This result is exact, since we are effectively taking the limit. The original integral can now be written in terms of t:

$$\int_a^b xe^{x^2}\,dx = \int_{t_1}^{t_2} \sqrt{t}\, e^t \left(\frac{1}{2}\frac{1}{\sqrt{t}}\,dt\right) = \tfrac{1}{2}\int_{t_1}^{t_2} e^t\,dt$$

$$= \tfrac{1}{2}(e^{t_2} - e^{t_1}),$$

where $t_1 = a^2$ and $t_2 = b^2$.

Some References to Calculus Texts

A very popular textbook is G. B. Thomas, Jr., "Calculus and Analytic Geometry," 4th ed., Addison-Wesley Publishing Company, Inc., Reading, Mass.

The following introductory texts in calculus are also widely used:

M. H. Protter and C. B. Morrey, "Calculus with Analytic Geometry," Addison-Wesley Publishing Company, Inc., Reading, Mass.

A. E. Taylor, "Calculus with Analytic Geometry," Prentice-Hall, Inc., Englewood Cliffs, N.J.

R. E. Johnson and E. L. Keokemeister, "Calculus With Analytic Geometry," Allyn and Bacon, Inc., Boston.

A highly regarded advanced calculus text is R. Courant, "Differential and Integral Calculus," Interscience Publishing, Inc., New York.

If you need to review calculus, you may find the following helpful: Daniel Kleppner and Norman Ramsey, "Quick Calculus," John Wiley & Sons, Inc., New York.

Problems

1.1 Given two vectors, $\mathbf{A} = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 7\hat{\mathbf{k}})$ and $\mathbf{B} = (5\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$, find: (*a*) $\mathbf{A} + \mathbf{B}$; (*b*) $\mathbf{A} - \mathbf{B}$; (*c*) $\mathbf{A} \cdot \mathbf{B}$; (*d*) $\mathbf{A} \times \mathbf{B}$.

Ans. (*a*) $7\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$; (*c*) 21

1.2 Find the cosine of the angle between

$\mathbf{A} = (3\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$ and $\mathbf{B} = (-2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$.

Ans. -0.805

1.3 The direction cosines of a vector are the cosines of the angles it makes with the coordinate axes. The cosine of the angles between the vector and the x, y, and z axes are usually called, in turn α, β, and γ. Prove that $\alpha^2 + \beta^2 + \gamma^2 = 1$, using either geometry or vector algebra.

1.4 Show that if $|\mathbf{A} - \mathbf{B}| = |\mathbf{A} + \mathbf{B}|$, then $\mathbf{A}$ is perpendicular to $\mathbf{B}$.

1.5 Prove that the diagonals of an equilateral parallelogram are perpendicular.

1.6 Prove the law of sines using the cross product. It should only take a couple of lines. (*Hint*: Consider the area of a triangle formed by $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, where $\mathbf{A} + \mathbf{B} + \mathbf{C} = \mathbf{0}$.)

1.7 Let $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ be unit vectors in the xy plane making angles θ and ϕ with the x axis, respectively. Show that $\hat{\mathbf{a}} = \cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}}$, $\hat{\mathbf{b}} = \cos\phi\hat{\mathbf{i}} + \sin\phi\hat{\mathbf{j}}$, and using vector algebra prove that

$$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi.$$

1.8 Find a unit vector perpendicular to

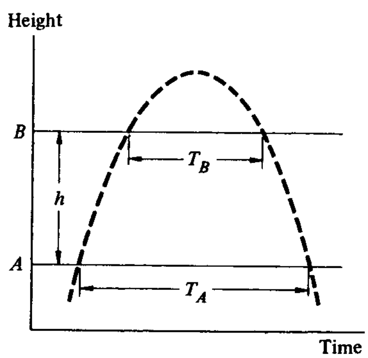
$$\mathbf{A} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) \qquad \text{and} \qquad \mathbf{B} = (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}).$$

Ans. $\hat{\mathbf{n}} = \pm(2\hat{\mathbf{i}} - 5\hat{\mathbf{j}} - 3\hat{\mathbf{k}})/\sqrt{38}$

1.9 Show that the volume of a parallelepiped with edges **A**, **B**, and **C** is given by $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})$.

1.10 Consider two points located at $\mathbf{r}_1$ and $\mathbf{r}_2$, separated by distance $r = |\mathbf{r}_1 - \mathbf{r}_2|$. Find a vector **A** from the origin to a point on the line between $\mathbf{r}_1$ and $\mathbf{r}_2$ at distance xr from the point at $\mathbf{r}_1$, where x is some number.

1.11 Let **A** be an arbitrary vector and let $\hat{\mathbf{n}}$ be a unit vector in some fixed direction. Show that $\mathbf{A} = (\mathbf{A}\cdot\hat{\mathbf{n}})\hat{\mathbf{n}} + (\hat{\mathbf{n}}\times\mathbf{A})\times\hat{\mathbf{n}}$.

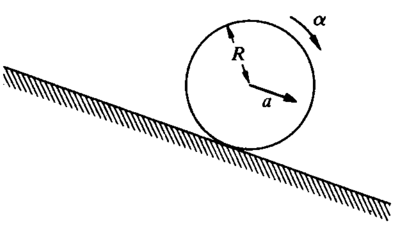


1.12 The acceleration of gravity can be measured by projecting a body upward and measuring the time that it takes to pass two given points in both directions.

Show that if the time the body takes to pass a horizontal line A in both directions is T_A, and the time to go by a second line B in both directions is T_B, then, assuming that the acceleration is constant, its magnitude is

$$g = \frac{8h}{T_A{}^2 - T_B{}^2},$$

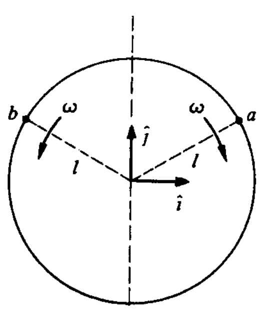
where h is the height of line B above line A.



1.13 At $t = 0$, an elevator departs from the ground with uniform speed. At time T_1 a boy drops a marble through the floor. The marble falls with uniform acceleration $g = 9.8$ m/s², and hits the ground T_2 seconds later. Find the height of the elevator at time T_1.

Ans. clue. If $T_1 = T_2 = 4$ s, $h = 39.2$ m

1.14 A drum of radius R rolls down a slope without slipping. Its axis has acceleration a parallel to the slope. What is the drum's angular acceleration α?



1.15 By *relative velocity* we mean velocity with respect to a specified coordinate system. (The term velocity, alone, is understood to be relative to the observer's coordinate system.)

a. A point is observed to have velocity $\mathbf{v}_A$ relative to coordinate system A. What is its velocity relative to coordinate system B, which is displaced from system A by distance **R**? (**R** can change in time.)

Ans. $\mathbf{v}_B = \mathbf{v}_A - d\mathbf{R}/dt$

b. Particles a and b move in opposite directions around a circle with angular speed ω, as shown. At $t = 0$ they are both at the point $\mathbf{r} = l\hat{\mathbf{j}}$, where l is the radius of the circle.

Find the velocity of a relative to b.