1) Consider a (free) electron in a constant magnetic field \( \vec{B} = B \hat{z} \) and a perpendicular electric field \( \vec{E} = E \hat{y} \). Find the energy eigenvalues and eigenfunctions (in terms of harmonic-oscillator eigenfunctions). Hint: Use Landau gauge \( \vec{A} = -By \hat{x} \).

2) A deuteron consists of one proton (with spin operator \( \vec{S}_p \)) and one neutron (with spin operator \( \vec{S}_n \)) bound together by nuclear forces. The ground state of the deuteron has orbital angular momentum \( \ell = 0 \) and spin angular momentum \( s_d = 1 \), where \( \vec{S}_d^2 = \hbar^2 s_d(s_d + 1) \), and \( \vec{S}_d = \vec{S}_p + \vec{S}_n \).

   a) Express the three deuteron spin states \( |s_d = 1, m_d \rangle \) as linear combinations of the states \( |m_p, m_n \rangle \), where \( m_d = -1, 0, 1 \) is the eigenvalue of \( (S_d)_z/\hbar \) and \( m_{p,n} = \pm \frac{1}{2} \) are the eigenvalues of \( (S_{p,n})_z/\hbar \).

   Consider the nucleon exchange operator \( P \): \( P \vec{S}_p P = \vec{S}_n \), \( P \vec{S}_n P = \vec{S}_p \), \( P^2 = I \).

   b) Show that \( P|1, m_d \rangle = |1, m_d \rangle \) for \( m_d = -1, 0, 1 \). Use these results to show that \( \langle 1, m'_d |(\vec{S}_p - \vec{S}_n)|1, m_d \rangle = 0 \) for all values of \( m_d \) and \( m'_d \).

Now consider the deuterium atom, which has one electron bound by the usual Coulomb potential to the deuteron nucleus. The hyperfine structure of the deuterium atom is governed by the hamiltonian

\[
H_{\text{HF},d} = - \frac{c_0}{a_0^3} \vec{\mu}_d \cdot \vec{\mu}_e ,
\]

where \( c_0 \) is a numerical constant, \( a_0 \) is the Bohr radius, \( \vec{\mu}_d = \mu_N (g_p \vec{S}_p + g_n \vec{S}_n) / \hbar \) is the magnetic moment of the deuteron, and \( \vec{\mu}_e = -2 \mu_B \vec{S}_e / \hbar \) is the magnetic moment of the electron. Here \( \mu_N = e\hbar c/2M_p c^2 \) is the nuclear magneton (\( M_p \) is the proton mass) and \( \mu_B = e\hbar c/2m_e c^2 \) is the Bohr magneton (\( m_e \) is the electron mass).

continued on page 2
c) Find the energy eigenvalues of $H_{HF,d}$ in units of $c_0 \mu_N \mu_B / a_0^3$, using $g_p = 5.56$ and $g_n = -3.82$. Hint: write $\vec{\mu}_d$ in terms of $\vec{S}_p \pm \vec{S}_n$.

In hydrogen, the hyperfine hamiltonian is

$$H_{HF,p} = - \frac{c_0}{a_0^3} \vec{\mu}_p \cdot \vec{\mu}_e,$$

where $\vec{\mu}_p = g_p \mu_N \vec{S}_p / \hbar$ is the magnetic moment of the proton, and the constant $c_0$ has the same value as it does in $H_{HF,d}$. In hydrogen, the transitions between the different spin levels produce radiation with a wavelength of 21.4 cm.

d) What wavelength (or wavelengths) of radiation are emitted by deuterium due to hyperfine structure?

3) Sakurai, p. 350, problem 18