1) Consider a one-dimensional harmonic oscillator with a spring constant that slowly weakens,

\[ H = \frac{1}{2m}p^2 + \frac{1}{2}m\omega^2 e^{-\gamma t}x^2. \]

The oscillator is in its ground state at time \( t = 0 \).

a) Use first-order time-dependent perturbation theory to calculate the probability that the oscillator will be found in any excited state at time \( t \). Assume \( \gamma \ll \omega \), and simplify your result as much as possible.

b) For what range of \( t \) is your calculation valid?

2) Consider the time-dependent hamiltonian \( H(t) = H_0 + e^{t/\tau}V \). At \( t = -\infty \), the initial state is \( |\psi(-\infty)\rangle = |n\rangle \), where \( H_0|n\rangle = E_n|n\rangle \).

a) Use time-dependent perturbation theory to first order to compute the state \( |\psi(0)\rangle \) at the “final” time of \( t = 0 \).

b) Take the limit \( \tau \to \infty \), and compare \( |\psi(0)\rangle \) with the result of first-order time-independent perturbation theory for the eigenstate \( |N\rangle \) of the hamiltonian \( H = H_0 + V \),

\[ |N\rangle = |n\rangle + \sum_{m \neq n} |m\rangle \frac{\langle m|V|n\rangle}{E_n - E_m}. \]

Your result is an illustration of the adiabatic theorem, which states that, for a time-dependent hamiltonian \( H(t) \) that is slowly varying, an initial eigenstate \( |n,t_1\rangle \) of \( H(t_1) \) at time \( t = t_1 \) will evolve into an eigenstate \( |n,t_2\rangle \) of \( H(t_2) \) at time \( t = t_2 \). (Up to a phase, which can be interesting: see Supplement I of Sakurai.)

3) Sakurai, p. 351, problem 22.