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# SPATIAL FILTERING

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In this experiment you have the opportunity to examine the fundamentals of spatial filtering. Spatial filtering beautifully demonstrates the technique of Fourier transform optical processing, which has many current applications, including the enhancement of photographic images and television pictures. Future applications include the optical data processor or optical computer. The basis of spatial filtering is Fraunhofer diffraction from the object whose image is to be spatially filtered.

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## BACKGROUND: PRINCIPLE OF OPERATION AND THEORY

In this section we aim to gain sufficient understanding of theory behind the Fourier transform and the Fraunhofer diffraction in order to make meaningful observations in our experiment.

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### FOURIER TRANSFORM

It is well known that a periodic waveform can be decomposed into a series of harmonic waves. The Dirichlet theorem states that if a function  $f(t)$  is a bounded function of period  $T$  with at most a finite number of extrema or discontinuities in a period, then the *Fourier series*

$$f(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} (a_m \cos m\omega t + b_m \sin m\omega t) \quad (1)$$

where the coefficients  $a_m$  and  $b_m$  are defined as

$$a_m = \frac{2}{T} \int_{t_0}^{t_0+T} f(t) \cos m\omega t \, dt \quad \text{and} \quad b_m = \frac{2}{T} \int_{t_0}^{t_0+T} f(t) \sin m\omega t \, dt$$

converges to  $f(t)$  at all points where  $f(t)$  is continuous and to the average of the right and left limits at each point where  $f(t)$  is discontinuous.

There is a different representation of the Fourier series. Using Euler's equation,  $e^{\pm i\theta} = \cos \theta \pm i \sin \theta$ , Eq. (1) can be put in a form

$$f(t) = \sum_{n=-\infty}^{+\infty} c_n e^{-in\omega t} \quad \text{where} \quad c_n = \frac{1}{T} \int_{t_0}^{t_0+T} f(t) e^{in\omega t} \, dt \quad (2)$$

Now, consider a *nonperiodic* function. Mathematically we may regard such a function as a periodic function with  $T \rightarrow \infty$ . In view of this Eq. (2) becomes

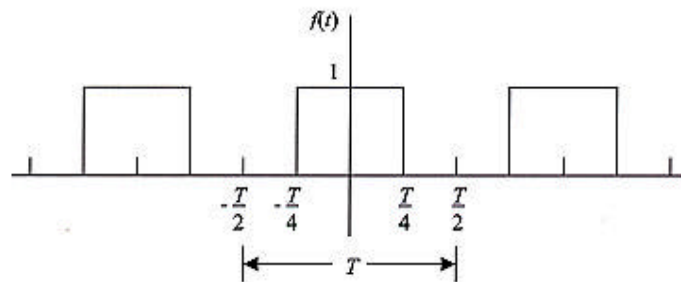
$$f(t) = \int_{-\infty}^{+\infty} g(\omega) e^{-i\omega t} \, d\omega \quad (3)$$

where the coefficient is given by

$$g(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(t) e^{i\omega t} \, dt \quad (4)$$

This is the oft-quoted result of the one-dimensional Fourier transform, where  $f(t)$  and  $g(\omega)$  are referred to as a *Fourier-transform pair*. Thus given a function  $f(t)$  we can obtain a frequency spectrum  $g(\omega)$  which constitutes the original function  $f(t)$ .

To illustrate how we interpret the result of the Fourier transform, consider a periodic function sketched in Figure **Error! Reference source not found.**



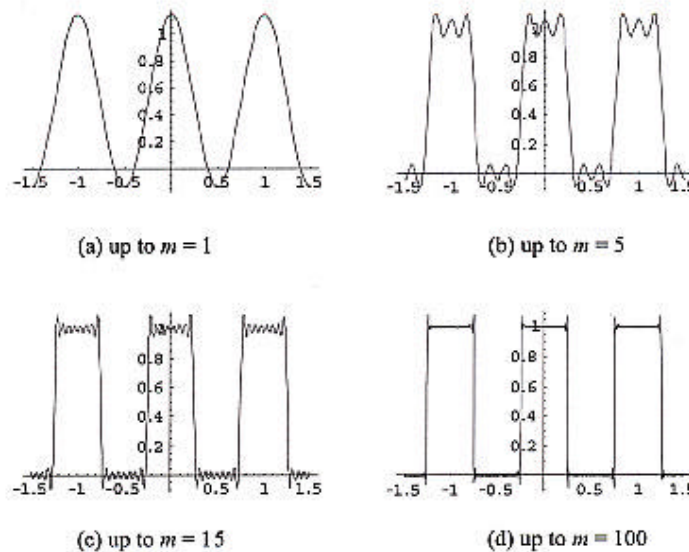
**Figure 1**

The first few terms of the Fourier series representation of this function are

$$f(t) = \frac{1}{2} + \sum_{m=1}^{\infty} \frac{2}{m\pi} \sin \frac{m\pi}{2} \cos m\omega t = \frac{1}{2} + \frac{2}{\pi} \cos \omega t - \frac{1}{3} \cos 3\omega t + \frac{1}{5} \cos 5\omega t + \dots \quad (5)$$

Notice that even a very simple square wave consists of an infinite series of harmonic functions of different frequencies and amplitudes.

We shall see how the terms in Eq. (5) add up to form  $f(t)$ . The plots in Figure 1 show how the series converges to  $f(t)$  as more and more higher frequency terms are added. (Note that  $T = 1$  in these plots)



**Figure 1**

Notice that the term with the fundamental frequency (i.e.,  $m = 1$ ) very roughly defines the outline of the original function. The finer features of the function  $f(t)$ , such as its corners of squares, can only be accomplished by adding or subtracting harmonic

functions of *higher* frequencies ( $m > 1$ ) than the fundamental frequency. Hence removing higher frequency components effectively eliminates such details from the original function.

**Exercise 1:** What happens to the function  $f(t)$  if you remove the  $m = 1$  term while retaining all the other terms? Sketch the resulting function.

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## FRAUNHOFER DIFFRACTION AND FOURIER TRANSFORM

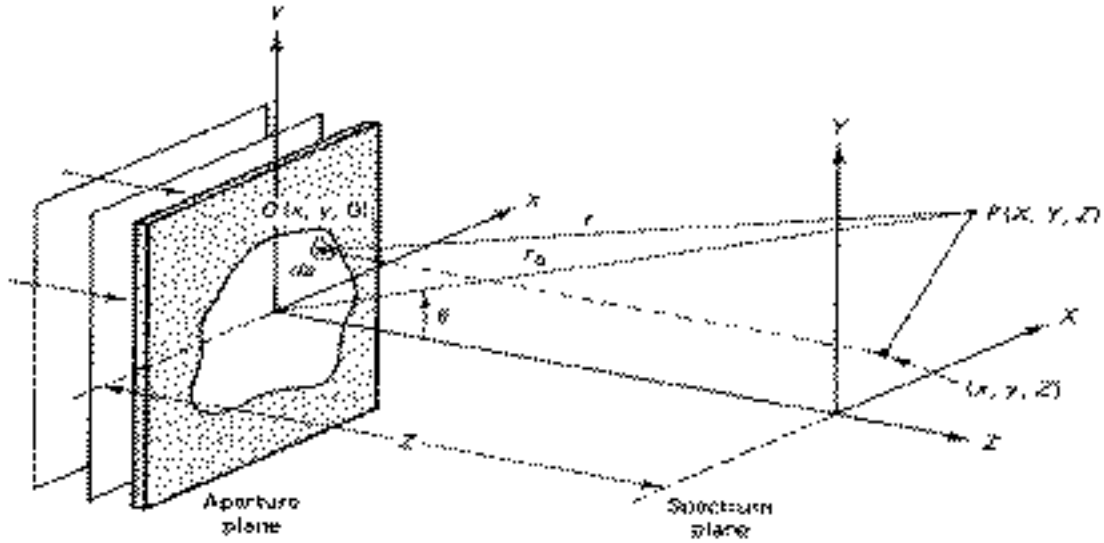
According to the Huygens principle, every point of a given wavefront of light can be considered a source of secondary spherical wavelets. Therefore the diffraction pattern at some point on a final screen consists of the superposition of the fields of all the secondary waves generated from such sources. The amplitude  $E(r)$  at the distance  $r$  from the source of a monochromatic spherical wave takes the form

$$E(r) = \frac{E_0}{|r|} e^{i(k|r| + \omega t)} \quad (6)$$

where  $E_0$  is the amplitude at  $r = 1$  (called the unit amplitude), and  $k$ ,  $\omega$ , and  $t$  are the propagation constant, angular frequency, and time, respectively.

The *Fraunhofer diffraction* is also referred to as *far-field diffraction*. It is a classification of diffraction effects that arises from the type of mathematical approximations possible in order to compute the resultant diffraction patterns. In particular, we consider a diffraction caused by a *coherent* light producing a final pattern on a screen placed effectively far enough that the incoming wavefront of light is plane.

In Figure 2, the diffraction pattern of the image at the aperture plane is observed in the  $XY$ -plane at a distance  $Z$  along the  $Z$ -axis.



**Figure 2**

At  $P$ , the light amplitude  $dE_p$  of the monochromatic spherical wave originating from an elemental area  $da$  at the point  $O$  at the aperture is given by

$$dE_p = \frac{E_s da}{|r|} e^{i(\omega t - k|r|)} \quad (7)$$

where  $r$  is the distance from the point  $O$  to  $P$ . The quantity  $E_s$  represents the amplitude per unit area of the aperture around the point  $O$ , and is called the *aperture function*. Simple geometry leads to the relations

$$r^2 = (X - x)^2 + (Y - y)^2 + Z^2$$

and

$$r_0^2 = X^2 + Y^2 + Z^2.$$

For the Fraunhofer diffraction, both  $x$  and  $y$  are negligible compared to  $r_0$ . Hence we can ignore the second order terms in  $x$  and  $y$ , and obtain

$$r = r_0 \left[ 1 - 2 \frac{(xX + yY)}{r_0^2} \right]^{1/2} \approx r_0 \left[ 1 - \frac{xX + yY}{r_0^2} \right] \quad (8)$$

where the approximation through the binomial expansion is used. Noting that  $r \approx Z$  is a reasonable approximation for the Fraunhofer diffraction, we reach an alternative expression of Eq. (7),

$$dE_P = \frac{E_s dx dy}{Z} e^{i\omega t} e^{-ik[r_0 - (xX + yY)/r_0]} \quad (9)$$

so that the integration over the area of the aperture (i.e., taking account the contributions from all the light sources on the aperture plane) leads to the integral

$$E_P = \int E_s(x, y) e^{ik(xX + yY)/r_0} dx dy \quad (10)$$

We can further simplify this integral by introducing the quantities called the *angular spatial frequencies*

$$k_X = \frac{kX}{r_0} \quad \text{and} \quad k_Y = \frac{kY}{r_0} \quad (11)$$

Using these Eq. (10) may be rewritten as

$$E_P(k_X, k_Y) = \int E_s(x, y) e^{i(xk_X + yk_Y)} dx dy \quad (12)$$

From this result, we immediately notice that  $E_P$  and  $E_s$  form a two-dimensional Fourier transform pair. Therefore it is also true that

$$E_s(x, y) = \frac{1}{(2\pi)^2} \int E_P(k_X, k_Y) e^{-i(xk_X + yk_Y)} dk_X dk_Y \quad (13)$$

What we demonstrated here is a proof that the Fraunhofer diffraction pattern described by  $E_P$  is the Fourier transform of the source pattern at the aperture plane, expressed by an aperture function  $E_s$ . (Of course, what you see projected on the screen is the intensity of light striking it,  $I$ , which is  $\propto (\mathbf{E} \times \mathbf{H}^*)$ , proportional to  $|\mathbf{E}|^2$ . 1 p.25)

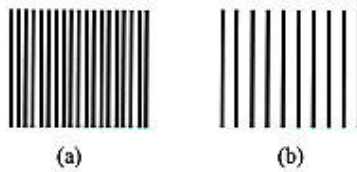
## SPATIAL FREQUENCY AND DIFFRACTION PATTERN

We defined angular spatial frequencies in Eq. (11). The terminology as well as the physical meaning is similar to the temporal analogs. The *spatial frequency* is the number of cycles per unit *length* (as opposed to time), or equivalently, how often the signal is repeated over a unit length. This definition suggests a unit of  $\text{cm}^{-1}$  (or  $\text{m}^{-1}$ ,  $\text{mm}^{-1}$ , etc.) for spatial frequency. Also, the angular spatial frequency  $k$  and the spatial frequency are related by

$$k = 2\pi\nu$$

as expected.

**Exercise 2:** Let us consider an example of spatial frequency. Figure 3 depicts two gratings consisting of repeating bars.



**Figure 3**

Measure the spatial frequencies of the gratings as accurately as possible. Which grating has a higher spatial frequency? How do you minimize the error in your measurements? If the black lines represent the open slits, how should the aperture function look like? Sketch the function.

We shall now go back to Eq. (11). Recall that  $k = 2\pi/\lambda$  where  $\lambda$  is the wavelength of incident light. The spatial frequencies are then given by

$$\nu_X = \frac{k_X}{2\pi} = \frac{X}{\lambda r_0} \quad \text{and} \quad \nu_Y = \frac{k_Y}{2\pi} = \frac{Y}{\lambda r_0}. \quad (14)$$

Since  $v_x$  is proportional to  $X$  and  $v_y$  to  $Y$ , the high (low) spatial frequencies appear at large (short) distance from the  $Z$ -axis, or the optical axis. It is quite general that the more localized a signal, the more spread out will be its transform. For our purposes, we can extend this to mean that the higher spatial frequencies (more localized signals) will lie at the periphery of the diffraction pattern and the lower spatial frequencies will be more centrally located. For example, if a diffraction peak appears at  $X$  on the one-dimensional spectrum plane, it corresponds to the term representing the frequency  $v_x = X/\lambda_0$  in the Fourier series.

**Exercise 3:** Consider a vertical grating with a spacing of  $d$  cm between lines placed at the aperture plane. What is the condition for the diffraction maxima to appear? (Derive an equation relating  $d$ ,  $v$  {refer to Figure 2}, and the wavelength  $\lambda$  of the incident light.)



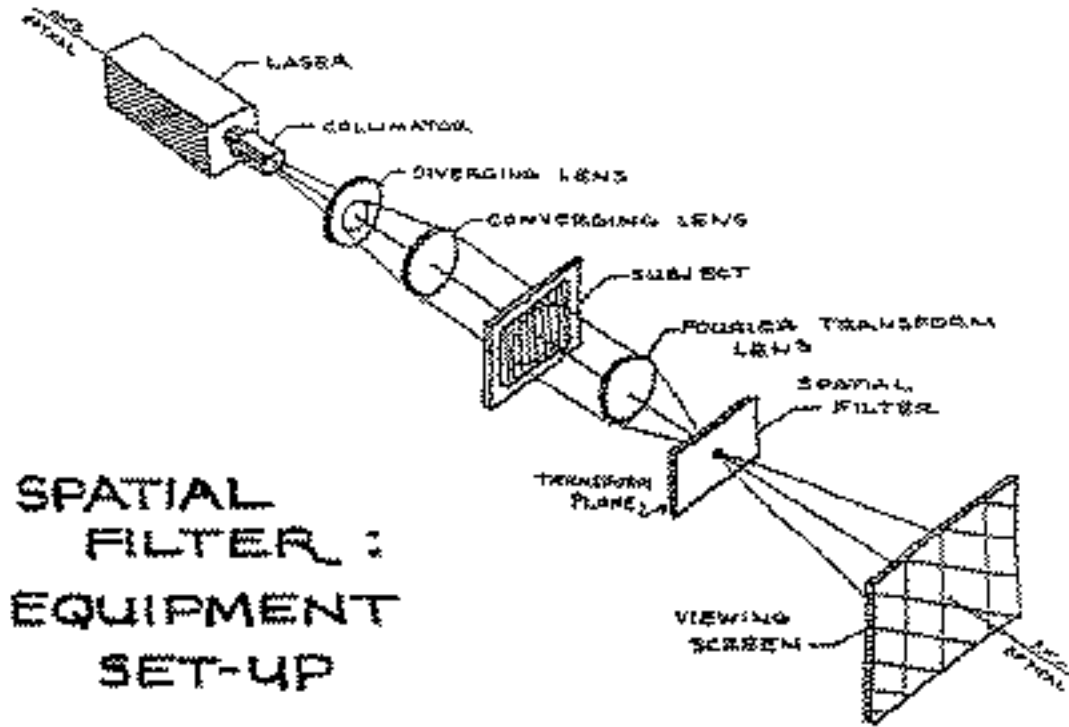
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# INSTRUCTIONS: EXPERIMENTAL PROTOCOL AND DESIGN

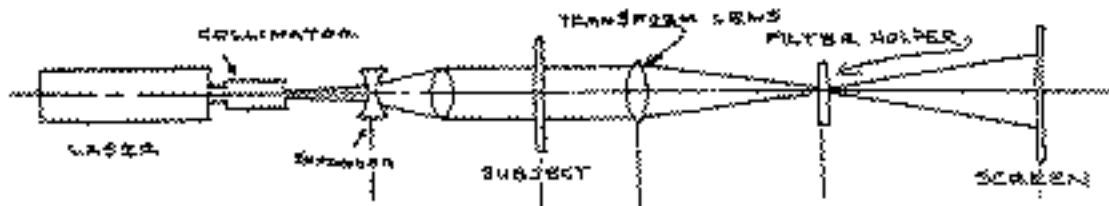
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## EQUIPMENT SETUP

Our goal is to set up the apparatus depicted in Figure 4.



**SPATIAL  
FILTER :  
EQUIPMENT  
SET-UP**



## Figure 4

Remove all the components from the optical bench, except the laser and the viewing screen. Light from the focused laser beam is spread out by a collimator, and then later by a diverging lens. The collimator is a small pinhole and when the laser passes through it, the light diffracts outwards.

Remove the collimator. Place a lens-holder on the optical bench, remove the lens, and replace it with an opaque object. By projecting the beam onto this object, (making marks on it) and moving the object back and forth along the optical bench, align, level, and center the laser beam with the center of the lens holder. You may have to translate it sideways, in addition to adjusting its direction

**Exercise 4:** Attach the collimator again and align the collimator with the laser beam using with the two knobs. When properly aligned, the light emerging from the collimator consists of concentric circles. By measuring the radius of the central spot estimate the diameter of the pin-hole (“D”) used in the collimator. (Recall the relationship  $\sin\theta = 1.22(\lambda/D)$ , for a circular aperture, <sup>1</sup> p.119)

Place the diverging lens directly in front of the collimator. Translate it vertically and horizontally in order to center the diverging cone of light, as much as possible in the original direction of the beam. (Use one of the lens holders with an extra knob which allows you to adjust its horizontal position.)

Add a converging lens as depicted in figure 5, and place it so that the parallel beams of light leave it pointing along the direction of the optical bench.

**Stop!** It is very important that the beam be collimated when it strikes the object. Test this by checking the beam diameter between the collimator and the transform lens with a piece of white paper. The diameter should be constant in this region. Why is it important to collimate the beam before hitting the subject? What happens if the expanded beam illuminates the subject without being collimated?

**Suggestion:** There are lenses with various focal lengths in the lab. We want a

strong even beam. Try different lenses. Choose one so it can be placed where that a large amount of light is projected uniformly across its surface.

**Think!** What is the role of the Fourier transform lens? Where does the Fraunhofer diffraction pattern form without the transform lens?

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## DIFFRACTION PATTERN

Place various subject transparencies on the subject plane and observe the diffraction patterns in the focal plane.

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## CCD CAMERA AND COMPUTER

The diffraction patterns often appear too small direct observations by projecting them onto a screen. A computer equipped with a CCD camera may also be used to capture the pattern. For this, you will need to remove the camera's focusing lens and image the diffraction pattern directly onto the CCD array itself. *Appendix A* explains this procedure.

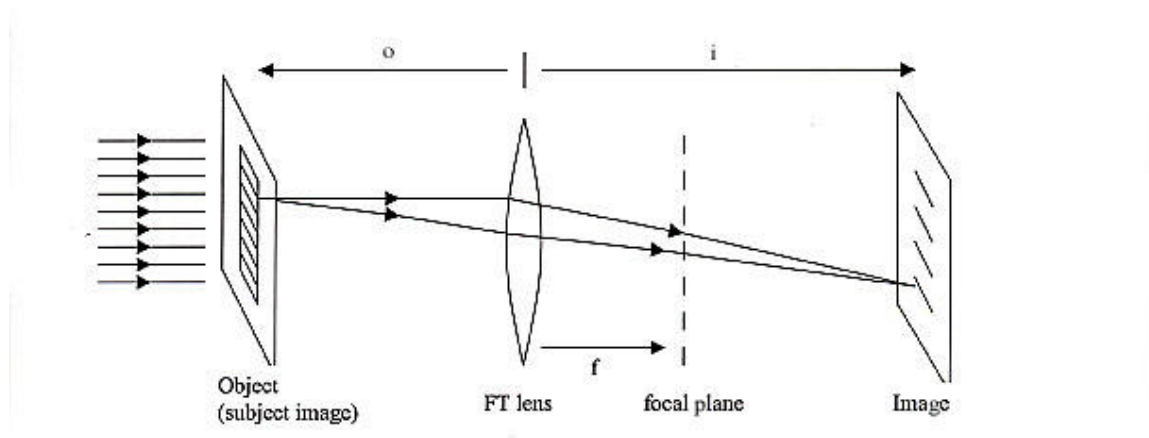
Use simple subjects such as the transparency consisting of alternating opaque and transparent bars, the fine wire mesh, a newspaper photograph, and so on. Try three (or more) different transparencies.

**Stop!** Remember to take notes of the features of your subjects, such as their spatial frequencies. (The starburst image, for example has a characteristic angular frequency  $w$  associated with the number of radial lines leaving the center. Write down whatever is relevant for your sample.) Either sketch or include a computer printout of both the subject images and their observed diffraction patterns in your lab book. Can you make any general conclusions about the correspondence between the subject images and their diffraction patterns?

**Exercise 5:** Assuming your diffraction pattern has well defined peaks, measure the physical distance between the peaks for the subject

transparencies that you used previously. If you used the CCD camera to enlarge the diffraction pattern, please include a print out of this pattern in your lab book. (The procedure for calibrating the length/pixels scale for the CCD is explained in Appendix A.) It is interesting to note that it is possible to obtain the Fourier transform (or inverse Fourier transform) of any of the images you capture with the digital camera, using the FFT feature of the image analysis software. (See appendix B.)

There's no reason we have to place the screen at the focal point of the lens. Now try moving the screen back a bit so that the right hand side of the apparatus in figure 5 (ie. from the subject-image on) resembles figure 6 below.



**Figure 6**

This is the traditional setup from geometric optics. The ray diagram looks different because, at this distance from the lens, all the rays of light that strike a particular point on the screen come from the same point on the subject image. What we were calling the “subject image” or “aperture” before is referred to as the “object” in this new context. In order to get a focused image, the distance from the object to the lens, “*o*”, and the distance from the lens to the image “*i*” must satisfy:

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f} \quad (15)$$

where “ $f$ ” is the focal length of the “Fourier transform” lens. The magnification (the ratio of the size of the final image to the original) will be  $i/o$ , and you will get an image of equal size to the original when  $o=I=2f$ . Of course, you will need to have the distance between the aperture and the “Fourier transform” lens ( $o$ ) be at least as far apart as the focal length ( $f$ ) in order to satisfy (15), for any  $i$ . You may want play around with different  $o$ ’s and  $i$ ’s to produce an images of a desired size. (To make it easier to photograph using the computer’s digital camera, for example.)

This is the setup we want for special filtering. For the effects we want to see, all the different beams of light striking a particular point on the screen should all be coming from the same spot on the subject image. It is important to place the screen in the right place so that this will happen. Oddly enough, in this lab, you can get a very focused-looking image even when equation (15) is violated. The ray-tracing arguments used to justify equation (15) ignore the wave nature of light and assume the light is scattering outwards evenly in all directions from the object. But in our setup, the angular spread of the beam tends to be very narrow after it scatters off of the object (or “aperture”). This is because the object is struck with coherent light {plane waves} whose wavelength is typically much smaller than the size of the holes and spaces within the object. Recall that diffraction from a single slit of width “ $a$ ” has an angular spread of approximately  $\lambda/a$ . ( $\lambda$  being the wavelength. If there are multiple slits, this just makes the diffraction even sharper.) Hence light propagating along directions other than the original direction of the beam tends to be very weak due to interference. The highly directional light leaving the aperture behaves similar to the light from a movie projector and, despite the lens, we will be able to get focused-looking images projected on the screen at just about any distance.

**Suggestion:** (Optional) We need to figure out where to put the screen. We could just calculate where using equation (15). Because of uncertainty about where the center of the lens actually is, this would only be accurate to about one or two centimeters. It would be nice to have some additional way to verify that the screen is in the right place. If we replace the laser light with an incoherent, omni-directional light source the aperture (say directly from a light bulb located near the aperture), geometric optics takes over, and we will only get a sharp image on the screen when it’s in the location predicted by equation (15). (Because the light no longer interferes with itself destructively at large angles.) You can simulate the effect of an incoherent light source by running the

collimated beam through a piece of bumpy, frosted or cloudy colored glass or plastic. Due to the varying thickness of this material, the beams of passing through different spots on the plastic material will be bent in different directions, and they will no longer be in phase with each other. (The plastic windows used in credit-card holders in wallets work well for this purpose. You get a brighter picture if you hold up the bumpy plastic close to the subject image.) Now move the screen back and forth until the image looks as sharp as possible.

In summary, with the screen placed at a distance “ $i$ ” from the lens, you will see a copy of the original image (upside down). If the screen is placed at the focal plane, and you are using a coherent light source, you will see the Fourier transform of the image.<sup>1</sup>

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## SPATIAL FILTERS

Spatial filters are used to block components of a diffraction pattern at the transform plane. To pass only the central peak of the diffraction pattern, we would need a hole filter, such as a metal sheet drilled with small holes. On the other hand, a transparency consisting of small dots can be used as a dot filter, which blocks the central peak. In this way, using spatial filters, it is possible to intentionally block or pass through the components of the Fourier series (i.e., the frequency spectrum of the subject pattern), if we know the shape of a diffraction pattern.

**Stop!** Place various spatial filters on the transform plane and observe the resulting images on the viewing screen. Compare them to the unfiltered images. Can you make any general conclusions about the effect of a dot filter? About a hole filter? Your measurements of diffraction pattern in

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<sup>1</sup> Comment: If you move the screen back and fourth between these two positions, you will see some intermediate stage between the two. This defines a continuous transformation between a function and its Fourier transform. Optional: Try this using the newsprint photo of the silly looking fellow’s head. Starting at the focal plane, describe what you see when the screen is moved away from the focal plane towards the image plane. You may be able to see the effect best by shining the light directly into the open CCD (with the lens removed), instead of trying to stare at the tiny pattern projected on a screen. It’s nifty.

the previous section should help you choose a filter of the right size.

**Suggestion:**

The task of placing tiny obstructions in front of selected portions of the diffraction pattern will be easier with a larger diffraction pattern. The size of the diffraction pattern is proportional to the distance from the lens to the Fourier Transform plane. Hence try using a lens with a large focal length. In this light, consider what kinds of images lend themselves well

- a fine mesh, or an image with fine structural detail, or
- a blocky, widely spaced mesh or pattern to spatial filtering:

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**EXPERIMENT AND ANALYSIS**

You will primarily use three kinds of subjects: a wire mesh, a starburst image, and a newspaper photograph.

1. Verify the relations described by Eq. (14) using the wire mesh and the starburst patterns. Note that  $r_0 = f$  where  $f$  is the focal length of the transform lens (why should this be true?).

**Think!** You would expect discrete diffraction peaks to appear for the wire mesh. Can you expect such a well-defined diffraction pattern for the starburst image? How do you go about verifying Eq. (14) if you cannot make direct measurements of diffraction pattern? Try hole filters with different hole sizes and observe how the final image changes.

2. Use the wire mesh as the subject. Eliminate vertical lines from the final image by using a suitable filter. (If end up using a spatial filter of considerable thickness, it's important to make sure the edge of this surface(s) needs to be located right at the

Fourier-transform plane where the diffraction pattern is most focused.<sup>2</sup> Optional: Once you have done this using the mesh, it's easy to swap the mesh with the starburst. If the spacing between the lines is similar to that of the mesh, you should see a similar effect.)

3. Now add some lower frequencies to the source image. One way is by affixing a few small objects to the wire mesh. Pieces of black tape cut and rolled up so as to be a few mm across work well. Another way is to generate a "wire mesh" using software on the computer in lab. (This is explained in Appendix C.) Using the computer, create a grid, superimpose it with dark spots and print it out on a transparency.<sup>3</sup> You may also want to try cutting holes in the grid pattern. (By superimposing white spots on top of the grid pattern. This is a bit easier than taking scissors to wire mesh. Think of both the holes and the blobs as large-wavelength fluctuations in the regular grid pattern.) Filter out the high frequencies so as to leave only these irregular objects visible. Now try filtering the low frequencies.<sup>4</sup>
4. As a source image, I recommend using the computer to generate a fine-grained mesh pattern with the average distance between lines in the mesh somewhere between 0.5 to 1 mm. It turns out to be easier to see the effect of filtering out the low frequencies if you cut holes in the mesh pattern rather than superimpose dark blobs on top of the

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<sup>2</sup> The following trick may be helpful: When you first place the spatial filter in the path of the beam, this will probably create a shadow on the image where the image is dimmer. As you move the spatial filter back and forth along the axis of the beam, as you approach the Fourier transform plane, this shadow will appear to get infinitely large. To center the spatial filter on the center of the diffraction pattern, it sometimes helps to move the spatial filter away from the Fourier-transform plane so that you can see the shadow in the image on the screen. Slide the spatial filter perpendicular to the optic axis until the shadow appears centered on the image. Once you have done this move the filter back to the Fourier-transform plane.

<sup>3</sup> Laser transparencies tend to be smoother and work better than inkjet ones.

<sup>4</sup> Warning: If you can't get this part of the experiment to work, you are in the majority. Students (and TAs) have found this is far more difficult than filtering out the high frequencies. Possibly this is because filtering out high frequencies has the effect of blurring the image, and just about anything else you inadvertently do to the beam causes this to happen.



mesh. (But you can experiment with both.) The holes should be approximately 2-10mm in size. (Note: You can also use the computer to generate a photo-negative of your source image. In Adobe Illustrator, create a large black square and click on the MAKE\_MASK menu item.)

**Think!** What is the shape of the spatial filter you should use?

**Suggestion:** You will get better results if you use a spatial filter that is, very opaque, and very symmetrically shaped. Spatial filters printed on the computer, or made by hand with blobs of ink or whiteout don't seem to work very well. (The ink seemingly blocks out only about 90% of the beam.) Also, it's easier to see results with larger spatial filters (6mm diameter) so perhaps you should start with them first. You should be able to get the best results using a filter as small as the hole you used to filter out the high frequencies in part 4.

There is a spatial filter made by one of the former 128a students made by depositing a thin layer of metal on a glass slide using the vacuum lab equipment. You may want to try out making your own slides as well.

5. Leaving the equipment the way it was in part 4, try the following:

(If you were unable to get part 4 to work read the comment below)

As you did when positioning the screen, place a rough bumpy translucent surface between the subject image and the “converging lens”. Place it up next to the subject image. As mentioned earlier this will destroy the coherence of the incoming beam.<sup>5</sup>

**Think!** What do you think will happen to the picture on the screen? Would a light-bulb make a good light source for spatial filtering?

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<sup>5</sup> Note: This is not required for lab, but you can use this method to test the quality of the inkjet transparency film we use in the lab. This can be done in a qualitative way by measuring how many layers of transparency film it takes to get the same effect as using the bumpy plastic from the credit card holder.

Note: You should get a focused image of some kind. If your image is too dim to see, then your spatial filter blocks out too much of the light.

*{Note: if you were unable to get part 4 of the experiment to work, you can still try this exercise. In that case try using the starburst your picture, and use a flat vertical toothpick 6 (or something with an equivalent shape) as your spatial filter. Unfortunately, this will not have a simple effect on the image, however it should preferentially filter out more of the frequencies along the vertical direction, leaving the horizontal lines more blurry. At least you should be able to make out a fuzzy hourglass shape at the center. This is pretty easy to see. Now try placing the rough translucent surface behind the starburst aperture and observe what happens to the pattern.}*

6. Use a newspaper photograph as the subject. Do some image processing based on your experience gained so far.

Report your findings in a short write-up. Make sure to discuss the reasoning of your conclusions based on the physics involved. Can you think of any improvements to the experiment?

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6 Again, to get this exercise to work, you need to have a spatial filter that blocks only a relatively small fraction of the light hitting the Fourier-transform plane. Hence, the toothpick.

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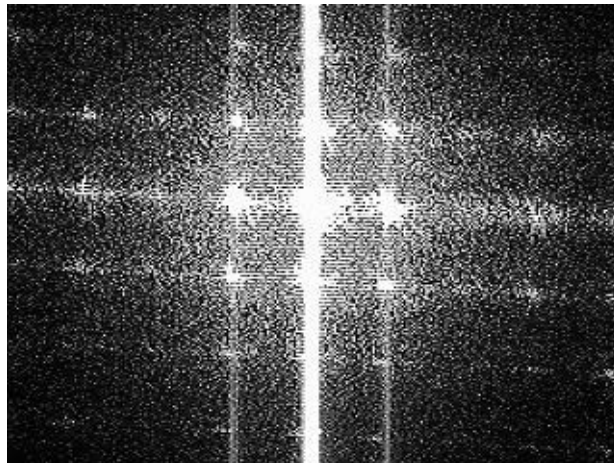
## Appendix A: Acquiring images from the digital camera

Launch “ViCAM TV” from the apple menu.

This will bring up a window named “Monitor” which should display the current view from the digital camera. If this does not happen, or you are asking “what camera”, then find one of the TAs. (If the contents of the window are all white or all dark, select the MONITOR→VIDEO\_SETTINGS menu option and adjust the “shutter speed”.) Otherwise, if the camera seems to be working, click on the MONITOR→FULL\_SIZE menu option. This will enlarge the image.

There are two ways of obtaining images with the camera.

1) If you want to take pictures the diffraction pattern, you will probably want to unscrew the lens in the camera and shine the light directly into the camera’s tiny CCD array. This is difficult because the CCD array is several centimeters behind the front of the camera. However, the diffraction pattern is typically too small to image any other way. If the diffraction pattern is too large to fit on the tiny green CCD square, using a Fourier-transform lens with a smaller focal length will shrink the image. The video image will be sharpest when the CCD array is positioned at the focal plane of the lens.



**Figure 7.** Diffraction pattern due to the subject image shown in Figure 8a) taken directly with the CCD

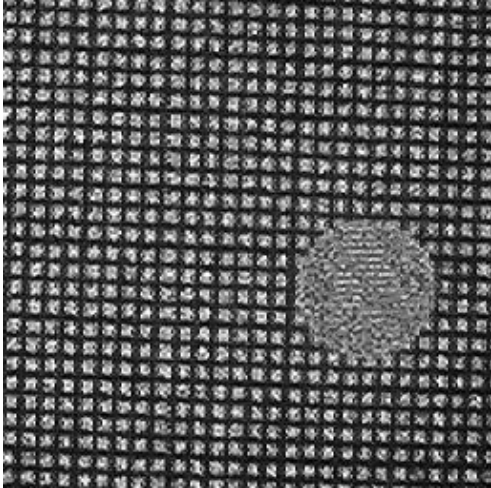
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**Calibration:**

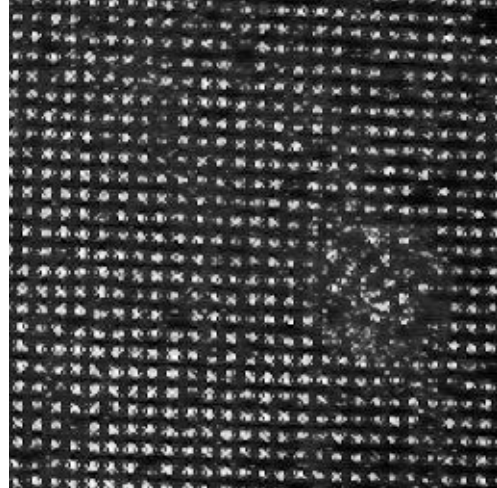
1. By placing the camera (with its CCD exposed) in the path of the columnated beam located between the converging lens and the subject image in Figure 5, you can block the beam and cast a shadow onto the CCD. By shining the light through a window of a known onto the CCD array, you can obtain the ratio of physical length/pixels on the CCD array. (Please include these calibration observations in your lab book.) The “barn door” aperture in lab is useful for creating a window of known size.
2. There is a 3x8” hunk of translucent glass in lab that makes a good projector screen. By positioning the camera behind this translucent glass, you can take pictures of images projected on this glass.

You will probably need to adjust the distance from the camera to the screen and/or the focus. The focus on the camera is adjusted by rotating the lens in front of the CCD. (Unfortunately the focus controls are crude and very sensitive. You may want to temporarily reduce the size of the monitor window as you course-adjust the focus. This enables the video to refresh more quickly.) If the picture remains blurry or hazy, the lens may need to be cleaned.<sup>1</sup>

Calibrate the computer monitor by putting a ruler on the viewing screen (i.e., obtain the length-to-pixel ratio). *Note that the calibration needs to be done whenever you change the distance between the CCD camera and the viewing screen.*



**Figure 8a.** Image of a mesh with a hole cut in the middle. This image was first projected from a screen and focused on the CCD using a lens.



**Figure 8b.** Image of a the same mesh after an attempt to remove low frequencies.

You may need to play with the camera's gain, shutter-speed, or gamma, in order to get a reasonable image. To do this, select the MONITOR→VIDEO\_SETTINGS menu option. You may want to play around with these settings.<sup>1</sup>

Once you have an image that you are happy with, select the EDIT→COPY menu option.

You can edit the image in another program, like NIH-image.

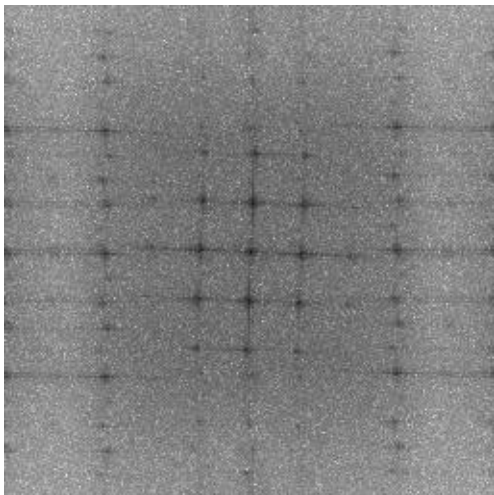
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## Appendix B: Using NIH-image

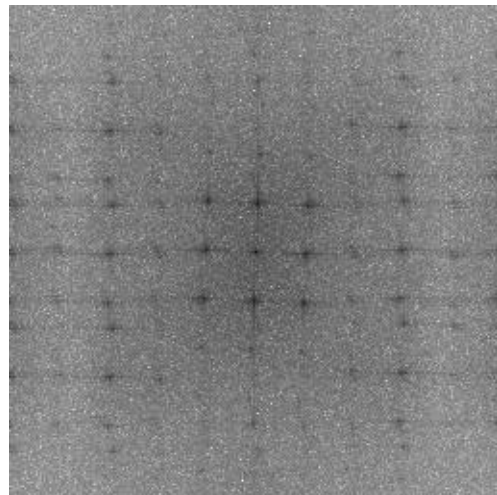
1. Select the FILE→NEW menu option. When the dialog box comes up asking you the size, enter 640 pixels by 480 pixels.
2. Assuming you have acquired an image using the directions above, select the EDIT→PASTE menu option. This fills the window with the image you just acquired.
- 3a. If the image is too dark, or the contrast is too poor, you can play with the brightness “map” tool. This tool resemble a square with a diagonal line running through it and two sliders below which you can manipulate. Once you are happy with the image, you can select the PROCESS→APPLY\_LUT menu option to make the changes permanent. (When comparing too different images, you will want to make sure they have a similar distribution of bright and dark pixels. You can obtain a histogram of brightness levels for your picture by selecting the ANALYZE→SHOW\_HISTOGRAM menu option.)
- 3b. Alternately, you can have the brightness map adjusted for you automatically by selecting the PROCESS→ENHANCE\_CONTRAST menu option.
- 4a. At several instances in the lab manual, you are asked to obtain the Fourier-Transform of an image using the computer.
- 4b. To obtain the Fourier-transform, you need to select a region from the image upon which to perform the Fourier-transform. This region must be a square and the length of each side must be a power of two. As you are dragging the mouse across your image, look to “Info Box” in the lower-left corner of the screen. This will tell you the current size (in pixels) of the box you have selected. (Holding the shift-key down as you make your selection, ensures that your selection is square, and not rectangular.)
- 4c. Select the PROCESS→FFT menu command. This will generate a new window containing the 2-D Fourier transform of your image (Note: phase information may

have been lost from the Fourier Transform.) The location of pixel in this new window is expressed in the “info box” in units of (1/pixels) using polar coordinates. Clicking on a pixel tells the spatial frequency it corresponds to. You can save this image to a file, and adjust it’s threshold and brightness settings as well.

- 4d. If you wish to compare the Fourier transforms of two different images (one with spatial filtering, one without), you may want to select a square at the exact same position within both images. To do this, display both images in NIH-image at the same time. Make the square selection from the first image, click on the window for the second and select the ANALYZE→RESTORE\_SELECTION menu command. The two images in figures 8a. and 8b. were selected from larger images in this way. The Fourier transforms of those two images are below.



**Figure 9a.** FFT of image in figure 8a.



**Figure 9b.** FFT of image in figure 8b.

(Admittedly these spectra are so similar, they don’t present a persuasive argument that spatial filtering actually worked. 10% Extra credit if you can generate an example of low frequency spatial filtering with better FFT images!)

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## Appendix C: Using Adobe Illustrator

Adobe Illustrator is a drawing program. You may use it to make source image transparencies; guinea pigs for the spatial filtering experiment. For the purpose of this lab, you are probably interested in using Adobe Illustrator to make grid shaped objects. That's what these instructions cover.

### A) Making grid objects:

1. First, create a tall, very thin filled black rectangle with the rectangle tool in the toolbox. (To give your rectangle these properties, you will need to select the hollow Border-Color square at the bottom and click on the button containing the red slash. Then you must select the Fill-Color square next to it, and click on the solid black square next to the red slash button. If it this square is not black, you may have to double click on it to change the color.)
2. Using the arrow (selection) tool from the toolbox, select the thin vertical rectangle you just created and select the EDIT→COPY menu option.
3. Paste many copies of this thin line on the using the EDIT→PASTE menu option or pressing CTRL-V. Don't yet worry about moving each line to the right place. Illustrator does this for you.
4. Select one of the tall thin lines and pull it over to the left or to the right of the other lines. This region between the lines you have copied defines the width of the grid you are about to make.
5. Select all of the tall thin lines, go to the Align Palette and click on Vertical Align Center. Then click on Horizontal Distribute Center. This will make the lines evenly spaced
6. Copy the entire group of lines and paste them. Rotate the second set by clicking on the TRANSFORM→ROTATE menu option. Then superimpose the set of horizontal on top of the vertical lines.



You can then paste “blobs” onto your mesh or cut “holes” in your mesh pattern by superimposing filled black and white circles and squares on top of your mesh.

## **B) Creating a “photo-negative”:**

(Assuming you have an image created already.)

1. Draw a filled black rectangle big enough to cover your image using the rectangle tool. Select it and click on the EDIT→CUT menu option.
2. Now select all the objects that make up your image and click on the FILTER→COLORS→INVERT\_COLORS menu option.
3. Click on the EDIT→PASTE\_IN\_BACK menu option and superimpose the black rectangle on your original image.

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## References

<sup>1</sup>G. Fowles, *Introduction to Modern Optics* (Dover, 1975).

<sup>2</sup>E. Hecht, *Optics* (Addison Wesley, 1990).