

# **Finite Difference Method**

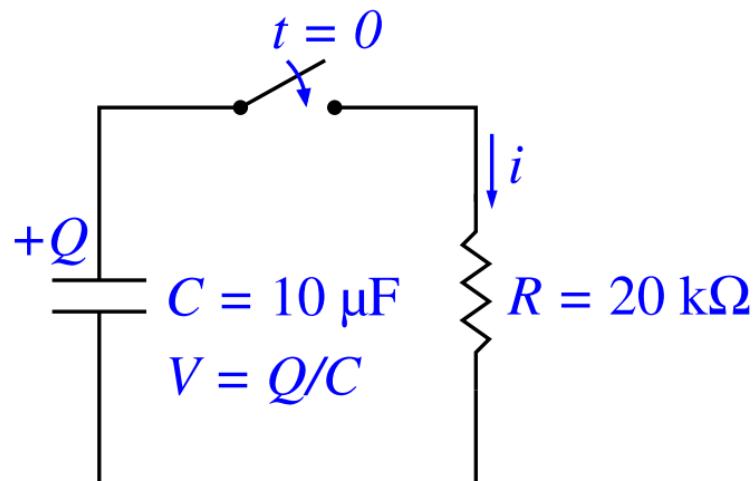
Physics 129L Fall 2021

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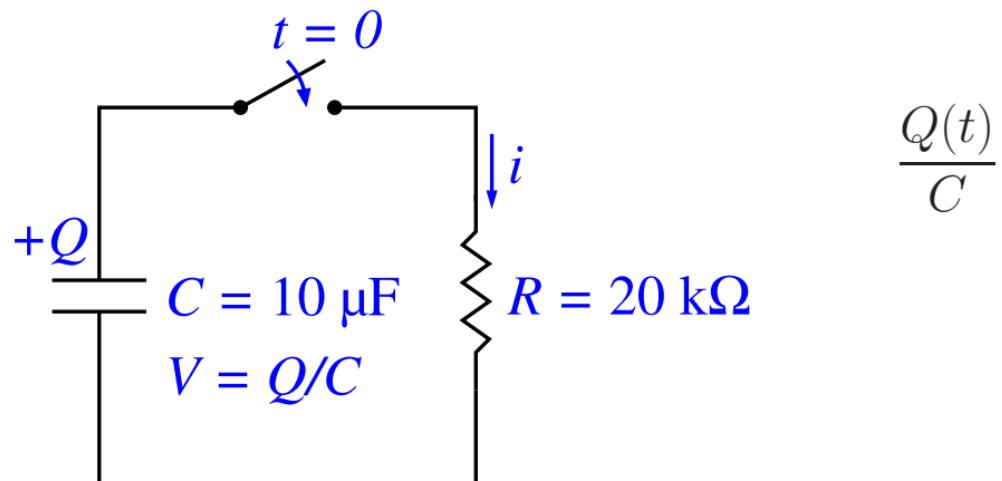
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# Finite difference method: RC circuit

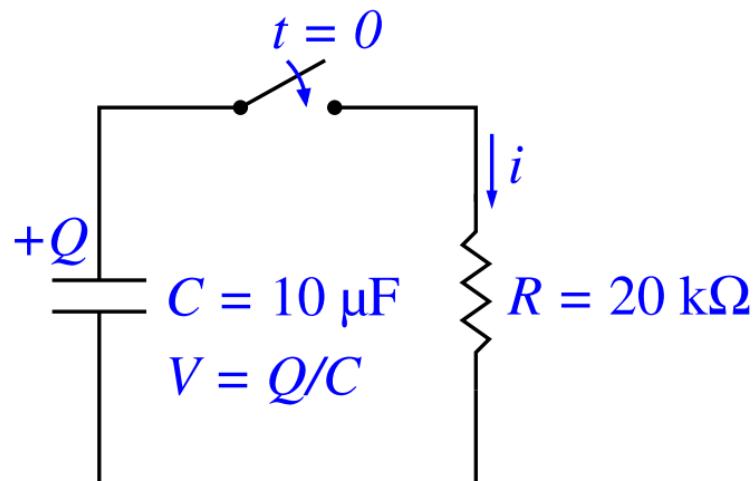


# Finite difference method: RC circuit



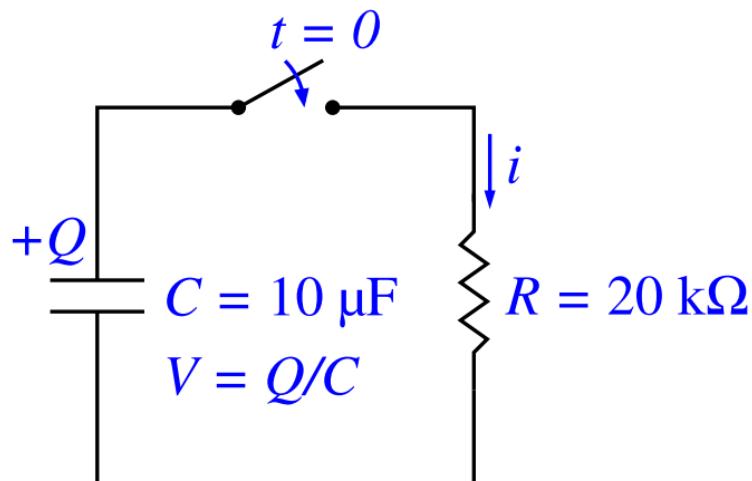
$$\frac{Q(t)}{C}$$

## Finite difference method: RC circuit



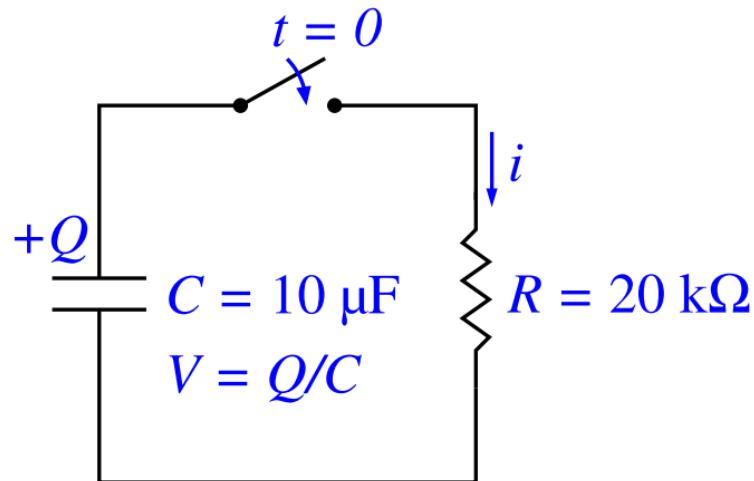
$$\frac{Q(t)}{C} - i(t)R = 0$$

## Finite difference method: RC circuit



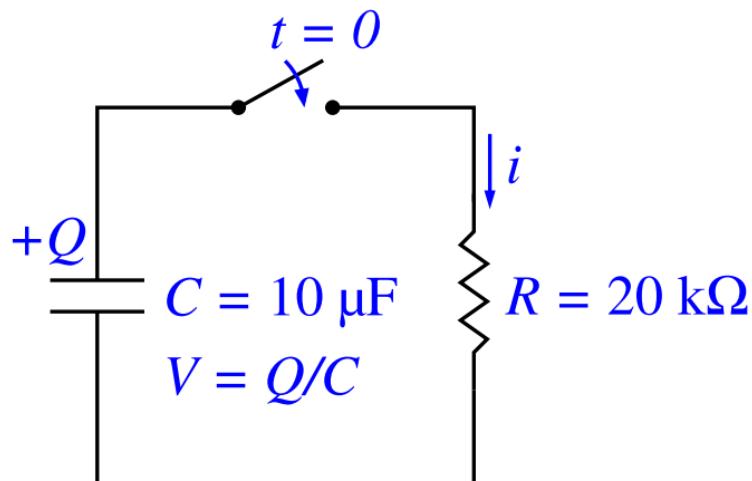
$$\frac{Q(t)}{C} - i(t)R = 0$$
$$i = \frac{dQ}{dt}$$

## Finite difference method: RC circuit



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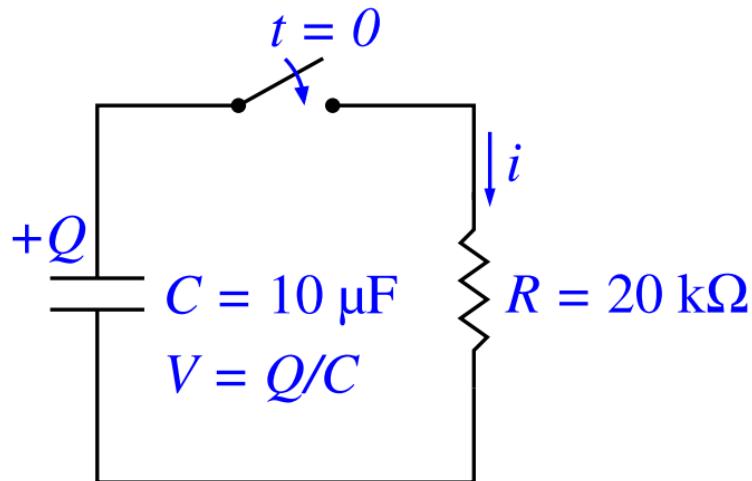


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## Finite difference method: RC circuit



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$$RC = 2 \times 10^4 \Omega \cdot 10^{-5} \text{ F} = 0.2 \text{ s.}$$

## Finite difference method: RC circuit

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$$Q(t + h_t) = \left(1 - \frac{h_t}{RC}\right) Q(t)$$

## Finite difference method: simple harmonic oscillator

$$m \frac{d^2x}{dt^2} + kx = 0$$

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$$x(t+2h_t) = 2x(t+h_t) - (1 + \omega^2 h_t^2) x(t)$$