Problem 1: Geometry of Curved Space – Metric

In ordinary three dimensional space, we can write the distance between two nearby points \((R, \theta, Z)\) and \((R + dR, \theta + d\theta, Z + dZ)\) as \(ds^2 = dR^2 + R^2 d\theta^2 + dZ^2\) using cylindrical coordinates. The equation \(R^2 + Z^2 = R_c^2\) describes a sphere of radius \(R_c\).

(a) Show that if the two points lie on this sphere, then the distance between them is

\[
{ds^2} = dR^2 (1 + \frac{R^2}{Z^2}) + R^2 d\theta^2 = R_c^2 [\frac{d\sigma^2}{1 - \sigma^2} + \sigma^2 d\theta^2]
\]  

where \(\sigma = R/R_c\).

(b) For what value of the curvature parameter \(\kappa\) does the Robertson-Walker metric resemble the equation above? Do we call this a closed, open, or flat geometry? Notice that a surface of constant \(\phi\) would look like a sphere of radius \(R_c\).

Problem 2 (3 pts): Non-Euclidean Geometry – Angular Diameters
Do Ryden 3.2.

Problem 3 (3 pts): Non-Euclidean Geometry – Measuring Curvature
Do Ryden 3.3