ASSIGNMENT #1
Due by Wednesday, January 13 at 5pm

1) Consider a three dimensional isotropic harmonic oscillator with frequency $\omega$.
   a) Determine the energy eigenstates, eigenvalues, and degeneracies by using Cartesian coordinates and reducing the problem to one dimensional oscillators.
   b) Determine the energy eigenstates by first writing the wavefunction in terms of angular momentum eigenstates: $\psi(r, \theta, \phi) = R(r)Y_l^m(\theta, \phi)$ and then solving the radial equation for $R(r)$. (Hint: Follow the procedure used in class for the hydrogen atom, i.e., first find the asymptotic behavior and then determine the recursion relations for the remaining polynomial.)
   c) Show that the eigenvalues and degeneracies obtained in part (b) agree with (a).

2) For the ground state of hydrogen
   a) Compute the uncertainty in radial position $(\Delta r)^2$.
   b) Find the wavefunction in momentum space $\psi(\vec{p})$.

3) The following basic properties of angular momentum will be useful in problem (4).
   a) Show that the vector operators $\vec{V} = \vec{X}, \vec{P}$ satisfy
   \[
   [L_j, V_k] = i\hbar \sum_l \epsilon_{jkl} V^l
   \] (1)
   Note that this is also true for $\vec{V} = \vec{L}$.
   b) Show that the scalar product of any two vector operators satisfying (1) commutes with $L_j$.

4) The Runge-Lenz vector is $\vec{M} = \frac{1}{2\mu} (\vec{P} \times \vec{L} - \vec{L} \times \vec{P}) - \frac{Ze^2}{r} \vec{X}$. We will show in class that one can determine the energy eigenvalues of hydrogen purely algebraically using the following properties of $\vec{M}$:
   a) Show that $\vec{M}$ is Hermitian: $\vec{M}^\dagger = \vec{M}$.
   b) Show that $\vec{M}$ is conserved, $[\vec{M}, H] = 0$, where $H = \frac{\vec{P}^2}{2\mu} - \frac{Ze^2}{r}$ is the Hamiltonian.
   c) Show that $[L_j, M_k] = i\hbar \sum_l \epsilon_{jkl} M_l$.
   d) Show that $[M_j, M_k] = -\frac{2i\hbar}{\mu} \sum_l \epsilon_{jkl} L_l$