1) (20 points) A one dimensional particle is governed by the hamiltonian

\[ H = \frac{1}{2m} p^2 + \frac{1}{2} m\omega^2 x^2 + \lambda x^4. \]

a) Compute the \( O(\lambda) \) correction to the ground state energy.
b) Compute the \( O(\lambda^2) \) correction to the ground state energy.
c) Show that perturbation theory is valid only if \( \lambda \ll \frac{m^2 \omega^3}{\hbar} \).

2) (15 points) Suppose one adds an electron to a Hydrogen atom. The original electron is in its ground state, and the second electron is placed in a 2p state.

a) What are the possible two particle states of the electrons? Include spin and statistics, and make sure your states are correctly normalized.
b) Suppose the atom is placed in a uniform magnetic field \( \vec{B} = B_0 \hat{z} \). What are the possible energy eigenvalues? Work to first order in \( B_0 \), and ignore spin-orbit coupling and electron-electron repulsion.

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3) (15 points) A three dimensional harmonic oscillator is in state $|n_x, n_y, n_z\rangle$ for $t < 0$. For $t \geq 0$, it is subjected to a time dependent, but spatially uniform force (not potential)

$$\vec{F}(t) = F_0 \hat{x} e^{-t/\tau}$$

where $\tau$ is a decay constant.

a) Using first order time dependent perturbation theory, compute the probability of being in any other state at time $t$.

b) Show that the transition rate goes to zero in the limit $t \to \infty$. Why is this physically reasonable?

4) (15 points) A square container of size $L$ contains liquid helium. The potential acting on an electron near the surface of liquid helium is approximately

$$V(x) = -\frac{Ze^2}{x}, \quad x > 0$$

and $V(x) = \infty$ for $x < 0$. The coordinate $x$ represents height above the surface, and $Z$ is a small constant reflecting the attractive force coming from an effective image charge.

a) Using your knowledge of the hydrogen atom, compute the energy eigenvalues for the electron. (Be sure to include the $y$ and $z$ directions in your analysis.)

b) What is the (normalized) ground state wavefunction?