Problems for HW 4

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1 HW4 Problem 1

A semi-infinite cylinder of radius $a$ has grounded walls. The base, at $z = 0$, is grounded as well, except for a very thin annular strip of radius $b < a$ and width $w < a$, at potential $V_0$. Find an expression for potential $V$ within the cylinder. You may assume $z >> w$.

It is desired to extend the potential as far into the cylinder as possible, while keeping $a$, $w$, and $V_0$ fixed. For what value of $b$ is the potential at a particular point on the $z$-axis, at $z_1 >> a$, as great as possible?

If a second ring is added, also with width $w$ but at potential $-V_0$, where should it be placed so as to maximize the potential at $z_1 >> a$? (Assume that $w < a/2$).

2 HW4 Problem 2

An ideal dipole resides outside a grounded, conducting sphere of radius $a$. Assume that the dipole lies on the $z$-axis at a distance $d$ from the sphere. The dipole moment is $\vec{p} = p_0(\cos \theta \hat{z} + \sin \theta \hat{x})$. Find the potential throughout space.

3 HW4 Problem 3

a) Prove that $Y_{\ell \ell}(\theta, \phi) \propto \sin(\theta)^\ell e^{i\ell \phi}$, for any nonzero integer $\ell$. Find the constant of proportionality.

b) Consider a conducting spherical surface, divided into $2N$ segments by $N$ planes. The planes all intersect on the $z$-axis and are equally spaced in $\phi$. The segments are kept at alternating potential: $+V_0, -V_0, +V_0, -V_0,\ldots$. Find the potential near the center of the sphere, to lowest order in $r$. 

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