P(q)$ as a function of the angle q of the polarizer. Make a graph of $P(q)$ vs. q and determine the degree of polarization of the laser light.

B) BEAM DIAMETER

In this part of the lab you measure the profile (the cross sectional intensity) of your beam, and see how it changes at various distances along the beam-line. You will also see how it varies after passing it through a lens. You will be asked to measure the width of the laser beam.

This involves obstructing a portion of the beam with further and further until a certain percentage of the incoming light has been blocked. The profile of the intensity of the TEM_{00} mode of a laser is (approximately) a Gaussian.

The Method :

Exercise IV.8: Let us take the direction of the beam as the z-axis. Write down the equation for the intensity *I(x,y)* of the beam as a function of the perpendicular distance $r = \sqrt{x^2 + y^2}$. From your equation for *I(x,y)*, calculate the total intensity of the beam.

Exercise IV.9: Measure the beam profile with a power meter, and an adjustable knife edge. (Because the laser beam is so narrow, you must use a sharp object like a razor. If you use an object with rounded corners, instead of blocking the beam, you will just be making it more blury.) Mount the laser, power meter, and knife edge on the optical bench. (For your first measurement, try it close to the exit hole of the laser. 1-2 cm should be fine.) Adjust the knife edge so that it can be moved accross the beam in the *x*-direction, thus partially blocking the beam.[7] Measure the total beam power *P(x)* (arbitrary units will do) as a function of the knife edge position, *x*. Make a graph of *P(x)*.

[7] Distances can be measured very precisely using the "barn-door" aperture from the spatial-filtering lab. You may want to borrow it and affix a razor blade to it. Turning the knob will move the razor blade back and forth along the *x*-axis with high precision.

Exercise IV.10: Derive a theoretical expression for the result of Exercise 3 from the equation of Excercise 2. Let x be the horizontal and y be the vertical distance from the beam axis.

Exercise IV.11: Let us define the beam diameter as the diameter at which the electric field has decayed to $1/e$ of its maximum value. From the measurements of Excercise 3, calculate the beam diameter. What fraction of the total power is inside the beam diameter?

Exercise IV.12: If you have to measure many beam diameters, you may not wish to measure the complete beam profile each time. Thus, calculate how much the total intensity is reduced when you move the knife edge within one beam radius of the beam of the beam axis. Next, calculate how much the intensity is reduced when you move the knife edge one beam radius beyond the beam axis. Using these intensity values, move the knife edge to the corresponding positions and determine the beam diameter. (For your first measurement, try this with the knife edge close to the output hole of the laser.)

C) BEAM DIVERGENCE:

Ideally the angular divergence of a laser beam is the result only of Fraunhofer diffraction caused by the exit aperture. The divergence is defined as the full angle $2\Delta q$. (Where Δq represents the angular spread of the beam due to diffraction, as measured from the optical axis.) Since the divergence is small unless the beam has passed through a lens with a short focal length (as in Fig. 6), it is usually expressed in milliradians (mrad).

Exercise IV.13: Using whatever method you prefer, measure the beam divergence by measuring the beam diameter both at a large distance (2 to 3 meters) from the laser. (Compare this to the diameter near the output port of the laser.) Does you result for $2\Delta q$ agree with the manufacturer's value?

The theory of Fraunhofer diffraction [6] tells us that

$$\Delta q = 1.22(l/D_0) \qquad (4)$$

where D_0 is the width of the laser beam as it exits the aperture.[8] (Which is what you measured in section IV, part B).

Exercise IV.14: Calculate the angular divergence predicted by equation (4) and compare with your measured value.

The divergence you are observing makes it impossible to focus the beam to a fine point using a lens.

D) BEAM WAIST

A TEM_{00} mode laser can be focused with a good lens to a certain characterstic minimum diameter *d*, called the "beam waist", as shown in Figure 6.

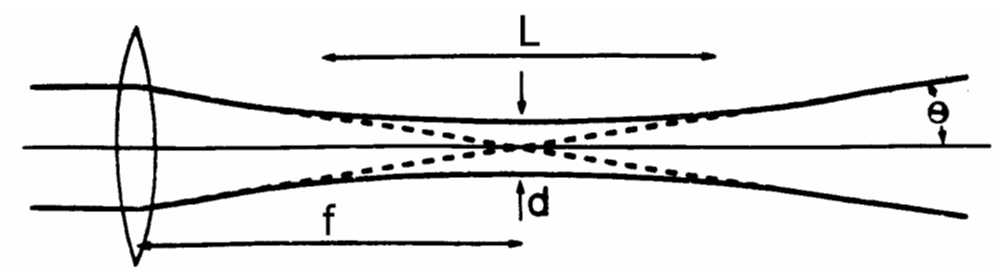

Figure 6 Illustration of the beam waist of the laser.

[8] This very common formula is typically used to characterize the diffraction-limited angular resolution of a telescope or microscope. If you do not know where it comes from, recall from your sophomore physics class that light passing through a slit of width D_0 spreads out in a diffraction pattern whose first minima (typically taken to be the "width" of the diffraction pattern) occurs at Dq @ l/D_0. However, in our case the light passes through a circular-hole aperture instead of a slit, hence the extra factor of 1.22. This formula is explained further in [6] or [7].

The beam will remain collimated around the beam waist for a distance L which is called the "depth-of-field". For a single mode TEM_{00} He/Ne laser, d and L can be shown to be given by:

$$d = \left(\frac{4l}{p}\right)\left(\frac{f}{D_0}\right) \tag{5}$$

and

$$L = 2\left(\frac{4l}{p}\right)\left(\frac{f}{D_0}\right)^2 \tag{6}$$

where D_0 is the diameter of the laser beam before entering the lens, and f is the focal length of the lens.

A derivation of equations (5) & (6) is sketched in Appendix B. The full derivation is given in [8].

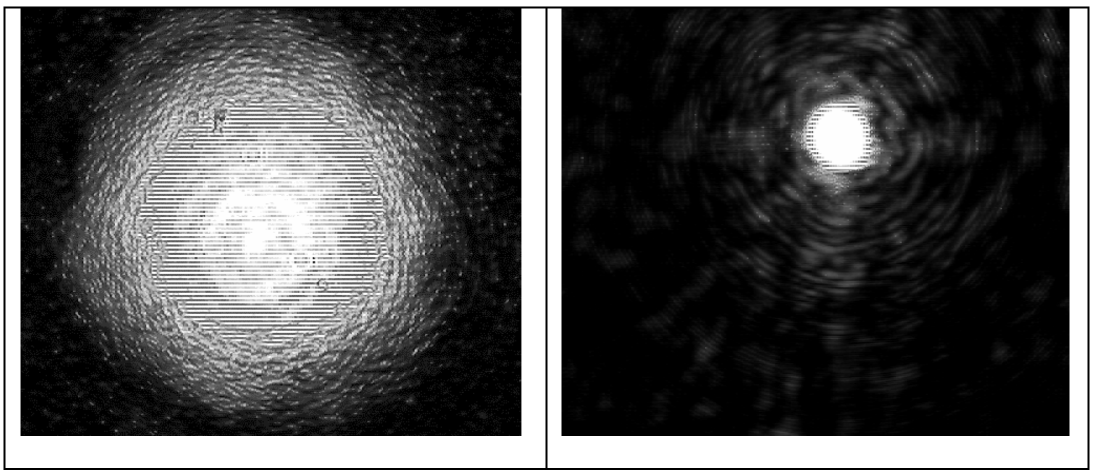

Figure. 7 Profiles of the laser beam made using the CCD array in Figure 5 a) before and b) after focusing the beam with a 25cm focal length lens.

Exercise IV.15: Calculate d and L for your laser and your lens.

Exercise IV.16: Pass the laser beam through a lens of known focal length and measure the beam diameter D(z) as a function of z at a number of axial positions. Plot the diameter as a function of z, and compare your results with the prediction given above.

E) THE FABRY-PEROT INTERFEROMETER

'Think' questions and Mode Stability, Polarization, and Spacing questions in this section <u>will not</u> be graded, but must be done in your lab notebook.

This device is actually a spectrometer in that it allows us to see the intensity distribution of the laser light as a function of frequency on an oscilloscope. The device is composed of a light detector, and two partially reflecting mirrors separated by distance x. (See Figure 8.) The laser light to be studied is directed into the Fabry-Perot (FB) interferometer in a direction perpendicular to the partially-reflective mirrors. When the distance between the these mirrors is an integer number of half-wavelengths, the rays which pass through the far mirror and make it to the detector are in phase and add constructively. The detector generates a voltage proportional to the intensity of the light striking it, and this signal is amplified and sent to an oscilloscope. Thinking of it another way, the two reflective mirrors form an optical cavity whose dimension can be modulated. When the dimension becomes equal to a integer number of half-wavelengths a resonance occurs, and a voltage proportional to the intensity of the beam (at that wavelength) is recorded by the oscilloscope.

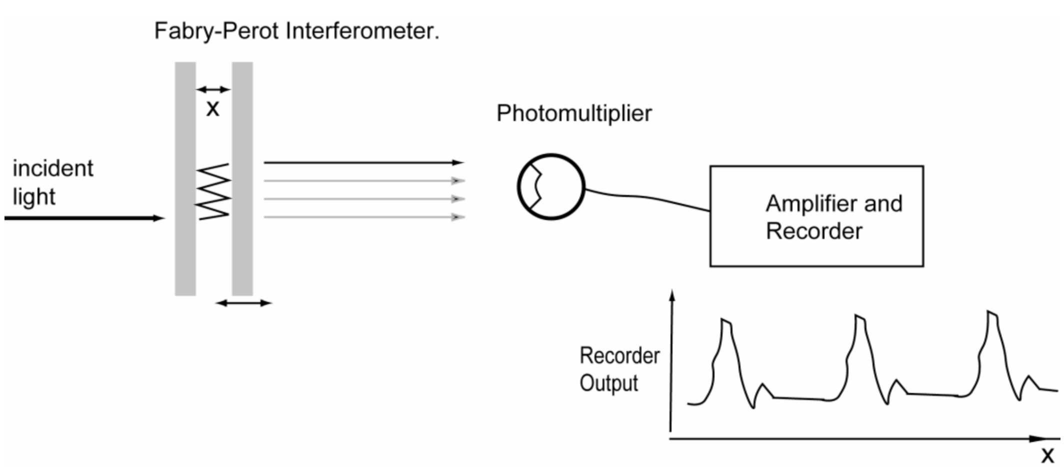

Figure 8 Fabry-Perot Interferometer

FP interferometers are discussed in more detail in [6] and [7].

Think The oscillascope will record a graph of the intensity of the light, $I(x)$, as a function of the distance between the mirrors, x. This graph will look periodic. Why? Hint: x is typically much larger than the wavelength $l=632.8nm$ and we will vary it over a distance of many l. What does $I(x)$ look like as you vary x from $n(l/2)$ to $(n+1)(l/2)$?

Think A Fabry-Perot interferometer is especially conveniant to measure the frequency distribution of highly monocrhomatic light narrowly peaked arount a central wavelength. Why? Suppose it were not so. If it helps, suppose you have two beams of radiation (A and B). Suppose first beam, A, contains radiation at wavelengths: $l_{A1}=100nm$, $l_{A2}=100.0001nm$, $l_{A3}=100.0003nm$, and beam B contains radiation of wavelenths: $l_{B1}=100nm$, $l_{B2}=100.0001nm$, $l_{B3}=200.0006nm$ (and suppose Fabry-Perot interferometer has a mirror spacing of $x=1mm$). Note that beam B is not narrowly peaked, and one would have to be careful not to confuse B with beam A when looking at the output of the FB interferometer.

Proceedure:

1. Turn on the Laser, Oscilloscope and "Spectrum Analyser Controller".
2. Align the Fabry-Perot interferometer with the laser light.
3. Hook up the FB interferometer to the scope as shown in Fig. 9

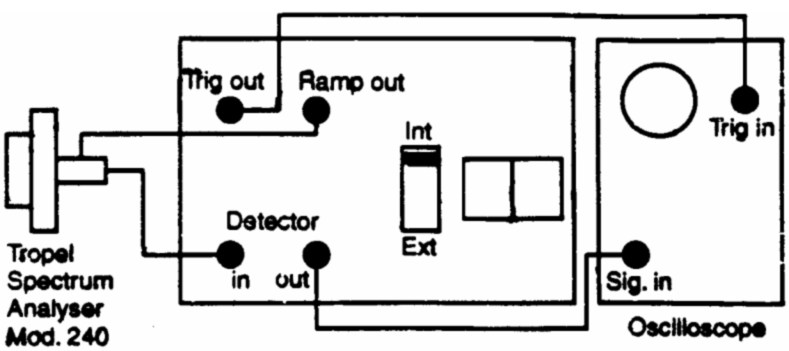

Figure 9 Schematic diagram of the interferometer and oscilloscope hookup.

4. Mode Stability: When the FB interferometer is properly aligned you should see a trace on the scope with several longitudinal modes lying within the Doppler-broadened gain curve, as shown in Figure 3. Note how the modes move. This is because the optical cavity of the laser is warming up and is expanding thermally. Recall how the frequencies of the modes depend on the length of the laser cavity? When the laser is thoroughly warm and the cavity length is stable, the modes will be stable. This is the reason one must allow the laser to warm up before doing work such as holography, which depends on a stable frequency output of the laser.

Use Sec/Div to make the signal wider and Volts/Div to make the signal taller.

5. Mode Polarization: The polarization of individual modes may be determined by rotating a sheet polarizer in front of the FB interferometer and observing the resulting intensity changes on the scop.e The results should be recorded. The digital oscilloscope is connected to a printer. Print out a trace from the oscilloscope. Do this at a couple different angles of the polerizer.

6. Mode spacing: To determine the longitudinal mode spacings, consider that the interferometer has a free spectral range of 7.5 GHz when the ramp input is swept from 0 to 40 volts. (From this information, can you estimate the maximum change in the length of the FB cavity?) Use the scope to check the scope's ramp output and determine what spectral range is swept. (Be sure to use the high-voltage probe when viewing signals larger than 10V on the oscillascope!) What is the mode spacing? What is the FWHM (width) of the Doppler broadened gain curve? What is the width of the spectrum $\Delta\lambda$?, $\Delta\nu$? (recall $n = c/l \leftrightarrow dn = -(c/l^2)dl \leftrightarrow \Delta n = -(c/l^2)\Delta l$). What is the coherence-length of your laser? Print out a trace from the oscilloscope and include it in your lab manual.

IV E) ZEEMAN EFFECT

The presence of a magnetic field changes the energy of various quantum states of the Ne atom (and/or causes degenerate energy levels to split into different energies levels.). The shift in energy levels may cause a shift in the frequecy of the different modes of light generated by the laser. (This effect is known to be about 1.82 MHz per Gauss for an He-Ne laser.) In addition to modifying the frequencies, a transverse magnetic field has a drastic effect on the polarization of the modes. See [9].

Mount the He-Ne laser between the poles of a magnet. (The magnetic field in this region should be on the order of 10 Gauss.) Observe what happens to the polerization of the modes and the mode structure as the magnetic field is increased. (If you cannot vary the strength of the magnet, for example if you have a permanent magnet, then move the laser cavity in and out of the region between the poles of the magnet, to simulate the same effect.) Print out a trace from the oscilloscope. See how the frequency distribution of your laser varies as you pass it through a polarizer and vary the angle. Do this for at least two different magnetic field strenghts (ie. with a magnetic field present, and with no magnetic field present). [9]

APPENDIX A: COHERENCE LENGTH

In section III, I mentioned that the lack of monochromaticity in the laser beam places limits on the geometry of the placement of objects that can be imaged in a hologram. Lets look at a simpler example. Consider a Michelson-Morely Interferometer:

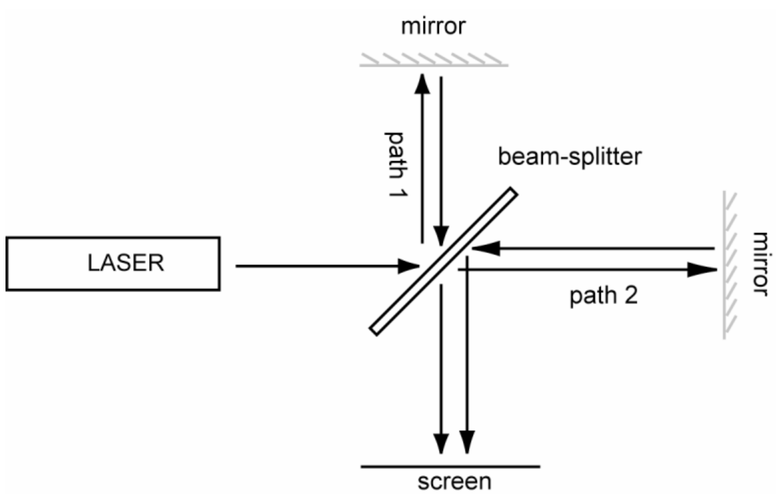

Figure 10 A Michelson-Morley interferometer.

In the paragraph that follows I will derive the usual formula for how the intensity of the light striking the screen depends on the difference in pathlength travelled by the two beams (equation (8)). This can be skipped without loss of continuity.

Assuming that the laser is completely monochromatic, the electric field coming out of the laser is a plane-wave and can be described by

[9] In many lasers, the mode structure collapses to a single longitudinal mode (whose transverse structure is TEM_{00}), when the Zeeman splitting becomes comparable with the laser cavity intermode spacing *c/2L*. This should occur near H = 300 Gauss for your laser.

$E(x,t) = A\cos(kx - wt)$.[10] Assume the beam passes through a beam-splitter, reflects off the mirrors and arrives at the screen as shown in Figure 10. Consider a particular point on the screen. Suppose that light from path #2 takes travels Δx meters farther than the light from path #1 to reach that point. (I'm ignoring details like phase changes due to reflections, and the index of refraction of the air.) The amplitude of the waves arriving at the screen is:

$$E(\Delta x,t) = \left[\frac{A}{\sqrt{2}}\cos(kx - v\,t) + \frac{A}{\sqrt{2}}\cos(k[x + \,?x] - wt)\right]$$

$$= Re\left\{\frac{A}{\sqrt{2}}e^{i(kx-wt)} + \frac{A}{\sqrt{2}}e^{i(k[x+\,?x]-wt)}\right\}$$

$$= Re\left\{e^{i(kx-wt)}\frac{A}{\sqrt{2}}\left(1+e^{ik\Delta x}\right)\right\} \qquad (7\,)$$

Dx (and *x*) will depend on the placement of the two mirrors, and where you are on the screen. The $\sqrt{2}$ was just added to insure that the total intensity of the light remains the same after being divided into two beams by the beam splitter.) This leads to the formula for the interference pattern generated by two interfering beams:

$$I(\mathbf{?\,x},t) \propto \left\langle E^2(\Delta x,t)\right\rangle = \left(1\!/\!2\right)\frac{A^2}{2}\left|1+e^{ik\Delta x}\right|^2$$

$$= I_0\left(1+\cos(k\Delta x)\right) \qquad (\,8\,)$$

Now consider using non-monochromatic light. To simplify the discussion, imagine that there are only two frequencies of light present in the laser beam, w_1 and w_2, (with wavevectors $k_1 = \omega_1/c$ and $k_2 = \omega_2/c$). When an electric field oscillating at frequency w_1 is superimposed with the an electric field oscillating at frequency w_2, the phase between two waves is constantly changing. Sometimes the two waves will be in phase (wave w_1 is at a crest when wave w_2 is at a crest). But $p/|w_1\text{-}w_2|$ seconds later wave w_1 and wave w_2 will be completely out of phase and cancelling out. This happens too rappidly for the eye to detect and what you see with your eye is a time-averaged intensity. Mathematically, suppose we add a wave of intensity I_1 (and max amplitude A_1) and frequency w_1 to a wave of intensity I_2 (max amplitude A_2) and frequency w_2. After taking the

[10] Of course, ***E*** and ***A*** should really be vectors (not scalars), but this is not an important detail.

square and averaging, the resulting intensity turns out to be the sum of the intensities I_{1+} I_2:

$$I_{w_1+w_2} = \left\langle \left[A_1 \cos(w_1 t) + A_2 \cos(w_2 t)\right]^2 \right\rangle$$

$$= A_1^2 \left\langle \cos^2(w_1 t) \right\rangle + 2A_1 A_2 \left\langle \cos(w_1 t)\cos(w_2 t) \right\rangle + A_2^2 \left\langle \cos^2(w_2 t) \right\rangle$$

$$= \left(\tfrac{1}{2}\right)A_1^2 + 0 + \left(\tfrac{1}{2}\right)A_2^2 = I_1 + I_2 \quad \text{(assuming } w_1 \neq w_2\text{)} \qquad (9)$$

So the intensity turns out to be the sum of the intensities due to the interference pattern generated by light of frequency w_1 and the interference pattern generated by light of frequency w_2. Suppose our He-Ne laser beam is composed of two frequencies in equal intensities (say, I_1 and $I_2 = I_0/2$). Plugging in equation (8), we get:

$$I(\Delta x, t) = \frac{I_0}{2}\left(1 + \cos(k_1 \Delta x)\right) + \frac{I_0}{2}\left(1 + \cos(k_2 \Delta x)\right) \qquad (10)$$

Because the two sources of light have different wavelengths, the two interference patterns generated by them are bright and dark in different places (ie. at different values of Dx). If Dx is large enough so that the bright portion of one of the interference pattern due to w_1 overlaps with the dark portion of the interference pattern due to w_2, you will see no interference pattern at all. This happens when $|k_1 - k_2|Dx = p$. **So, in order to see a well defined interference pattern, Dx should be far less than**

$$\Delta x << L_c \equiv \frac{p}{|k_1 - k_2|} = \frac{c}{2|n_1 - n_2|} \qquad (11)$$

This distance is called the **coherence length**.

What we see is that using non-monochromatic light has the effect of blurring or smearing out the interference pattern for large Dx. In the derivation we assumed we started with a bichromatic source of light, and we saw that the two interference patterns blurred together (equation (10)). A more general formula for the coherence length for any beam of non-monochromatic light [7] is:

$$L_c = \frac{c}{\Delta n} \qquad (12)$$

where Δn is the variance in the frequency distribution of the light source. The two formulas give different ways to estimate L_c and have the same dependence on *Dn*. (Although they differ by a constant, this constant factor is near unity.)

APPENDIX B: BEAM WAIST

Our laser beam is on the order of 1mm thick. However when we think of a plane wave, we think of a wave that extends to infinity in all directions. When the laser is in the TEM_{00} mode the light in the laser-tube has an approximately gaussion intensity profile. However you can get geometrical insight by making the cruder assumption that the laser beam was a perfect plane-wave with uniform intensity before before it passes through the hole at the end of the laser cavity. (This cruder approximation will introduce a small error into our answer, but it does not effect the physics of what is going on.) We can think of the hole at the end of the laser cavity as a circular aperture with diameter D_0. Again, Fraunhofer diffraction causes the beam to spread outward according to $\Delta q = 1.22(l/D_0)$. Because the light in the beam is not all travelling in the same direction, the beam cannot be focussed to a point by a lens. Instead the beam will have a finite size "waist" of diameter *d*.

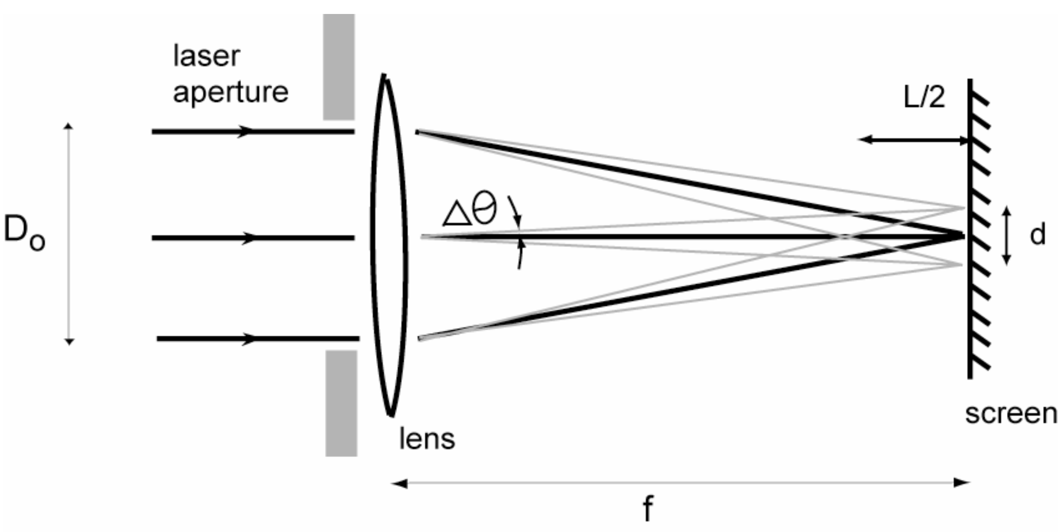

Figure 11 Approximate relationship between the width of the beam waist, and the angular spread in the beam due to diffraction. *f* is the focal length of the lens. The angular spread of the beam *Dq* (which is measured from the black line to the grey line) is due to diffraction from the aperture, not the lens, and would be present if the lens were absent. (Not to scale.)

Looking at it another way, the very fact that the beam has finite width implies that there are inherent uncertainties in the direction that the beam is travelling. If we pass the beam through a lens, uncertainty in the direction of the beam translates into uncertainty in where the beam is focused. Recall that beams which travel through the center of the lens (for example, the three beams next to the *Dq* symbol in Figure 11) are undeflected by the lens. The spread in the direction of these three undeflected beams this gives us a crude idea how wide the spot will be at the focal length of the lens (see Figure 11). Consider the right triangle between the central undeflected beam (black) and one of the beams at angle *Dq* (grey). This triangle has height *d/2* and base *f* (and an interior angle of *Dq).* Using *tan(Dq) = (d/2) / f* and substituting in for *Dq* you get:

$$d = 2\,f\tan(\Delta q) \approx f\,2\Delta q \approx 2\times 1.22\,f\left(\frac{l}{D_0}\right) \qquad (13)$$

(Comparing equation (13) with equation (5), we see that our answer is too large by an overall factor of about 2. Again this comes from the approximation we made by saying that the beam had a uniform intensity before exiting the laser cavity. See [8] for a more rigorous treatment.)

Include the following in your lab book:

- Exercises 1 through 16
- Zeeman Effect Questions and Oscilloscope Print out
- Any Appropriate Diagrams or Pictures
- Procedure and Conclusion

REFERENCES

[1] "Helium Neon Laser Guide" Melles Griot (1980) [(*Author unknown.* -A pretty good, short, exposition of many of the topics that are discussed in this lab manual. *This should be among the reprints lying on the bench next to the laser.*]

[2-3] The manuals for the "Coherent Spectrum Analyser Controller", and the "Gauss-meter". [*These are in the book reprints lying on the bench next to the laser aparatus.*]

[4] Banwell, C.N., McCash, E.M., "Fundamentals of Molecular Spectroscopy" (1994, 4th Edition), McGraw-Hill (Presently downloadable from: http://laser.inha.ac.kr/grad/Spect/), [Appendix B begins with a short & sweet list of the energy levels of the Ne atom and the transitions between them. Ch 5 explains some of the spectroscopic notation used in these energy-level diagrams.]

[5] Garrett, C.G.B., "Gas Lasers" (1967), McGraw-Hill, pp.62-75 [This book contains a more detailed discussion of the various energy levels of the Ne and He atoms. It also discusses producing laser radiation using transitions between other energy levels than the ones that generate 632.8 nm visible light, as well as how to avoid this.]

[6] Fowles G.R., "Introduction to Modern Optics" (1989), Dover, pp.99-119 [This book has a short explanation of Fabry-Perot interferometers. *This should be among the books in the senior-lab library. Please use the sign-out list if you check it out.*]

[7] Akhmanov, S.A., Nikitin, S. Yu., "Physical Optics" (1997) Clarendon Press, pp. 185-208, 211-222 [This book has a lengthy discussion of the coherence length of a laser beam. It also has a discussion of Fabry-Perot inteferometers.]

[8] Sinclair, D.C., Bell W.E., "Gas Laser Technology", Holt Rinehart (1980) [This book contains (among other things) a more rigorous derivation of the width of the beam waist of a TEM_{00} mode laser with a gaussian intensity profile than I gave in Appendix B.]

[9] Ferguson, J.B., Morris, R.H. "Single Mode Collapse in 6238 Angstrom Helium Neon Lasers", Aplied Optics, (1978) [This contains a discussion of Zeeman splitting and its effect on the radiation from 632.8nm He-Ne lasers.]

[10] Liboff, "Introductory Quantum Mechanics" (2nd edition), Addison-Wesley (1992), pp. 685-690 [The end of this book contains a discussion of a simple two-state laser. It covers stimulated and spontaneous emission of photons and the same material I talked about in section II A. In many ways, the discussion is less intimidating than Einstein's article (see below). Although it mentions time-dependent-perturbation-theory, you can get something out of reading it even if you haven't seen this yet in your quantum mechanics class. The newer (3rd) edition of this book should also work fine.]

[11] Einstein, A., "On the Quantum Theory of Radiation" (1917), taken from "LASERS: Selected Reprints", published by the American Association of Physics Teachers, Editors: O'Shea D.C., and Peckham D.C. (1982) [This covers the same material in Liboff [3]. Unlike Liboff, it doesn't assume knowledge of quantum-mechanical perturbation theory. (It was written before quantum mechanics was fully developed.) However it does assume some knowledge of (quantum) statistical mechanics. *This should be on the bench next to the laser.*]

[12] Merzbacher, "Quantum Mechanics" (3rd Edition), Wiley John & Sons, Inc. (1998), Chapters 21 & 23 [For future reading only. If you are compelled to know why atoms decay and spontaneously emit photons, this book will tell you. However, this is quite an advanced book and Chapters 21 and 23 are difficult, time-consuming reading.]

[13] Metrologic He-Ne Laser Catalog (100 ways to use a laser)