Midterm Solutions
Phys 131

1) (a) $$\tau = \int \left[\left(\frac{dt}{d\lambda}\right)^2 - \left(\frac{dx}{d\lambda}\right)^2 \right]^{1/2} d\lambda$$

$$= \int \left[(25 - 9)\, e^{2\lambda} \right]^{1/2} d\lambda$$

$$\boxed{\tau = 4e^{\lambda}}$$

Can also use the fact that the worldline is straight : $x = \frac{3}{5}t \Rightarrow d\tau = (1-v^2)^{1/2} dt$

$$= \frac{4}{5} dt \Rightarrow \tau = \frac{4}{5}t$$

(b) $t = \frac{5}{4}\tau$, $x = \frac{3}{4}\tau \Rightarrow$ $\boxed{\begin{aligned} u^t &= \frac{dt}{d\tau} = 5/4 \\ u^x &= \frac{dx}{d\tau} = 3/4 \end{aligned}}$

Check: $\underset{\sim}{u} \cdot \underset{\sim}{u} = -\frac{25}{16} + \frac{9}{16} = -1$ ✓

(c) 4 momentum of particle $\underset{\sim}{p} = m(1,0,0,0)$
Since $\underset{\sim}{u} = (5/4, 3/4, 0, 0)$, the energy is

$$\boxed{E = -\underset{\sim}{p} \cdot \underset{\sim}{u} = 5m/4}$$

2) (a) Clocks on the roof and ground are related by:

$$\Delta\tau_{ground} = \Delta\tau_{roof}\left(1 - \frac{gh}{c^2}\right) \qquad \text{(eq. 6.10)}$$

So

$$\boxed{\Delta\tau_{ground} = T\left(1 - \frac{gh}{c^2}\right)}$$

(b) From above: $\frac{1}{\Delta\tau_{roof}} = \frac{1}{\Delta\tau_{ground}}\left(1 - gh/c^2\right)$

So $\boxed{\omega_{roof} = \omega\left(1 - gh/c^2\right)}$

At infinity, we must replace gh with the change in Newtonian potential GM/R_E. So

$$\boxed{\omega_\infty = \omega\left(1 - \frac{GM}{R_E c^2}\right)}$$

3 (a) Metric is indep of $x \Rightarrow \underset{\sim}{\xi} = (0,1)$ is a Killing vector. $\boxed{\underset{\sim}{\xi}\cdot\underset{\sim}{\xi} = g_{xx} = t^4}$

(b) $dT = 2t\,dt$, so $\boxed{ds^2 = -dT^2 + T^2 dX^2}$

(c) $\text{Area} = \int \sqrt{-g_{tt}}\sqrt{g_{xx}}\,dx\,dt = \int 2t^3\,dt\,dx = \frac{t^4}{2}\Big|_0^1$

So $\boxed{\text{Area} = 1/2}$

This is the same in any coordinates.