Physics 133: Extragalactic Astronomy and Cosmology

"That wraps it up -- the mass of the universe."

Week 2 – Spring 2018
Previously:

- Empirical foundations of the Big Bang theory. II:
  - Hubble’s Law ==> Expanding Universe
  - CMB Radiation ==> Universe was hot and dense in the past
  - Baryonic matter is found to have a very regular chemical composition, mostly H, He and tiny amounts of heavier matter. The large amount of He implies the universe was much, much hotter and denser in the past
  - The mass-energy density in CMB radiation is
    \[ \rho_{\text{rad}} = a_{\text{rad}} \frac{T^4}{c^2} = 4 \sigma \frac{T^4}{c^3} \]
    \[ \rho_{\text{rad}} = 4.6 \times 10^{-31} \left( \frac{T}{2.725K} \right)^4 \text{ kg/m}^3 = 4.6 \times 10^{-34} \text{ g/cm}^3 \]
This Week: Theoretical Foundations of the Big Bang Theory

• Gravity is the dominant force on the scale of the Universe
  – Newton vs. Einstein on gravity
  – Generalized metric for isotropic and homogenous space

• The Robertson-Walker metric
  – Proper Distance
  – Cosmological Redshift

• Dynamics of the Universe:
  – Friedmann Equation
The four forces

• In our current understanding of physics all interactions are due to 4 forces:
  1. Gravity
  2. Electromagnetic
  3. Strong Interactions
  4. Weak Interactions
Electromagnetic force

• Main properties:
  – Long range
  – Attractive and repulsive
  – Much stronger than gravity but effectively “shielded over long distances”
  – Exchange Boson: photon
  – NB: E&M is unified description of electricity and magnetism

• Examples of systems:
  – Atoms (electrons and nuclei)
  – Electromagnetic waves: light, cell phone…
**Weak force**

- **Main properties:**
  - Short range
  - Responsible for change of flavor of quarks (e.g. neutron decaying into proton)
  - Exchange Boson: $W^+, Z_0$

- **Examples of systems:**
  - Neutrino interactions
  - Beta decays
Strong force

- Main properties:
  - Short range
  - Holds quarks (and nuclei) together
  - VERY STRONG!!! (keeps protons together even though they have the same electric charge)
  - Exchange Boson: gluons

- Examples of systems:
  - Nuclei of atoms
Gravity

• Main properties:
  – Long range
  – Only attractive
  – Very weak force
  – Consider the ratio of the gravitational and electric attraction between a proton and an electron:
    – \( F_G = G \frac{m_p m_e}{R^2} \)
    – \( F_{EM} = k \frac{q^2}{R^2} \)
    – \( \frac{F_{EM}}{F_G} = 10^{39} \)
  – Exchange boson: graviton

• Example of systems dominated by gravity?
  – Universe
  – Black hole
Gravity is the dominant force on large scales (i.e. in cosmology)

- Weak and strong interactions are short range
- There are negative and positive charges so that charges and currents tend to shield each other and neutralize each other over long distances.
- Gravity, is the winner!
Newton’s gravity

- The gravitational force on a body depends on its gravitational mass
  \[ F = -G \frac{M_g m_g}{R^2} \]
- The acceleration of body depends on its inertial mass
  \[ F = m_i a \]
- Experiments that measure the acceleration due to gravity,
  \[ a = G \frac{M_g}{R^2} \left( \frac{m_g}{m_i} \right), \]
  find that the inertial and gravitational mass are the same to within 1 part in \(10^{12}\)
- All objects accelerate in the same way on the Earth’s surface (Galileo’s experiment)
- Space is Euclidian.
Einstein’s gravity.
Equivalence principle

Equivalence of inertial and gravitational mass is exact!
Einstein’s gravity. Propagation of light.

- If the elevator is accelerating the beam of light appears to bend as it propagates through the elevator.
- The **equivalence principle** says we can replace the accelerated box by a box experiencing constant gravitational acceleration.
- This means the path of the photons is curved downward in the presence of a gravitational field.
Einstein’s gravity. Fermat’s principle.

- A fundamental principle of optics is that a light ray travels along the path that minimizes travel time (actually the extrema of the time delay surface)
- In a vacuum, this means light takes the shortest path between two points. In flat Euclidian space these are straight lines.
- What about the accelerating elevator? The path taken by the light is not a straight line.
- *Aha! Space must not be Euclidean.*
Newton vs Einstein:

- **Newton**: Gravitational mass tells gravity how to exert a force. Force tells inertial mass how to accelerate. Inertial and gravitational mass are the same just by coincidence.

- **Einstein**: Mass-energy tells space-time how to curve. Curved space time tells mass-energy how to move.
Einstein’s gravity. The bottom line.

1. Since light does not travel along straight lines, space is not flat!
2. Light rays follow null geodesics in a curved space.
3. Energy and mass are equivalent (E=mc²).
4. Energy and mass curve space time.

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Flat Space or Euclidean Geometry: Examples in 2D Plane

1. In Euclidean geometry. The angles of a triangle add up to $\pi$
2. The distance between two points is given by:
   $$ds^2 = dx^2 + dy^2$$
3. In polar coordinates:
   $$ds^2 = dr^2 + r^2 d\theta^2$$
4. These expressions are called the metric

3.2.1 Right-angled triangle
The theorem of Pythagoras
In a right-angled triangle, the square on the hypotenuse is the sum of the squares on the other two sides.
Pythagoras (about 580 to 500 BC)
Curved Space: Example of Non-Euclidean Geometry in 2D. Sphere.

1. On the surface of a sphere, the angles of a triangle add up to \( \pi + A/R^2 = \alpha + \beta + \gamma \).
   Does not depend on the location on the sphere. The curvature is isotropic and homogeneous.

2. The distance between two points is given by:
   \[ ds^2 = dr^2 + R^2 \sin^2(r/R) d\theta^2 \]
   [Define \( r \), \( \theta \).]

3. The surface is finite, and there is a maximum distance.

4. The surface of a sphere, as all spaces where the angles of a triangle add up to more than \( \pi \), is positively curved.

5. Write down the metric for a positively curved 2D surface.
Non-Euclidean geometry.
Examples in 2D. Hyperboloid.

1. On the surface of a negatively curved surface the sum of the angles is $\pi - \frac{A}{R^2}$
2. The distance between two points is given by: $ds^2 = dr^2 + R^2 \sinh^2\left(\frac{r}{R}\right)d\theta^2$
3. The surface is infinite, and there is no maximum distance
Non-Euclidean geometry.
3D isotropic surfaces.

1. Curvature is a local property. Isotropic and homogenous spaces need to have a constant curvature, which can be zero (flat), negative, or positive. Call the curvature constant $\kappa$.

2. All the properties of the surface are described by the sign $\kappa=-1,0,+1$ and radius of curvature $R$.

3. The metric in spherical coordinates is given by:
$$ds^2=dr^2+S_\kappa(r)^2 \, d\Omega^2$$
$$=dr^2+S_\kappa(r)^2 (d\theta^2 + \sin^2\theta \, d\phi^2 )$$

2D analogs

$$S_\kappa[r] = \begin{cases} 
R\sin(r/R) & (\kappa=+1) \\
r & (\kappa=0) \\
R\sinh(r/R) & (\kappa=-1) 
\end{cases}$$
Space-time in special relativity

- In special relativity, space and time are united in a (flat) 4D space-time.
- The metric of this space is:

\[ ds^2 = -c^2 dt^2 + dr^2 + r^2 d\Omega^2 \]

- Minkowski’s:

\[ ds^2 = -c^2 dt^2 + dr^2 + r^2 d\Omega^2 \]
- A photon travels at the speed of light. This means photons travel along null geodesics, \( ds^2 = 0 \).
- When gravity is added, the permissible space-times are more interesting.
Space-time in general relativity. Homogeneous and isotropic universe

- If the universe is homogenous and isotropic at all time, and distances are allowed to expand (or contract) as a function of time, then we can separate the time part of the metric from the space part of the metric. [Again, on scales $> 100$ Mpc.]

- Howard Robertson and Arthur Walker first realized, independently in fact, that these conditions allow just three possibilities for the curvature of space -- flat everywhere, positive curvature everywhere, or negative curvature everywhere. The Robertson-Walker metric can be written in a concise form.
Space-time in general relativity.

Properties of RW metric

\[ ds^2 = -c^2 dt^2 + a(t)^2 \left( dr^2 + S_\kappa(r)^2 (d\theta^2 + \sin^2\theta d\phi^2) \right) \]

• The RW metric describes the universe over large scales (100 Mpc or more).

• \( \kappa \) describes the geometry
  - Positive curvature (hyper-sphere)
  - Negative curvature (hyper-hyperboloid)
  - Zero curvature (flat Euclidean space)

• The spatial variables (x, theta, phi) or (r, theta, phi) are called **comoving coordinates** of a point in space. They remain constant in time for objects at rest, i.e. those not acted upon by perturbative forces. The distance between objects at rest increases as \( a(t) \) due to the Hubble flow.
Robertson-Walker Metric

Homogeneous and isotropic universe

\[ ds^2 = -c^2 dt^2 + a(t)^2 \left( dr^2 + S_\kappa(r)^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right) \]

- Space component is a hyper-sphere, hyper-hyperboloid or a flat Euclidean space (\( \kappa \) and R) up to a constant scalar factor \( a(t) \) that depends on time.
- The time variable is \textit{cosmological proper time, or cosmic time}, and it is measured by an observer who sees the universe expanding uniformly, i.e. an observer at rest with respect to the Hubble flow.
- The spatial variables \((x, \theta, \phi)\) or \((r, \theta, \phi)\) are called \textit{comoving coordinates} of a point in space. They remain constant in time for an objects at rest, i.e. those not acted upon by perturbative forces. The distance between objects at rest increases as \( a(t) \) due to the Hubble flow.
Space-time in general relativity. The RW metric and the Universe

\[ ds^2 = -c^2 dt^2 + a(t)^2 \left( dr^2 + S_\kappa(r)^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right) \]

- The time variable is **cosmological proper time, or cosmic time**, and it is measured by an observer who sees the universe expanding uniformly, i.e. an observer at rest with respect to the Hubble flow.
- The kinematics of the universe is described by \( a(t) \). As we will see, one can write dynamical equations for \( a(t) \) and solve them, thus reconstructing the past and future of the universe.
- We know the local derivative of \( a(t) \). What is it? **The Hubble constant!**
1. Find separation of events in a spherical 3D space using cylindrical coordinates. Compare form of metric to 2nd form of RW metric.
Distances in the Universe: A number of definitions are useful

• What is the distance between two objects in a RW metric?

• There are several kinds of distance. e.g.:
  – Proper distance
  – Luminosity distance
  – Angular size distance

• In an expanding universe, the distance between objects is increasing with time. If we want to assign a spatial distance between two objects, we must specify the time at which the distance is the correct one.
Distances in the Universe.
Proper distance

- The proper distance is the distance between two sets of comoving coordinates at a given cosmic time.
- This is given by the spatial part of the metric at fixed $a(t)$.
- From the RW metric
  $$d_p = a(t) \, r$$
  where $r$ is the comoving distance to the object.
- When are proper distance and comoving distance equal?
- What is $v_p$? [Blackboard]
Direct limits on the geometry of the Universe

• The curvature of the (homogeneous, isotropic, and expanding) universe is described completely by the curvature constant $\kappa$, the radius of curvature at the present moment $R_0 = R(t_0)$, and the scale factor $a(t)$. Note that we define $a(t_0) = 1$, so $R(t) = R_0 a(t)$

• We could in principle measure the curvature constant by drawing a big triangle and measuring the angles, and the radius of curvature by measuring the area of the triangle. [HW#2 R 3.3, 3.2]

• If the universe is positively curved, the radius cannot be much smaller than the Hubble Length, otherwise photons would have had time to go around the surface in circles and we would see periodical images, i.e. $c \left(1/H_0\right) < 2\pi R$
Expansion factor and redshift.

- What is the distance to the objects in the Hubble Ultra-Deep Field?
- Given the redshift, which is easy to measure, we can infer $a(t_e)$ at the time the light was emitted.
- Once we find a model for $a(t)$, the redshift will give us the distance(s)
Expansion factor and redshift.

Propagation of light.

- Light travels along null geodesics: $c^2 dt^2 = a(t)^2 dr^2$
- Imagine a wave of light. As time goes by, between one crest and another, the universe expands. So that the distance between wave crests appears longer to the observer.
- Light is redshifted!

[Blackboard]

- **Cosmological Redshift**
  
  $$\lambda_0/\lambda_e = a(t_o)/a(t_e) = 1 + z$$
Dynamics of the Universe. 
Friedmann Equation

- The dynamical evolution of the universe is described by $a(t)$.
- Einstein’s field equations link the geometry of the universe ($\kappa, R_0, a(t)$) to the contents of the universe ($\varepsilon(t)$ and $P(t)$). Poisson’s equation is the closest Newtonian analogy.
- The equations were first worked out by Alexander Friedmann in 1922, a full 7 years before Hubble’s Law was discovered. Einstein didn’t accept the expanding universe until Hubble made his discovery.
- Newtonian analog [blackboard]
- What are the shortcomings of this approach?
Dynamics of the Universe.
Friedmann Equation

- **Friedmann Equation** (1922) is the correct form with energy instead of mass and curvature instead of internal energy.

[Blackboard]

- Compare to Newtonian analog:
  1. Rest-mass contributes most of the energy-density of non-relativistic particles
     \[ \rho \rightarrow \frac{\varepsilon}{c^2} \]
  2. Curvature of space related to total energy
     \[ 2 \frac{C}{r_s^2} \rightarrow \kappa \frac{c^2}{R_0^2} \]
  3. H(t) = adot(t) / a(t)

1888 - 1925
Dynamics of the Universe: Critical Density

- The Friedmann equation tells us how the curvature of space at any time is related to the mass-energy density of the contents of the universe at that time.
- The critical density is the amount of stuff that makes the geometry of the universe flat.
- By convention, a subscript “0” indicates the value of a time-varying quantity at the present.

\[ H_0 = H(t_0) = 71 \text{ km/s/Mpc} \]
\[ \rho_{0,\text{c}}(t) = \rho_{\text{c}}(t_0) = 9 \times 10^{-27} \text{ kg/m}^3 = 9 \times 10^{-30} \text{ g/cm}^3 = 1.4 \times 10^{11} \text{ Msun/Mpc}^3 \]
Dynamics of the Universe: Density Parameter

\[ \Omega \equiv \frac{\rho}{\rho_c} = \frac{8\pi G}{3H^2}\rho \]

- It is often convenient to use not the energy density, but the ratio of the energy density to the critical density. **This ratio is called the density parameter**, \(\Omega = \varepsilon(t) / \varepsilon_c(t)\)

- In a negatively curved universe, an empty universe produces minimum curvature, i.e. \(R_0 = c / H_0\); so \(R_0\) must be greater than the Hubble distance.

- The Friedman equation can be written in terms of the density parameter. **Notice that the curvature can’t change sign**! If you know the density parameter at any time, you know the sign of the curvature.
Matter density of the Universe.

1: Radiation

- Blackbody: $\rho_{\text{rad}} = a_{\text{rad}} \frac{T^4}{c^2} = 4 \sigma \frac{T^4}{c^3}$
- Where $c$ is the speed of light, $T$ is the temperature, $\sigma$ is the Stefan-Boltzmann constant), $5.67 \times 10^{-8} \text{ W m}^{-2} \text{K}^{-4}$
- So $\rho_{\text{rad}} = 4.6 \times 10^{-31}$ $(T/2.725 \text{K})^4 \text{ kg/m}^3$
  $= 4.6 \times 10^{-34} \text{ g/cm}^3$
Matter density of the Universe.

1: Radiation in critical units

- It is convenient to write this down in terms of the critical density, the amount of energy/matter needed to “close” the universe.
- Defined as:
  - \( \rho_{\text{crit}} = \frac{3H_0^2}{8\pi G} \)
  - \( = 9.5 \times 10^{-27} \text{ kg/m}^3 \) or \( 9.5 \times 10^{-30} \text{ g/cm}^3 \)
- The density of radiation is \( 4.8 \times 10^{-5} \rho_{\text{crit}} \)
- This can be written as \( \Omega_{\text{rad}} \sim 5 \times 10^{-5} \)
Matter density of the Universe.
2: Neutrinos

- Limits on neutrino mass density come from:
  - Oscillations (lower limit; superkamiokande)
  - Large scale structures (upper limits; CMB+2dF; Sanchez et al. 2006)
  - Cosmic rays striking atmosphere

- In critical units neutrino mass density is between:
  $0.0010 < \Omega_\nu < 0.0025$
Matter density of the Universe.

3: Baryons

- People have counted the amount of mass in visible baryons.
- Baryonic inventory (total=0.045+0.003 from nucleosynthesis and CMB):
  - Stars $\Omega_*=0.0024\pm0.0007$ (comparable mass in neutrinos and stars!)
  - Planets $\Omega_{\text{planet}} \sim 10^{-6}$
  - Warm intergalactic gas 0.040+0.003
- Most of baryons are in intergalactic medium, filaments in the cosmic web,
Matter density of the Universe.

4: Dark matter

- Dark matter is harder to count, because we can only “see” it via its gravitational effects.
- One way to count it is for example to measure the dark matter to baryon ratio in clusters.
- Assume that this number is representative of the Universe because the collapsed volume is large.
- Take the fraction of baryons (from BBN) and multiply.
- This and other methods give $\Omega_{dm}=0.23$.
- The total amount of matter is given by: $\Omega_m=\Omega_{dm}+\Omega_b=0.27$. 
Matter density of the Universe.

5: Dark energy (or Λ)

- As we will see most of the energy in the universe appears to be of a mysterious form called dark energy.
- Dark energy repels instead of attracting, and therefore causes the expansion of the universe to accelerate.
- One form of dark energy is the cosmological constant (Λ), introduced by Einstein a long time ago, and this is a purely geometrical term… We will explain this later.
- According to current measurements $\Omega_{de} \sim 0.72$ or $\Omega_{\Lambda} \sim 0.72$. 
Mass + Curvature Models: Measuring $\Omega_0$ and $H_0$ Determines Curvature

$$H_0^2 (1 - \Omega_0) = -\kappa \, c^2 / R_0^2$$

Measuring Curvature Determines $\Omega_0$ and $H_0$
End Week 2

- You should have read through the end of chapter 4.
- Homework due Friday by 1:30pm.
- Start reading chapter 5.
Summary for Week 2

• The dominant force on the scale of the Universe is gravity

• Gravity is accurately described by the theory of general relativity
  – Mass-energy tells spacetime how to curve and spacetime tells mass-energy how to move
  – Distances are measured along geodesics.

• There are only three possible geometries for the universe. Their metric is the Robertson-Walker metric.
Summary for Week 2

- Proper distance is the distance we measure along a geodesic with a ruler at some particular cosmic cosmic time.
- Redshift is a measure of distance.
- Hubble’s law is not the result of “motion” through space (because with respect to the comoving coordinates objects are not moving) but of the expansion of space time.
- Cosmological redshift results from the stretching of photon wavelength by the cosmic expansion.
- The dynamics of a model universe are described by solving Einstein’s field equations. When we use the Robertson-Walker metric, they simplify to the Friedmann equation.
Summary for Week 2

- Friedmann Equation describes the evolution of $a(t)$ depending on content and geometry of the universe.
- Critical density is the mass-energy density required to make the geometry of the universe flat. Note that an empty universe has negative curvature.
- Most of the mass-energy density consists of an unknown type of particle!